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ORDER AND CHAOS

by

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The mechanistic world view

When Isaac Newton derived Keplers laws of planetary motion and the laws of terrestrial mechanics from his system of axioms he achieved a historical unification of the realms of the world. The physics of the aetheral heavens and the physics of the terrestrial elements earth, water, air and fire, which had been strictly seperated since the times of Aristotle, were subsumed under one system of mechanics by which God seemed to govern the universe.

A few problems remained unsolved, however. In order to prevent the planets to fall into the sun it was neccesary for them to have initial angular velocities. Where did these initial velocities come from? For Newton the answser was obvious as he wrote in his fourth letter to Bishop Bentley:

"In my former (letter) I represented that the diurnal rotations of the Planets could not be derived from gravity, but required a divine arm to impres them. And tho Gravity might give the Planets a motion of descent towards the sun, either directly or with some little obliquity, yet the transverse motions by which the revolve in their several orbs, required the divine arm to impres them according to the tangents of their orbs" (1).

God as a cosmic guest worker

While the laws might be "laws of nature", the initial conditions had to come from somewhere else, from a "first mover" who started it all. But even beyond this first initial push god was needed continuously to keep the system going, as Newtons pupil Clarke discussed in his famous exchange of letters with Leibniz. In these letters he argued that the solar system would become disorganized periodically by the mutual attractions of the planets and that god would have to intervene personally about once in 10000 years to rearrange the system and to keep everything in shape. It was mainly the "great anomaly of Saturn and Jupiter" that led to this conclusion - the fact that these planets seemed to deviate more and more from their initial orbits designed for them in the creation of te universe.

It was only 100 years later that Laplace was able to provide a satisfactory answer to this great challenge. He argued that the mutual perturbation of Jupiter and Saturn was a resonance phenomenon that would lead the planets away from and back to their original orbits quite automatically every

1000 years. God had lost his role of a "cosmic guest worker" and an entirely mechanical conception of the world and the heavens began to predominate.

The most famous presentation of this mechanical world view is "Laplace's demon", who, given the initial positions and velocities of all molecules, could calculate the past history and future fate of the universe in all detail.

The demon becomes entangled in paper work

It is easy to show how absurd this conception of a omniscient demon - who would achieve the perfect reduction of all sciences to mechanics without any further effort - really is. In order to calculate the future of the universe the demon would have to know the initial conditions of all molecules exactly - i.e. to infinitely many decimal places. To write these initial conditions down or to store them into a computer would exceed the capacity of the universe for storing information.

On the other hand it is easy to show that limited accuracy is insufficient to achieve the desired capacity for prediction or retrodiction. One can show e.g. that a set of billiard balls arranged on a table at an average distance of about 1m and hit by an additional ball becomes indeterministic due to quantum effects after only 8 collisions - i.e. even if the balls are arranged and fixed in whatever manner the player choses he cannot be certain whether he will hit or miss the ninth ball.

Laplaces Demon is a lousy historian

The example of the billiard balls shows that even macroscopic systems become indeterministic due to quantum effects. The situation is even worse if we consider microscopic systems such as molecules in a gas. In this case quantum indeterminacies prevent one from predicting whether a given molecule will hit or miss the very first target molecule at the mean free path length typical for gases under standard conditions. One might argue that the individual fate of a molecule is of no particular interest and averages are all that will be needed. That this is not quite true can be shown by considering the famous library of Alexandria which was burned destroying many of the most famous Greek manuscripts. Couldn't one reconstuct this library from the smoke that should still be around? In this case it is obviously insufficient to find the average position of the letters - more details will be needed.

Physics and technology limit your vision

Frogs have a very selective vision of the world. They see moving objects only, since these are the "socially relevant" phenomena from their point of view. In the same manner human vision and the human perception of the world works very selectivly. Not motion is the criterion here, but the regular, the repeatable event. Human vision tends to see patterns and regularities even if there are none. As with the frog, questions of survival and orientation in this world lie at the origin of this selective vision.

Physics and technology enhance this evolutionary pattern. The codification of the regular, the repeatable is the task of physics. The use of reliable structures with foreseeable reactions lies at the heart of technolgy.

Physics education is in part an education for adaption to the system of knowledge created during the course of several centuries. It is a school of seeing - a school of selective vision. The paradigmatic examples provided by textbooks have to be studied by every student. The world as a planetary system, the world as a harmonic oscillator - this is the vision of the world for which he is trained. It is a regular, predictable, causal (and boring) world, dominated by Laplace's demon.

Roulette and die - the exceptions to the rule

A few exceptions disturb and perturb this universal world view of Newtonian mechanics. Roulette and casting the die are examples for these exceptions. In these cases unpredictability is not only accepted, but demanded by society (and people capable of predictions in these cases are usually seperated from the rest of mankind by police).

These exceptions do not appear to be very serious at first. One might hope that better control of the variables involved might help. But there are other systems that come to mind where such a better control of the dynamical patterns have been tried for decades - without too much success. The weather forecast is but one example (economical forecasts might be another). These are examples of systems in which slight deviations from the initial conditions, slight changes in the starting positions and velocities of the atoms, molecules, clouds ... involved lead to drastic consequences.

Causality is at stake - and some systems are no systems

"Similar causes have similar consequences" - this is the principle of causality as formulated by David Hume. This principle does not hold for the systems mentioned above. Similar causes, a similar start of the ball in a game of Roulette, a similar cast of the die - they can have drastically different consequences. These systems are indeed so sensitive to small changes in the initial conditions that even a fly jumping around on Sirius or some other star might change the behaviour of the system appreciably.

In this case the very notion of a "physical system" breaks down. These systems are supposed to be isolated parts of the universe that can be studied on their own - without taking into account what happens in the rest of the world. The super-sensitive "chaotic systems" are no physical systems for which such an isolation and separation from the rest of the world would be possible. The unity of the whole universe has to be taken into account when detailed predictions about the behaviour of these systems are needed. The example of the library of Alexandria has shown that such predictions are indeed needed in some, and probably in many cases. It is only our tacit knowledge about the impossibility of such predictions that prevents us from asking such "stupid questions".

The exception becomes the rule and the rule is the exception

Roulette and die seem to be rather exceptional cases at first. The rule seems to be the regular and predictable planetary system or the harmonic oscillator. These are the canonical examples contained in all textbooks and generations of students have learned their view of the world from these books.

Theorems proved by Siegel in 1941 and 1954 have shaken this mechanically stable universe (2). His results have shown that among the class of all possible physical (Hamiltonian) systems the regular and predictable ones are the exceptions (form a class of measure zero) and the unpredictable, chaotic ones are the rule (generic case). "Allmost all" mechanical systems defy the physicists attempt to calculate and predict their behaviour since even the smallest cosmic perturbation is sufficient to throw these systems out of their calculated path.

This result is not quite as bad for physics as it might seem. There are many important systems that do show regular behaviour and even the chaotic ones are regular for large regions of initial coordinates and velocities. Hydrodynamics is an example, where a regular "laminary" part of phase space exists, while the system becomes turbulent and thus chaotic for other ranges of velocities.

Order behind the chaos

Unpredicablility is the rule and regularity is the exception. This is the lesson that the recent history of physics has taught us. Chaos everywhere - a rather negative outlook at least at a first glance.

But behind the chaos there is order again. It is a new type of order - spontaneous order, soft order, very different from the rigid order of Laplaces demon or the "rigor mortis" of the crystalline world of mineralogy. It is the "soft order" of snowflakes - a type of order characterised by a delicate balance of regularity and individuality. Each snowflake has its own characteristic individuality and no two snowflakes are exacly alike. At the same time they are all snowflakes - all of them are characteristic members of a easily recognizable familiy of objects. Only due to this regularity we can assign a common name to this set of natural objects.

How can order be found behind the chaos? Is this not in contradiction with the second law of thermodynamics? Is this not a violation of the universal "heat death" of all matter? How can the regular pattern of a snowflake emerge from the chaos of a cloud? It is here that a new dicipline sets in. It has been called "synergetics" by Hermann Haken, one of its main proponents who has written several excellent expositions of this new science.

Prigogines road to order

There have been several independent paths towards the understanding of spontaneous order emerging from chaos. One of the main ideas is due to Ilya Prigogine and his school (3). His idea is that thermodynamics far from equilibrium is the key to structure.

Ordinary thermodynamics is really thermostatics. It deals with equilibrium states with simple and definite properties that can be related to one another with the help of thermodynamic arguments (making use mainly of the interchangability of second derivatives) or can be calculated from the microscopic structure of matter with the help of statistical mechanics.

"Thermodynamics" can deal with dynamical processes such as the flow of a gas from a container only by relating the static initial and final configurations. Slightly better is the standard "nonequilibrium thermodynamics". It enables one to calculate transport coefficients such as electrical or thermal conductivities, diffusion coefficients etc. In its "classical form" it deals with slight deviations from equil brium, e.g. with small temperature differences where the distributions of the coordinates and momenta of the particles do not deviate appreciably from their equilibrium values.

What happens when the deviations from equilibrium are no longer small? This is the problem underlying the "thermodynamics far from equilibrium" which forms the center of Prigogines work. The canonical example for the surprising effects that can be expected in this case are the "Benard cells". When a liquid is heated from below its thermodynamic equilibrium is disturbed. When the heat current streaming through the liquid is small no very exciting effects are observed. Here we are in the range of the classical nonequilibrium thermodynamics which discribes the heat current through the liquid by a thermal conductivity.

When the temperature difference across the liquid exceeds a critical value a completely new phenomenon sets in. The molecules rise no longer in small irregular patches transporting the heat from the bottom to the top of the liquid but in regular convection cells. This phenomenon can not be described by classical equilibrium or nonequilibrium thermodynamics. It is due to a "phase transition far from equilibrium". Such phase transitions are obsered when (heat, electric et.) currents through a system exceed a certain critical value.

The critical energy current

The general idea behind this and other spontaneous transitions from disorder to order, from irregular individual molecular motions to highly regular collective motions, is that these transitions are possible only when the energy flow through a system exceeds a certain value characteristic for the individual system considered. In this case the transition from disorder to order - from a state of high entropy to a state with low entropy - does not contradict the second law of thermodynamics. The reason for this is that the incoming energy E1 carries only a small amount of entropy S1, while the outgoing energy E2 (which will usuallly be equal to E1) carries very high entropy S2. The total entropy S of system plus energy flow can increase due to the increased entropy contained in the energy, while the entropy of the system can even decrease at the same time due to the spontaneous creation of order within the system.

It is the flow of solar energy through our ecosphere that provides the explanation for the possiblity of evolution i.e. of the creation of order on earth. The earth receives solar energy in the form of highly ordered radiation at a temperature of 6000 K and re-radiates this energy in the form of infrared radiation at a temperature of 300 K. The entropy of the outgoing radiation is more than 20 times higher than the incoming entropy! The difference explaines how "the French Academy of Science could form spontaneously from a sea of tadpoles".

Thermodynamics far from equilibrium is one of the key concepts for the understanding of the emergence of new properties and of spontaneous order. Prigogine has tried to build a completely general theory of thermodynamic processes far from equilibrium. This attempt was, however, only partially successful. The phenomena are too rich, too complicated, and too interesting to fit into a thermodynamic theory based on a few general principles. In order to deal with relevant problems it was soon necessary to use more specific dynamical considerations - i.e. reaction kinetics. This leads to the approach pioneered by Eigen and Schuster.

The Eigen-Schuster road to order

It has often been argued that the origin of life is an extremly improbable event. Arranging the atoms of a single biomolecule correctly by chance would require incredibly many Universes filled completely with matter. Is there some principle unknown to physics and chemistry at work to create the order of the living organism? In a fascinating series of papers Eigen and Schuster have shown that such conclusions are unwarranted (4). Order can spontaneously originate from chaos when non-equilibrium conditions are prevalent, as the ideas of Prigogine have shown. Rather than trying to formulate this into a general thermodynamic theory Eigen and Schuster wrote down the specific equations for the reaction kinetics of autocatalytic processes. This became the basis for a theory of evolution, in which concepts like "survival of the fittest" and "selection" could be formulated quantitatively (although only in rather restricted models).

The dynamical equations of Eigen and Schuster are non-linear equations, since the probability for the collision of molecules - leading to the formation of new molecules - is proportional to the numers A and B of molecules already present:

Probability for formation of A+B = const.* A * B

This lead to rather general studies of such nonlinear equations, i.e. to the theory of dynamical systems which has become one of the most fascinating research topics of the past decade. Here we find also the connection to the approach to order studied mainly by Hermann Haken.

Hakens road to order

For Haken the laser was the prototye of a spontaneous transition to order (5). When the atoms within a laser are excited by a weak flow of energy the radiation is emitted individually by each atom and the usual "natural light" results. It is incoherent, i.e. uncoordinated. When the energy flow through the laser exceeds a critical value the behaviour of the system is changed drastically. Photons emitted by one atom induce transitions in other atoms and a single, intense light wave is emitted. The connection with Prigogines ideas is obvious here.

The formulation of the dynamical equations of this system is rather easy. The number n of photons within the laser changes due to

induced emission of photons   a * N * n

loss at the endface of the laser   - b * n

Here a and b are constants and N is the number of excited atoms in the laser. This number N is given by

Number of excited atoms   N = Nx - c * n

where Nx is the number of atoms excited by the external energy source while the term - c * n takes into account that some of these atoms have returned to the ground state after emitting photons.

The dynamical equation of the laser is thus given by

$$dn/dt = A * n - B * n * n .$$

This is a non-linear differential equation containing two constants $A = a * Nx - b$ and $B = - a * c$. The behaviour of its solutions depends strongly on the sign of the constant A. When Nx, the number of the externally excited atoms, is small, A is negative. In this case any photons that were initially present are quickly radiated away and no laser action takes place.

When the energy flow through the system and thus Nx is sufficiently large to make A positive a completely new behaviour is observed. In this case a new equilibrium value $n = A/B$ is obtained besides $n = 0$. The laser begins to work and the atoms that had been radiating independently start their coherent actions. Order has emerged from disorder.

Hakens swimming pool

Before we study the mathematics of this system any further it is useful to describe a simple analogy to this transition to atomic order. Consider a swimming pool where a number of people swim in a random, uncoordinated manner. Frequent collisions will take place and the "mean free path length" of the swimmers will be rather small. When some of the people begin to swim in a circular pattern by chance they will soon be joined by others since this regular pattern diminishes the number of collisions and eases the motion. Order has emerged from chaos.

The order that has emerged is the soft order of the snowflake. While the circular pattern might proceed to the right on one day it could proceed to the left the next morning. On some other day two "convection cells" could form - one for the girls and one for the boys. That order will emerge is a necessity when the equations of the systems are suitable. Which type of order will emerge can not be predicted from the basic equations. "Chance and necessity" form a perfect union in this example. Individual and regular features of the system are in harmony.

Instability and broken symmetry

Hakens swimming pool is an example of a broken symmetry. The initial arrangement of swimmers prefers no sense of direction - neither to the left nor to the right. When one of the circular patterns emerges the left- right symmetry is broken. When the double pattern is observed it is the up - down symmetry that has been violated. A new feature of the system has thus emerged spontaneously that had not been observed before. This new property of the system is either handedness or a prefered direction.

Which property will emerge in an individual case can not be predicted. Each such property is consistent with the basic dynamical equations but which one will be realized in an individual event can only be determined afterwards. This fact is due to the basic dynamic instability of the system. Even small perturbations will suffice to induce either the rotation to the left or the one to the right. Before the order emerges it is impossible to predict which one it will be.

New properties emerge

Let us emphasize this central point again. It is not only undetermined what the specific numerical value of the newly emerging property will be. Such a numerical value might be +1 when the swimmers rotate to the right and -1 when a rotation to the left sets in. The emergence of new propertiess means much more than this: It is undetermined whether the system will develop the property "handedness" or the property "prefered direction". Not only the numerical value but the very type of property that will emerge is undetermined. It is in this very strong sense that genuinly new properties emerge.

Here we begin to see the connections to the instabilities and to the chaotic properties of systems that we discussed before. "Soft order" and individuality can emerge only when instabilites are present - otherwise everything would be rigidly determined. It is the order behind the chaos which we observe and only systems that are linked to all of the Universe due to their sensible reactions to external perturbations are capable of displaying new and unpredictable properties.

Back to the laser

Let us return now to the example of the laser. In this case it is the linear term in the basic equation

$$dn/dt = A * n - B * n * n$$

that causes the instable behaviour and thus the transition to the new and ordered state. If this A - term were the only one present n would increase exponentially and the system would "explode". Such systems were usually ignored by the pre-computer world of physics, since their exponential and instable character made them appear to be rather unsuitable models of the real world.

It is only the non-linear term that stabilizes the system. This term lets the number of photons level off when the equilibrium value n = A/B has been reached. But non-linear systems were hard to deal with in the pre-computer age of physics. Therefore they were usually ignored and nonlinearities were linearized (or discussed) away. It was only with the availablity of computers that the importance of nonlinear dynamical systems was recognized and their properties were investigated in detail. The computer has turned out to be a genuine research instrument of great heuristic value in this case.

Non-linear dynamical systems are also the priciple uniting the approaches of Prigogine, of Eigen and Schuster, and of

Haken. They have been arrrived at with different methods, different motivations, different backgrounds, and different interpretations. They have turned out to be some of the most important innovations in mathematicals physics or even in "natural philososphy", if we may use this old fashioned term.

Synergetics and the reduction problem

Let us turn now to the reduction problem. Can chemistry be reduced to physics? Has this been achieved already in quantum mechanics? Can biology be reduced to chemistry? Has this already been achieved by molecular biology? Can psychology be reduced to biology? Has this already been achieved by.....??

Problems of central importance to philosophy of science are raised by these questions. What can synergetics contribute to answering these questions?

What we have learned is that instable dynamical systems (systems that are stabilized by nonlinearities only) possess completely new properties. These properties emerge in systems far from equilibrium and lead to new types of order. The decisive point is that not only the specific values of these properties can not be foreseen, but also that the type of property that emerges can not be predicted.

This fact is of relevance for all disciplines in which structures and processes do have a strong historical component, i.e. are not determined almost exclusively by differential equations but mainly by initial conditions.

Examples for such disciplines are geology; the origin of the solar system; and biology. In this respect the question of the reduction of biology to chemistry is analogous to the reduction of e.g. geology to physics (and not to the reduction of chemistry to physics).

Has chemistry been reduced to physics?

The electron distribution in many molecules has already been calculated from first principles on the basis of quantum mechanics. The results are in excellent agreement with experiment and it is only a question of time and money to extend these calculations to larger and larger molecules. Has chemistry been reduced to physics?

1. We can argue that axioms - such as the axioms of quantum mechanics - do not specify which theorems (or numerical results) can be derived. The motivation and the idea for such derivations has to come from outside. Furthermore the axioms provide no algorithm for the derivation of the theorems.

To use a specific example: The axioms of number theory do not contain any motivation for the proof of the great Fermat theorem nor do they provide an algorithm for the automated proof of this or any other theorem. In this sense the axioms are necessarily incomplete and any highly formalistic point of view concerning the nature of scientific proofs provides no answers to these questions. Motivation, skill, experience and luck are elements that play an important role even in the most theoretical science.

In a similar manner the axioms of quantum mechanics contain no instruction to calculate e.g. the structure of the water molecules and no hints how to do it.

2. The calculation of molecular structures is an important task of theoretical chemistry. But this is not all of chemistry. While the theoretical language of chemistry - or at least a major part of it - may actually have been reduced to physics there is still the empirical language. This language contains words like ion exchanger, catalyst, solvents, spectrographs etc.. These expressions belong to the specific idiom of chemistry and have not been reduced to physics. According to Bohrs version of the Kopenhagen interpretation of quantum mechanics there is no chance to reduce these terms to the theoretical terms of quantum mechanics since these classical expression preceede quantum mechanics logically and are presupposed there.

3. Even theoretical and empirical language taken together are not equivalent to the complete structure of a scientific discipline. Another part of a science that must not be underestimated is its canonical set of problems and textbooks. These books do not start with powerful and general axioms and proceede from there to the elementary problems of introductory laboratories. The specific method by which a field is being learned and taught is as much part of a scientific discipline as its empirical and theoretical terms. This part of chemistry has never been reduced to physics. It is probably due to the highly formalistic conception of science prevalent in early philosophy of science that these important parts of the conception and tradition of science have been ignored.

4. There is one more argument for the fact that the reduction of chemistry to physics is only a formal one. It is the old problem of what is meant by "understanding" in a field of science. Let us start with a specific problem again.

It is well known that charged particles spiral around magnetic field lines. An example for this are the electrons in the van Allen belts. What is the influence of the earths gravitational field on these particles? A naive argument might sugggest that they will fall in this field in a downward spiral. The calculation shows that this is incorrect. The particles will spiral horizontally across the field lines and not downward!

This is the result of the calculation. But has one understood the strange behaviour of these electons? Can one transfer this answer to other, similar situations? A very different approach is needed to reach this level of understanding. Only qualitative reasoning about the behaviour of the electrons in velocity space will be useful to achieve this.

This qualitative reasoning is characteristic and specific for each science. It is the tool needed for a wide ranging insight and indispensable for problem solving as educational research has shown.

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