

COMMITTEE I
Unity of Science: Organization and
Change in Complex Systems

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PARTICLE PHYSICS AND THE EARLY UNIVERSE*

by

Harald Fritzsch
Professor of Theoretical Physics
University of Munich
and
Max-Planck Institute of Physics
Munich, WEST GERMANY

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1. Cosmology: The Standard Model

Modern cosmology is based on the idea of the "hot big bang" first discussed by George Gamow. It is based on a few, but important experimental facts:

a) The expanding universe

Distant galaxies are receding from us with a speed proportional to their distance from us. Since the discovery of this phenomenon in 1929 by Hubble and collaborators the experimental data have become rather precise at present. If we relate the velocities of the distant galaxies determined by redshift measurements and the corresponding distances by the linear "Hubble law":

$$v = H_0 \cdot r \quad (1.1)$$

(we use the conventions $c = \hbar = 1$) the Hubble parameter H_0 is determined within a factor of two:

$$H_0 \approx 50 \dots 100 \text{ km s}^{-1} \text{ M}_{\text{pc}}^{-1} \quad (1.2)$$

The galaxy recession is commonly interpreted as a consequence of the expansion of the universe, which according to the measured values of H_0 started about 10 ... 20 billion years ago.

b) Photons in intergalactic space

The observation of an isotropic electromagnetic radiation with a Planck spectrum of 2.8 K supports strongly the idea that the universe has been very hot in the past. In the average there are about 400 photons per cm^3 ,

carrying an energy of 0.25 eV/cm^3 . In the standard "hot big bang" scenario these photons are relics of the big bang. The number density of photons in the universe is much larger than the number density of nucleons. One estimates

$$\frac{n(\gamma)}{n(N)} \sim 10^9 \dots 10^{10} \quad (1.3)$$

c) Primordial helium

It is well-known that the heavier elements are produced by star burning. This explains why the relative abundances of these elements vary greatly throughout the universe. On the contrary helium is very abundant (slightly more than 1/4 of all matter in the universe is helium), and distributed rather uniformly. The only satisfactory explanation of this phenomenon is to suppose that the helium was formed shortly after the big bang, when the matter in the universe was present in form of a hot isotropic and homogeneous plasma of mostly nucleons and electrons. By the time the temperature has dropped to less than 1 billion degrees, all neutrons combine with protons to form deuterons. The latter react quickly among each other such that within about 200 seconds virtually all deuterons disappear and helium nuclei are produced. The relative amount of helium produced depends only on the relative number of neutrons present at the time when the temperature drops below 1 billion degrees such that deuterons can be formed without being broken up shortly afterwards by the radiation. On the other hand the number of neutrons depends solely on the n-p-mass difference and to some extent on the time elapsed since the formation of the neutrons shortly after the big bang (free neutrons are not stable (half-life $\tau_{1/2} = 10.6 \pm 0.2 \text{ min.}$). At the time of deuteron formation one expects that of every 16 nucleons, about 14 are protons and 2 are neutrons.

During the helium production 4 of every 16 nucleons form a helium nucleus. Thus about 25 % of all nuclear matter is transformed into helium. During the past 10 or 20 billion years the burning of hydrogen into helium in the stars led to an increase of the helium content of the universe of another 2 ... 3 %. Thus about 90 % of the helium observed in the universe today is of primordial origin; it is a relic of the hot and dense plasma which filled the universe hundreds of seconds after the big bang.

It is assumed that the dynamics of the gravitational force, of space and time, is governed by Einstein's field equations of General Relativity:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (1.4)$$

($R_{\mu\nu}$: contracted curvature tensor; $R = R_{\mu}^{\mu}$; $g_{\mu\nu}$: metric tensor; Λ : cosmological constant).

The cosmological constant Λ was introduced originally by Einstein in order to obtain a static solution of (1.4). After the discovery of the expansion of the universe the original motivation for introducing the Λ -term was gone, and in cosmological considerations Λ is usually set to zero. As we shall see, a Λ -term might have been present shortly after the big bang and might have played a crucial rôle in the dynamics of the universe ("inflationary universe").

Assuming that the universe on a large scale is isotropic and homogeneous, the line element can be written as:

$$ds^2 = dt^2 - R^2(t) d\sigma^2 \quad (1.5)$$

where R is a scale factor ("radius"), related to the Hubble parameter

$H = \dot{R}/R$, and $d\sigma^2$ is the line element of a three-dimensional space of constant curvature (independent of time). Introducing radial and angular variables, (1.5) can be rewritten as:

$$d\sigma^2 = \frac{dr^2}{1 - k \left(\frac{r^2}{R^2} \right)} + r^2 (d\theta^2 + \sin^2\theta dp^2) \quad (1.6)$$

where the parameter k can assume three different values:

- a) $k = +1$ (positive curvature, finite universe)
- b) $k = 0$ (flat space, infinite universe)
- c) $k = -1$ (negative curvature, infinite universe).

Einstein's field equations lead to the Friedman equations for R :

$$\left(\frac{\dot{R}}{R} \right)^2 = H^2 = \frac{8\pi G}{3} \rho + \frac{1}{3} \Lambda - \frac{k}{R^2} \quad (1.7)$$

$$\frac{d}{dt}(\rho R^3) + p \cdot \frac{d}{dt}(R^3) = 0 \quad (1.8)$$

(ρ : matter density in universe).

In order to determine $R(t)$ from eq. (1.7), one needs a relation between ρ and R , which can only be derived from the specific properties of the matter in the universe at the corresponding time.

At the present epoch one can neglect the pressure terms in the matter-energy-momentum-tensor, in which case one obtains from (1.8):

$$\frac{4\pi}{3} \rho = \frac{M}{R^3} \quad (M: \text{constant}). \quad (1.9)$$

One finds that the sign of k is related to the density in the universe:

$$\begin{aligned}
k &= +1 && \text{for } \rho > \rho_C \\
k &= 0 && \text{for } \rho = \rho_C \\
k &= -1 && \text{for } \rho < \rho_C,
\end{aligned}
\tag{1.10}$$

where the critical density is given by

$$\rho_C = \frac{3 H^2}{8\pi G} \quad (\text{H: Hubble parameter}).$$

In the early universe where ρ is large the spatial curvature term $k R^{-2}$ in (1.7) can be neglected, and (1.7) reduces to:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3} \rho + \frac{1}{3} \Lambda
\tag{1.11}$$

It is important to stress that both the Einstein equations (1.4) and its consequence, the Friedman equations (1.7, 1.10) are valid only in the range where quantum effects can be neglected. This is the case if

$$\rho \ll \dot{M}_p^4, \text{ or } (\dot{R}/R) \ll t_p^{-1},
\tag{1.12}$$

where M_p is the Planck energy:

$$M_p = (\sqrt{G})^{-1} = 1,22 \cdot 10^{19} \text{ GeV}
\tag{1.13}$$

and t_p the Planck time:

$$t_p = 5.4 \cdot 10^{-44} \text{ s.}
\tag{1.14}$$

Thus the Friedman equation (1.11) cannot be integrated backward to the time at which ρ exceeds M_p^4 . At present it is not possible to make statements about the very early universe ($t < t_p$), i.e. about the period at which the

quantum fluctuations of space and time are strong enough to destroy our conventional notions about space-time structure. Throughout this paper I will identify the beginning of the cosmic time with the time at which the energy density drops below M_p^4 .

If we set $\Lambda = 0$ and define the curvature $K = k/R^2$, the Hubble parameter $H = \dot{R}/R$ and the deceleration parameter $q = -\ddot{R}/RH^2$, one finds:

$$K = H^2(2q-1), \quad 4\pi G\rho = 3 H^2 q. \quad (1.15)$$

If space is flat ($k = 0$), the deceleration parameter is fixed: $q = \frac{1}{2}$, and one has:

$$8\pi G\rho = 3 H^2. \quad (1.16)$$

In this case the age of the universe is given by H : $t = \frac{2}{3} H$. The "radius" R varies in time as $R/R_0 = (t/t_0)^{2/3}$.

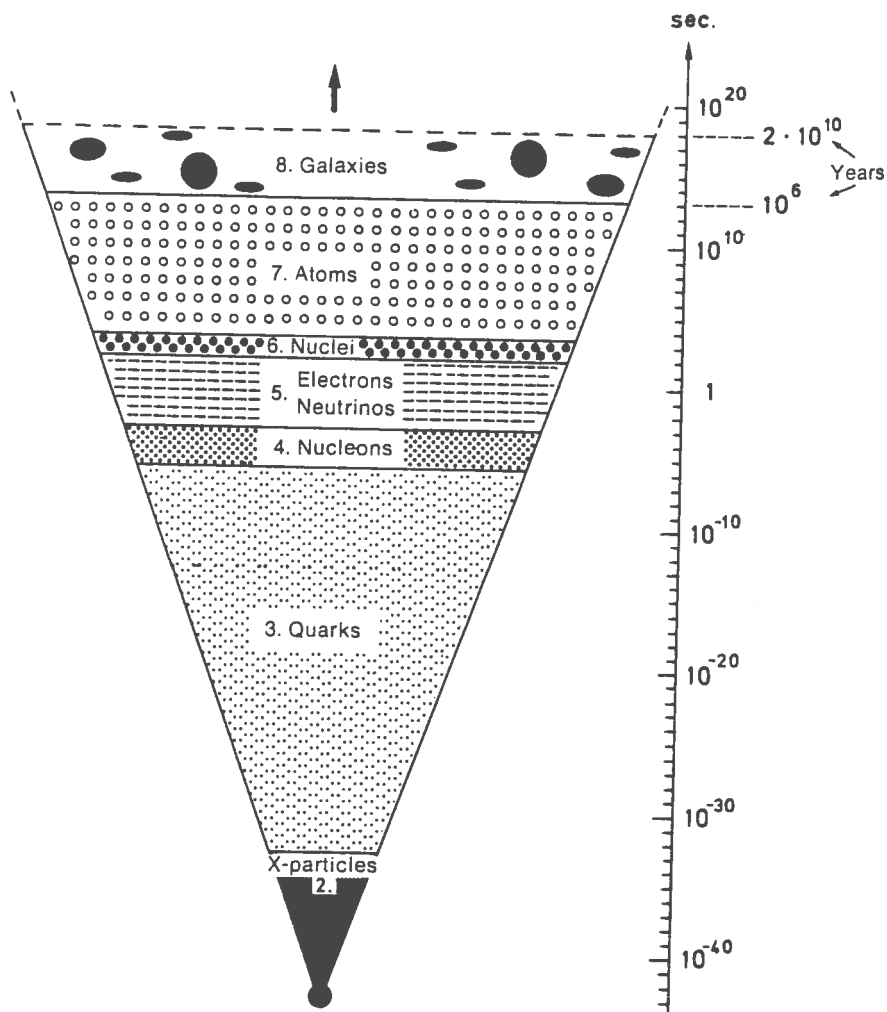
According to our present view about the structure of matter the world consists of a finite number of particle species: leptons, quarks, bosons. If the energy is high enough, these objects act like massless quanta of spin 0, $\frac{1}{2}$, 1, Therefore the matter density ρ in the early universe is related in a simple way to the temperature:

$$\rho = \frac{\pi^2}{30} g \cdot T^4 \quad (1.17)$$

(g effective number of spin degrees of freedom: $g = n_b + \frac{7}{8} n_f$, where n_b , n_f denote the number of bosonic, fermionic degrees of freedom - spin states and antiparticles are counted separately).

The entropy density of the universe is given by:

$$s = \frac{2\pi^3}{45} g \cdot T^3 \quad (1.18)$$



A schematic drawing of the evolution of the cosmos. Eight epochs of cosmic evolution are represented, beginning with the first 10^{-43} seconds following the Big Bang to the present epoch, which is marked by the presence of galaxies.

From: H. Fritzsch, The Creation of Matter Basic Books Inc., New York 1984 (Reprinted with permission of the publisher).

The entropy in a volume R^3 is constant, i.e. $S = R^3 \cdot s = \text{const.}$, and it follows that the temperature in the early universe behaves as R^{-1} . According to (1.16) and (1.11, $\Lambda = 0$) one finds:

$$\rho \sim \frac{1}{R^4}$$

$$\frac{\dot{\rho}}{\rho} = -4 \cdot \frac{\dot{R}}{R} = -4 \cdot \left(\frac{8\pi G}{3} \cdot \rho \right)^{\frac{1}{2}} \quad (1.19)$$

$$\rho(t) = \frac{3}{32\pi G t^2} \sim t^{-2}$$

This relation can be rewritten as follows:

$$t = \left(\frac{45}{16\pi^3 G} \right)^{\frac{1}{2}} g^{-\frac{1}{2}} T^{-2} = 2.41 \mu\text{s} g^{-\frac{1}{2}} (T[\text{GeV}])^{-2}. \quad (1.20)$$

At the CERN collider particles with masses up to 100 GeV can be produced. Taking into account the number of observed leptons, quarks and bosons (γ , weak bosons) g is of the order of 100. The time corresponding to $T = 100$ GeV is therefore $t \approx 10^{-11}$ s. High energy physics allows us to describe the development of the universe from 10^{-11} s until today. Further extrapolation (from 10^{-11} s to $t_p \approx 10^{-43}$ s) can only be made by relying on specific theories of elementary particle physics, which have not yet been tested experimentally.

2. Particle Physics: The Standard Model

Matter consists of quarks and leptons. Using high energy accelerators, one has been able to explore the structure of matter down to distances of the order of 10^{-16} cm. All phenomena of particle physics can be described in

terms of the standard model, i.e. in terms of the electroweak gauge theory of electromagnetic and weak interactions, and in terms of quantum chromodynamics (QCD), the gauge theory of quarks and gluons, which has turned out to be the correct field theory of the strong interaction.

Hadrons and QCD

QCD is the gauge theory of colored quarks transforming as triplets under the color group $SU(3)^C$, and of massless gluons, representing the gauge particles and transforming as $SU(3)^C$ -octets. The QCD gauge force acts universally on all quarks. The gluons mediate the strong force and are responsible for the permanent binding of the quarks and gluons to $SU(3)^C$ -singlet hadrons. The strong nuclear forces among hadrons are indirect manifestations of the color force.

During the last few years the most impressive success in high energy physics has been the check of the predictions of quantum chromodynamics for hadronic physics. QCD has developed from a hypothetical scheme to a realistic theory of the hadrons and their interactions. The quarks u and d relevant for the stable matter in the universe are essentially massless. The other quarks (s , c , b , t) have masses ($m_s \approx 0.2$ GeV, $m_c \approx 1.2$ GeV, $m_b \approx 4.8$ GeV, $m_t > 23$ GeV).

In QCD the hadronic mass scales, e.g. the proton mass, have nothing to do with the intrinsic masses of the quarks, but with the mass scale entering into the theory in a non-perturbative way via the QCD coupling constant.

The strong interaction coupling constant g has been determined recently in the lepton-hadron scattering experiments and in e^+e^- -annihilation with a

relatively good accuracy. One finds at an energy of about 30 GeV:

$g_s^2/4\pi = \alpha_s \approx 0.15$, i.e. $g_s \approx 1.4$. Both in the lepton-hadron scattering experiments and in e^+e^- -annihilation one observes effects (e.g. scaling violations) due to gluon radiation off quarks, which are the QCD analogs of the well-known QED bremsstrahlung. Furthermore one has found evidence for the production of quark-antiquark pairs by gluons (the QCD analog of the Bethe-Heitler process), e.g. by observing the production of charmed or b-flavored particles in hadronic collisions. In addition there are indications that gluons couple directly to gluons, as predicted by the non-Abelian nature of the QCD gauge theory. One can draw this conclusion by investigating the change of the gluon distribution function of the proton at increasing energies.

We conclude: the agreement between experiment and the theoretical predictions based on perturbation QCD is excellent. According to the theory the QCD coupling constant α_s at high energies behaves like

$$\frac{1}{\alpha_s(q^2)} = \frac{1}{\alpha_s(\mu^2)} + \frac{11 - \frac{2}{3}n_f}{4\pi} \ln \left(\frac{q^2}{\mu^2} \right) = \left(11 - \frac{2}{3}n_f \right) \ln \left(\frac{q^2}{\Lambda^2} \right) \quad (2.1)$$

(μ : arbitrary renormalization scale,

n_f : number of quark flavors).

The experiments give $\Lambda \approx 100 \dots 200$ MeV.

It is remarkable that the numerical value of Λ is close to the inverse radius of the nucleon (e.g. the (charge radius)² of the proton is 0.7 fm^2). This suggests that Λ^{-1} is related in a simple way to the confinement length of the theory, i.e. to the length at which the force between the quarks becomes strong. Furthermore the proton mass must be directly related to Λ :

$M_p = c \cdot \Lambda$, where c is a numerical constant which is in principle calculable from QCD and which is of the order of 6.

As far as astrophysics and cosmology is concerned, we can say that the hadronic matter at high densities and temperatures looks much simpler than previously thought. In a good approximation it is simply a free gas of spin 1/2 quarks and spin 1 massless gluons.

The question which remains completely open within the framework of QCD is the relation between the QCD mass scale Λ (or, what is the same, the proton mass) and the gravity scale. What determines the weight of the proton? Presumably an answer can be found only within a unified theory of QCD and general relativity.

Electroweak forces

The gauge theory of the electroweak forces describes two different kinds of weak interactions, the charged current interaction mediated by W^\pm -exchanges as well as the neutral current interaction mediated by Z-exchange, and the electromagnetic interaction mediated by photons. The gauge symmetry is $SU(2) \times U(1)$. It is valid only at energies much above 100 GeV. At low energies it is spontaneously broken. As a consequence the W and Z-particles are massive (masses of order 100 GeV). These masses are determined by the vacuum expectation value of a scalar Higgs field φ , which in turn is determined phenomenologically by the observed value of the Fermi constant G_F :

$$\frac{1}{2\langle 0|\varphi|0\rangle^2} = \frac{G_F}{\sqrt{2}} \quad (\langle 0|\varphi|0\rangle = 246 \text{ GeV}). \quad (2.2)$$

The SU(2) gauge force acts universally on all lefthanded quarks and leptons which transform as SU(2) doublets. The righthanded quarks and leptons are singlets. As a consequence parity is violated maximally. It is unknown why this is the case, nor does one know whether parity conservation is restored at some high energy scale.

The U(1) force acts differently on the various kinds of quarks and leptons. The coupling strength is determined by the electric charge. Charge quantization is not required. Thus on the level of the electroweak gauge theory it remains unclear why e.g. the u-quark charge is exactly equal to 2/3 of the positron charge.

A remarkable feature of the observed leptons and quarks is the classification into three different families:

$$\begin{array}{l}
 \text{I} \quad \left(\begin{array}{c|ccc} \nu_e & u_r & u_g & u_b \\ e^- & d_r & d_g & d_b \end{array} \right) \\
 \text{II} \quad \left(\begin{array}{c|ccc} \nu_\mu & c_r & c_g & c_b \\ \mu^- & s_r & s_g & s_b \end{array} \right) \\
 \text{III} \quad \left(\begin{array}{c|ccc} \nu_\tau & t_r & t_g & t_b \\ \tau^- & b_r & b_g & b_b \end{array} \right).
 \end{array}$$

(r, g, b: red, green, blue).

Within each family the sum of the electric charges vanishes. The existence of the three types of families, in particular the fact that the leptons and quarks of these families seems to have identical properties, apart from the

various lepton- and quark masses, remains one of the great puzzles in high energy physics.

3. Unity of Quarks and Leptons

A popular way to extrapolate to the physics at energies much above 100 GeV is to suppose that a unified description of the leptons and quarks can be achieved following a similar line as within the electroweak theory which gives a unified description of the electromagnetic and weak forces due to the spontaneous breaking of the electroweak gauge symmetry. It might well be that quarks and leptons are related by a large symmetry, which however is valid only at very small distances. The differences between leptons and quarks observed at distances of the order of 10^{-16} cm or larger must then be attributed to a spontaneous breakdown of the symmetry. Our aim is to embed both the QCD gauge group $SU(3)^C$ and the electroweak gauge group $SU(2) \times U(1)$ in a larger group G :

$$SU(3)^C \times SU(2) \times U(1) \subset G. \quad (3.1)$$

The "observed" quarks and leptons can be divided into three generations:

$$\begin{pmatrix} \nu_e & | & u \\ e^- & | & d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu & | & c \\ \mu^- & | & s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau & | & t \\ \tau^- & | & b \end{pmatrix} \quad (3.2)$$

(We note that the t quark has not been observed yet; its mass must be larger than ~ 18 GeV).

If we disregard the family problem and consider only the first family (all lefthanded fermions and antifermions, the corresponding righthanded fermions are provided by CPT) we are dealing with the representation

$$(1,2, -\frac{1}{2}) + (1,1,1) + (3,2, \frac{1}{6}) + (\bar{3},1, \frac{1}{3}) + (\bar{3},1, -\frac{2}{3}) \quad (3.3)$$

$$=(\nu_e, e^-)_L + (e_L^+) + (u, d)_L + (\bar{d})_L + (\bar{u})_L$$

under the group $SU(3)^C \times SU(2) \times U(1)$. Fifteen lefthanded fermions are needed. One may add one further fermion, the lefthanded antineutrino $\bar{\nu}_{eL}$, in which case one obtains sixteen fermions. The maximal symmetry of the free fermion Lagrangian is $SU(15)$ or $SU(16)$ respectively. However these groups cannot be used as gauge groups, due to the existence of anomalies which spoil the renormalizability of the theory. One may ask the question whether there exists a group such that

- a) it is a subgroup of $SU(15)$ or $SU(16)$,
- b) the 15 or 16 fermions form a representation of this group which has the desired transformation property under $SU(3)^C \times SU(2) \times U(1)$,
- c) there exist no anomalies.

The solutions to this problem are well-known. In the 15-fermion case one finds $SU(5)$, while in the 16-fermion case the solution is $SO(10)$. In both schemes the color group $SU(3)$ and the electroweak group $SU(2) \times U(1)$ is embedded in the larger group $SU(5)$ or $SO(10)$. In particular the $U(1)$ -generator is represented by one of the generators of $SU(5)$ or $SO(10)$, implying that the electric charges of the fundamental fermions are quantized. Furthermore the coupling constants g_3 , g_2 , and g_1 are equal (up to group factors of order 1) to the fundamental gauge coupling g . This implies in the symmetry limit:

- a) The value of the $SU(2) \times U(1)$ mixing angle is given by $\sin^2 \theta_W = 3/8 = 0.375$.
- b) $\alpha_S = \frac{g_3^2}{4\pi} = 8/3 \cdot \alpha$ (3.4)

These values are in disagreement with the values of the coupling constant observed in the experiments. The only way to achieve consistency is to assume that the energy at which the symmetry limit is obtained is far away from the region studied in the present experiments. The renormalization effects increase both $\sin^2\theta_W$ and α_s , and a remarkable agreement with experiment is achieved, if the unified symmetry sets in at about 10^{15} GeV.

In the SU(5) scheme the 15 Fermions are described by the reducible representation $\bar{5} + 10$, which decompose under the subgroup SU(3) x SU(2) x U(1) exactly as needed:

$$\bar{5} = (\bar{3}, 1, \frac{1}{3}) + (1, 2, -\frac{1}{2}) = \bar{d}_L + (\nu_e, e^-)_L \quad (3.5)$$

$$10 = (\bar{3}, 1, -\frac{2}{3}) + (3, 2, \frac{1}{6}) + (1, 1, 1) = \bar{u}_L + (u, d)_L + e^+_L.$$

In the SO(10) scheme the 16 fermions are described within the irreducible 16 - dimensional spinor representation of SO(10). Under the subgroup SU(3) x SU(2) x U(1) one finds the decomposition:

$$16 = (1, 2, -\frac{1}{2}) + (1, 1, 1) + (3, 2, \frac{1}{6}) + (\bar{3}, 1, \frac{1}{3}) \quad (3.6)$$

$$+ (\bar{3}, 1, -\frac{2}{3}) + (1, 1, -1).$$

The group SU(5) is a subgroup of SO(10), and one can decompose the fermions with respect to SU(5) as follows:

$$16 = \bar{5} + 10 + 1,$$

i.e. one obtains the fermions of the SU(5) scheme and a SU(5) - singlet, the righthanded neutrino. It is interesting to observe that the SO(10) group is

able to accommodate a left right symmetric gauge theory of the weak interactions:

$$SO(10) \supset SU(3)^C \times SU(2) \times SU(2)_R \times U(1) \quad (3.7)$$

Both in the SU(5) and the SO(10) schemes quarks, leptons and antiquarks appear in the same representation. Hence baryon number is not conserved and the proton is unstable.

In the SU(5) scheme the lifetime of the proton can be estimated rather precisely to

$$\tau(\text{proton}) = 4 \cdot 10^{31} \pm 1.3 \left(\frac{\Lambda}{0.16 \text{ GeV}} \right)^4 \text{ years} \quad (3.8)$$

The best value of the QCD Λ -parameter is about 0.16 GeV, hence the proton lifetime is expected to be about 10^{31} years, in disagreement with experiment which suggests $\tau > 10^{32}$ yrs if the decay branching fractions are deduced according to the SU(5)-theory.

In the SO(10) scheme the decay of the proton proceeds essentially along the same lines as in the SU(5) scheme, if the SO(10) symmetry is broken at $\sim 10^{15}$ GeV down to $SU(3)^C \times SU(2)_L \times U(1)$. The life time of the proton is equal to the one predicted within the SU(5) scheme. However the situation is different if the left-right symmetric electroweak theory $SU(2)_L \times SU(2)_R \times U(1)$ is valid down to much smaller energies, say a few hundred GeV. In this case the proton life time is typically larger than 10^{31} years and may even be above 10^{33} years. Thus at present it seems that the SU(5) theory is unlikely to be correct. However the SO(10) scheme may still be the correct way of achieving a unification of QCD and the electroweak interactions.

Probably the most interesting application of the idea of a unified interaction becoming relevant at a large energy scale ($> 10^{15}$ GeV) is the idea of spontaneous baryon number generation.

In particular the decays of superheavy gauge bosons or superheavy scalars can produce a net baryon number. The idea is that soon after the "big bang" the baryon number violating interactions came into equilibrium, and any initial baryon asymmetry (if present) was washed out. If the expansion rate of the universe is fast compared to the decay rate of the superheavy particles, the latter drop out of equilibrium, and a net baryon number is generated, as discussed later.

The SU(5) and SO(10) schemes discussed here are the simplest schemes in which a unity of quarks and leptons is achieved. Many other theoretical schemes have been discussed in the recent years, in particular supersymmetric gauge theories. At present it is unclear whether nature indeed follows this path of introducing higher and spontaneously broken gauge symmetries.

Another way of achieving a unity of quarks and leptons is to consider composite models. Quarks and leptons may consist of smaller constituents. Experimental constraints require the bound state radii of quarks and leptons to be less than about 10^{-16} cm. This length scale corresponds to an energy scale of about 1 TeV. In some models of this type even the weak bosons are considered as bound states, in which case the 1 TeV energy scale arises as the natural scale for the inverse radii of the bound states. Such ideas have direct implications for cosmology. Shortly after the big bang, when the temperature of the universe was still above 1 TeV, the matter in the universe would have been present in form of a gas of lepton-quark- constituents.

Furthermore the mechanism for the generation of a non-zero baryon number in the universe would depend on details of the dynamics of the constituents.

Which way high energy physics might go in the future, depends in particular on the new insights which will be gained at the end of the decade when new accelerators like LEP (CERN, Geneva) and HERA (DESY, Hamburg) start to operate.

4. The Universe Backward in Time

High energy physics allows us to extrapolate the development of the universe backward in time until about 10^{-11} s after the big bang, a time at which the temperature of the universe was about 100 GeV. Further extrapolations depend on specific theoretical frameworks, which have not yet been tested. In this chapter I shall describe the development starting from our present time.

At present matter consists of nucleons, electrons, photons, neutrinos and perhaps other neutral particles not yet detected. There is no evidence that antimatter is present in large quantities.

The gravitational dynamics of the universe is governed by the total energy density ρ , which can be decomposed in its different components:

$$\rho = \rho_n + \rho_{e^-} + \rho_\gamma + \rho_\nu + \hat{\rho} \quad (4.1)$$

nucleons	electrons	photons	neutrinos	other yet unknown particles
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The energy density provided by the nuclear matter is in the average about 400 eV/cm^3 . This corresponds to a ratio

$$\frac{\text{number of photons}}{\text{number of nucleons}} \approx 10^9 \dots 10^{10}. \quad (4.2)$$

The nuclear matter density dominates the energy density, unless massive neutrinos or other neutral particles are important. It is much less ($\ll 10\%$) than the critical energy density, which on the basis of the observed value of the Hubble parameter is estimated to be $\sim 10^3 \dots 10^4 \text{ eV/cm}^3$.

If neutrinos are massive, one expects the neutrino sea (analogous to the photon sea) to contribute to the energy:

$$\rho_\nu \sim 230 m_\nu \text{ cm}^{-3} \quad (\text{each species}). \quad (4.3)$$

The restriction $\rho_\nu < \rho_e$ leads to the constraint:

$$\sum_{\text{neutrinos}} m_\nu \ll 50 \text{ eV}. \quad (4.4)$$

Today the energy density ρ is dominated by nonrelativistic particles (nucleons, neutrinos?, ...). At earlier times the relativistic particles take over, since $\rho \sim T^4$ ($T \sim R^{-1}$), hence $\rho \sim R^{-4}$, to be compared with $\rho \sim R^{-3}$ for nonrelativistic matter. The energy density of the photon sea exceeds the nuclear density until about 500 000 years after the big bang, when the temperature dropped below 0.3eV.

During time $10^{-3} \text{ s} \dots 5 \cdot 10^5 \text{ years}$ a number of important changes took place in the universe, which will be mentioned here briefly:

- a) Nucleosynthesis (formation of deuterium and helium $t \sim 3 \text{ min.}$)
- b) The disappearance of the positrons by e^+e^- annihilation ($t \sim 10 \text{ s}$)
- c) The decoupling of the neutrinos ($t \sim 1 \text{ s}$)
- d) Decay of all strongly interacting particles except nucleons.

As already mentioned, the nucleosynthesis, generating in particular a sizeable amount of helium, is one of the important features of the hot big bang cosmology.

At about $t = 10^{-3}$ s the universe was filled with a plasma ($T \sim 30$ MeV) of electrons, positrons, photons, neutrinos, and of nucleons. If we go back in time ($t \sim 10^{-6}$ s, $T \sim 1$ GeV), further particles come to play a rôle (π -mesons, muons). If one treats the pions naively as pointlike objects, one arrives at the conclusion that at $T \sim 1$ GeV the tiny volume of one cubic fermi ($1 \text{ fm} = 10^{-13}$ cm) would be inhabited by fifty pions. According to our present understanding of the strong interactions this is impossible, since pions like any other strongly interacting particles consist of quarks which are confined inside a finite volume with the extension of about 1 fm. We conclude that during the time interval (10^{-6} s ... 10^{-4} s) the hadronic matter went through a phase transition. At $T \gtrsim 1$ GeV no free hadrons were present. Instead the hadronic matter was represented by a gas of nearly free quarks and gluons, which I like to call chromoplasma. It was in thermal equilibrium with the other light not strongly interacting particles (photons, e^- , e^+ ...). The chromoplasma consisted of quark and antiquarks of the u, d and s- flavor. The densities of s-quarks and \bar{s} -quarks were exactly the same, while for u and d-flavors there was a tiny excess of quarks over antiquarks:

$$\frac{n(u) + n(d)}{n(\bar{u}) + n(\bar{d})} - 1 \sim 10^{-9} \quad (4.5)$$

The baryon- antibaryon asymmetry observed in our present universe can be traced back to this tiny excess of quarks in the chromoplasma.

At $T \sim 1$ GeV only the three light quark flavors u, d, s are present in the chromoplasma. At higher temperatures ($t \sim 10^{-8}$; $T \sim 30$ GeV) the new quark flavors c and b contribute, and at $T > 50$ GeV presumably the sixth quark flavor t starts to play its rôle. (Thus for the existence of the t -quark has not been confirmed by experiment, however there exist some indications from the collider experiments at CERN that its mass is of the order of 40 GeV.)

If the temperature of the universe drops significantly under 1 GeV, the chromoplasma starts to disappear, and individual hadrons form. Due to our lack of understanding nonperturbative aspects of QCD dynamical details of this phase transition are not known. Preliminary studies indicate that this may be a first order phase transition, the transition temperature being of the order of 150 ... 250 MeV.

Details of the phase transition can be obtained by new experimental studies, notably by investigating high energy collisions of heavy nuclei. One expects that in a frontal collision of two heavy nuclei part of the colliding hadronic matter undergoes a transition to the chromoplasma phase provided the critical temperature is reached. The plasma would cool down rapidly, until the condensation of the quarks, antiquarks and gluons into individual hadrons (mostly pions) sets in. Possible signals for the formation of the chromoplasma would be the production of an unusual number of γ -rays and of e^+e^- - or $\mu^+\mu^-$ -pairs by the annihilation of the quarks and antiquarks in the chromoplasma.

The first experimental data will be available soon, after the start of an exploratory program at the CERN-SPS involving ions up to oxygen with energies up to 225 GeV/nucleon. The best prospects of creating substantial

amounts of chromoplasma exist if one were able to perform colliding ion beam experiments with energies ~ 50 GeV/nucleon (e.g. using a machine like the one discussed at the Brookhaven National Laboratory).

The chromoplasma in the universe must have undergone the transition to the hadronic phase around $t \sim 10^{-5}$ s. Presumably a large number of pions have been formed during the phase transition, and, of course, the nucleons as the constituents of the nuclear matter in our present universe. The pions decay weakly or electromagnetically, and these decays mark the end of the life of the antiquarks in the universe. Afterwards the hadronic matter is present only in form of nucleons consisting of quarks.

It is well-known that cosmic rays contain a small flux of antiprotons. Although the exact magnitude of the \bar{p} -flux in the primary cosmic rays is still controversial, it is consistent to suppose that all antiprotons observed in the universe today are products of high energy collisions of nucleons (either cosmic ray collisions or collisions in man-made accelerators). The antiquarks which belonged to the most prominent inhabitants of the universe at $t \ll 10^{-5}$ s, have all disappeared suddenly at the time of the transition chromoplasma \rightarrow hadrons.

In view of our limited knowledge both from experiment and theory about the phase transition it cannot be excluded that besides the nucleons other exotic pieces of electrically neutral quark matter, in particular matter involving a non-vanishing density of strange quarks, have survived the chromodynamic phase transition. As speculated by a number of theorists, this new type of nuclear matter could manifest itself in the existence of neutral "quark nuggets" of macroscopic size. They could even provide the dominant

part of the "missing matter" in the universe.

During the period $10^{-12} \text{ s} < t < 10^{-6} \text{ s}$ the temperature of the universe dropped from 1 TeV to about 1 GeV. This drop of temperature affected mostly the electromagnetic and weak properties of matter. At energies above 100 GeV, i.e. above the masses for the W and Z bosons, one expects the weak interaction to be of essentially the same strength as the electromagnetic interaction. This follows from the SU(2) x U(1)-theory of electroweak interactions. In that theory the mass differences of the bosons (Z, W, γ) are generated by a spontaneous breaking of the gauge symmetry. The masses are proportional to the vacuum expectation value of the Higgs field. This type of symmetry breaking has the interesting property that it disappears at sufficiently high temperatures. This effect is similar to the disappearance of the spontaneous magnetization of a ferromagnet at high temperatures. The net effect would be that the Higgs fields act as if their vacuum expectation values would be zero. As a result at $t \ll 10^{-12} \text{ s}$ all four gauge bosons Z, W^\pm and γ are massless, likewise the quarks and leptons. If this picture is correct, the appearance of masses is due to a new phase transition, the electroweak phase transition. If the temperature drops below $\sim 1 \text{ TeV}$, the symmetric Higgs phase is replaced by the asymmetric one, i.e. a non-zero vacuum expectation value is developed, and the bosons and fermions acquire their values measured today.

The dynamical details of the electroweak phase transition depend greatly on whether the SU(2) x U(1) theory is indeed a microscopic theory of the fermions and bosons. In various schemes of composite leptons, quarks and bosons the 1 TeV energy scale corresponds to the inverse radii of the leptons and quarks. If this is the case, the matter in the universe at $T > 1 \text{ TeV}$

would essentially be a gas of the lepton- quark constituents, a type of matter which I like to call hyperchromoplasma. The electroweak phase transition would be rather similar to the chromodynamic phase transition:

$T > 1 \text{ TeV}$: Hyperchromoplasma

Phase Transition: Formation of quarks, leptons and weak bosons by the hypercolor interaction among constituents.

$T < 1 \text{ TeV}$: Plasma of quarks, leptons, massive weak bosons, massless photons.

Which picture of the electroweak phase transition, if any, is correct, remains to be seen.

5. The Genesis of Baryons

Today there exists clear evidence that the luminous matter in the universe consists of u and d quarks and of electrons. Large quantities of antimatter do not seem to exist.

In the standard $SU(3) \times SU(2) \times U(1)$ model the number of quarks minus the number of antiquarks is exactly conserved. This conservation law implies the conservation of baryon number. During the hot phase of the chromoplasma the quarks dominated slightly over the antiquarks by a tiny amount: $(1: 10^9 \dots 10^{10})$. As long as the physics of matter is described by the standard model, there is no way to explain the net baryonic charge of the universe. It must rather be considered a boundary condition for the big bang.

It would be much more satisfactory to obtain the baryon number as a result of the dynamics of matter shortly after the big bang. Any framework

doing so must incorporate three features:

- a) Baryon number cannot be strictly conserved.
- b) Charge conjugation (C) and the CP-symmetry must be violated.
- c) The relevant processes cannot be in thermal equilibrium.

The first two conditions are easy to understand, since the aim is to generate more quarks than antiquarks, violating both the C and CP symmetry. The third condition is necessary in order to avoid the CPT-theorem which implies equal masses for particles and antiparticles, hence strictly equal abundances, in thermal equilibrium.

A simple mechanism for baryon number generation exists if we suppose that the idea of grand unification makes sense. Around 10^{-36} s after the big bang the temperature drops below the grand unification mass scale of the order of $10^{14} \dots 10^{15}$ GeV. As long as the temperature is still above that energy, the matter in the universe will be present in form of a hot plasma of leptons, quarks, their antiparticles as well as photons, gluons, weak bosons, and the new gauge bosons called X which carry both color, electric and weak charges. The latter are responsible for the unification of the strong and electroweak interactions.

As the temperature of the universe drops below 10^{15} GeV, the bosons decay, thereby violating the conditions of thermal equilibrium. These decays will violate both C and CP, since those symmetries are violated by the electroweak interactions. An drastic way to envisage what happens is as follows. Suppose we start out from a pair $X \bar{X}$. The X-particle may, for example, decay into two u-quarks, the \bar{X} -particle into a d-quark and an electron:

$X \rightarrow u u \quad \bar{X} \rightarrow d e^-$. The net result is: $X \bar{X} \rightarrow (u u d) + e^-$. From equal amounts of matter and antimatter we obtain effectively a proton and an electron, i.e. hydrogen.

Although the chain discussed above is a very simplified one, it clearly displays the generation of baryon number. The violation of CP will guarantee that the other reaction $X \bar{X} \rightarrow \bar{u} \bar{u} \bar{d} + e^+$ is slightly suppressed in rate. Unfortunately quantitative estimates of the amount of baryonic charge generated by this mechanism depend on unknown parameters, e.g. the amount of CP-violation in the X-decay, or the X-mass, which in turn determines the decay rate of the proton. Guesses in the literature range from $\frac{n_B}{n_\gamma} \sim 10^{-6} \dots 10^{-12}$, the observed value being $\sim 10^{-9} \dots 10^{-10}$.

It is interesting to note that the picture of baryon genesis discussed above is intimately connected with the idea of the unification of quarks and leptons at some high energy scale. We do not know whether the theories of unification based on the symmetries SU(5) or SO(10) give an adequate description of what happens at high energy. Perhaps the unity of quarks and leptons is achieved in some other way, e.g. within the composite model approach. But the chances are good that the baryon number of the universe and the unification idea are closely related. Consequently the proton should not be stable. Eventually the decay should be observed in laboratory experiments.

6. Cosmological Puzzles and Inflation

a) The problem of entropy (flatness problem).

The total entropy S in the universe today in a volume given by R^3 is very large: $S > 10^{87}$. It can be related to the departure of the actual matter

density from the critical one as follows. We define $\Omega = \rho/\rho_c$. The cosmological equations give in the radiation dominated universe:

$$\frac{\Omega-1}{\Omega} = 0.2 \cdot \frac{k}{g^{1/3} S^{2/3}} \cdot \left(\frac{m_p}{T}\right)^2 \quad (5.1)$$

(m_p : Planck mass). Due to the large value of S it follows:

$$\left(\frac{\Omega-1}{\Omega}\right) < 10^{-59} g^{-1/3} \left(\frac{m_p}{T}\right)^2. \quad (5.2)$$

Already at $T = 1 \text{ MeV}$ one obtains $|\Omega - 1| < 10^{-15}$, i.e. the higher the temperature, the closer the density approaches the critical one. At $T \sim 10^{14} \text{ GeV}$ one has $|\Omega - 1| < 10^{-49}$. It is unlikely that these constraints are an accident.

The most satisfactory solution would be $\rho = \rho_c$, i.e. $\Omega = 1$.

b) The horizon problem

The photon sea in the universe is very isotropic (accuracy one part in ten thousand). On the other hand a quick calculation shows that only regions in the sky which are less than 2° apart could have been in contact with each other 300 000 years after the big bang, i.e. at the time when the photons decoupled. In view of the large scale inhomogeneities of the luminous matter in the universe the isotropy of the 3 K - radiation is a puzzle.

A popular way to solve these puzzles is the idea of the inflationary universe. Suppose the symmetry of a grand unified theory of quarks and leptons is broken by a scalar Higgs fields which acquires a vacuum expectation value. The latter depends on specific properties of the effective potential for the ω -field, which plays the rôle of an order parameter of the system. At temperature above a certain critical temperature T_c the vacuum expectation value

$\langle 0|\phi|0\rangle$ is zero - the symmetry is unbroken.

If the temperature drops below T_c , a phase transition takes place, and due to a quantum mechanical tunneling phenomenon the vacuum expectation value becomes different from zero: $\langle 0|\phi|0\rangle = \sigma \neq 0$. The system makes a transition from a symmetric phase to an unsymmetrical one. Suppose we identify the latter phase with the state of the universe today. From observations we know that the energy density of the vacuum, i.e. the cosmological term Λ , must be very small (if different from zero at all). In any case it must be significantly less than the critical density, which is $\rho_c \approx 10^{-46} \text{ (GeV)}^4$.

However the vacuum energy density of a system involving a Higgs field ϕ depends on the effective potential V_{eff} and hence on σ . The potential difference ΔV_{eff} between the unbroken and broken phase describes the difference of the vacuum energy density between the two phases. Since in the broken phase $\Lambda \approx 0$, it follows that in the unbroken phase the cosmological term $\Lambda = 8\pi G \Delta V_{\text{eff}}$ must be given by $T_c \sim 10^{14} \text{ GeV}$: $\Delta V_{\text{eff}} \sim (10^{14} \text{ GeV})^4$. (However one must bear in mind that this argument does not explain why Λ is zero or nearly zero in the universe today. The smallness of Λ today, i.e. in the broken phase of the universe, is a great puzzle.)

If we insert in the Friedmann equation such a big Λ -term, it implies a constant Hubble parameter

$$\hat{H} = \left(\frac{8\pi \Delta V_{\text{eff}}}{3 m_p^2} \right)^{\frac{1}{2}} \approx 10^{10} \text{ GeV.} \quad (5.3)$$

This leads to an exponential expansion of the universe: $R(t) \sim e^{\hat{H}t}$. The typical time scale for this expansion is of the order of $\hat{H}^{-1} \sim 10^8 \dots 10^9$

Planck time units.

During the inflationary era which is estimated to last about 10^{-32} s, the universe increased by a factor of about 10^{50} . Afterwards the transition to the broken symmetry phase takes place. The tremendous energy density of the unbroken phase was released in form of production of particles.

The inflation of the size of the universum by a factor as large as 10^{50} solves the problem of the isotropic 3 K radiation. Before inflation the region which later on turns into be universe observable today is small enough such that a thermal equilibrium can be achieved.

If an inflation took place early in the history of the universe, one expects the ratio $\Omega = \rho/\rho_c$ to be very close to one. The inflation causes the space to become nearly flat, i.e. the universe today should be of the type $k = 0$ ($\Omega = 1$). Although many observers believe Ω to be significantly less than 1, e.g. $\Omega \approx 0.1$, the case $\Omega = 1$ is certainly not yet excluded.

In the simplest inflation scenario described about serious problems exist. In particular one expects, in analogy with similar situations in solid state physics, that the phase transition at $T_c \sim 10^{14}$ GeV would proceed via a nucleation of bubbles of the new phase, resulting in the creation of severe inhomogeneities, which are not seen in the universe today. These problems can be solved in the model of the so-called new inflationary universe. Here a very special type of Higgs potential is required such that a particular smooth phase transition occurs, allowing a sizable supercooling effect. The reheating temperature depends on specific details of the effective potential. However under reasonable assumptions it is slightly below T_c . In such a

scenario the energy density of the universe is interpreted as the latent heat produced by the phase transition. Since the reheating temperature is not much below T_c , the baryon dominance is generated by the same mechanism as in the cosmological picture, discussed previously.

The idea of the inflationary universe is very promising, since it realizes an interesting point of view. In this model the evolution of the universe is essentially independent of the details of the initial conditions. All details of the latter are washed out by the inflation. It is quite possible that the universe started out from a highly chaotic state. Certain parts would undergo inflation, thus creating regions which are fairly smooth and regular. Due to huge factors of order 10^{50} playing a rôle, such a region can be large enough to encompass the entire observable universe.

Another interesting point of the inflationary scenario is the idea that the matter and energy of the universe are relics of a phase transition, representing the latent heat liberated by the transition. In such models it is quite plausible that the gravitational energy of the universe cancels all other energy such that the universe has zero energy. A creation of the universe out of nothing may well be possible.

7. Concluding Remarks

It is quite clear that during the recent decades cosmology has turned from a speculative field to a real science of the dynamical evolution of the universe. Due to the progress made in our understanding of the structure of matter and the dynamical issues of the forces in nature one is able to trace back the history of the universe to very early times. Thus far the structure

of matter is explored to a distance of the order of 10^{-16} cm, corresponding to an energy scale of about 100 GeV. The standard cosmological model implies that the temperature of the universe dropped below 100 GeV at a time of about 10^{-11} s after creation. Thus we may say that the cosmological development is reasonably well understood since $t \approx 10^{-11}$ s. As far as earlier times are concerned, only theoretical extrapolations can be made. However the chances are good that at least a part of the speculations we discussed like grand unification and the associated phase transition, the mechanism of baryogenesis via X-particle interactions, or the inflation of the universe may have contact with reality. On the other hand I am not satisfied with the way one has to rely on the standard mechanism for the dynamical symmetry breaking. Thus far no scalar Higgs particles have been observed, and chances are that the symmetry breaking is caused by dynamical effects such that no explicit Higgs particles are needed. If this is the case, the theoretical ideas with respect to baryogenesis or inflation have to be modified. Furthermore a serious problem remains entirely unsolved. If indeed inflation is caused by a mechanism which can be described effectively by a cosmological term, it is completely unclear why this term is absent in our observed universe.

Combining both theoretical speculations and knowledge from experimental high energy physics, one may divide the cosmological development into eight periods. This "eightfold way" of the evolution of the universe is shown in the figure. The first epoch is the epoch at which quantum gravity is important. Not much is known about it. It follows the period at which the ideas of grand unification are relevant (perhaps inflation, baryogenesis). The third epoch corresponds to the epoch of the electroweak and chromodynamic plasma. In the

fourth epoch protons and neutrons are formed. The final traces of antimatter (positrons) disappeared during the fifth period. The sixth epoch is the one of nucleosynthesis. During the seventh epoch atoms are formed. The photon sea decouples from matter. Today we live in the eighth epoch in which macroscopic structures like galaxies, stars and planets came into existence.

The principal feature of the present, eighth epoch of cosmic evolution, is structure - a characteristic of the universe not found in the first million years. During its first seven eras the structure of the cosmos underwent little change. Energy and matter were distributed homogeneously; the universe was filled with uniform radiation. The only evidence of temporal evolution was found in the expansion of the cosmos and the constant drop in temperature.

At the beginning of the eighth epoch the universe was composed of a relatively hot gas made up of hydrogen and helium atoms. Today, shortly before the end of the second millennium of human time, about 20 billion years or 10^{18} seconds after the Big Bang, the cosmos is filled with galaxies, stars, planets, and such complex structures as we ourselves. The eighth epoch can rightly be called the epoch of structures.

As far as the very early universe is concerned, many questions remain to be answered, in particular the questions about the details of the generation of baryon number and about the possible rôle of the spontaneous symmetry breaking during the yet hypothetical inflationary period. We have not mentioned the important rôle, which massive neutrinos or other massive neutral particles might have played in the early universe in providing a background for the development of the large scale structure of the universe (galactic clusters etc.).

Although many uncertainties are still present, the prospects seem good for explaining the observed features of the universe and its development on the basis of the physical laws of nature, without taking reference to a special set of initial conditions. The question remains: Who invented the physical laws, the rules which govern the behaviour of matter and the space-time structure of the universe not only today, but also in the very early universe?

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