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THEORIES OF SPACE AND TIME

by

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1. The Structure of Space and Time

The notions of space and time, perhaps more than any other concepts, are of fundamental importance both for science and for philosophy, for they are the common ground on which philosophical speculation and physical deliberation intertwine with each other.

In ancient and classical thought space and time were generally conceived as disjoint and unrelated entities. Aristotle's theory of time, although based on the concept of motion and change, as developed in Book Four of his *Physics*, is completely unrelated to his theory of space or "place" ("topos"). Euclid's *Elements* describes the structure of geometrical space without making any reference whatever to the notion of time. In Newton's *Principia* absolute space and absolute time are fundamental constituents of physical reality but, precisely because of their absoluteness, totally separated from each other. Even in Kant's *Critique of Pure Reason* space and time, being pure intuitions or forms of sensibility ("Anschauungen") and hence *a priori* conditions for the possibility of experience, are physically unrelated, for they are not properties of the things-in-themselves but only different forms of perceiving them.

Since, however, as Minkowski once said, nobody "has ever noticed a place except at a time, or a time except at a place"[1], it may be expected that some union of these two notions should have been conceived quite early in the history of human thought.

In fact, an early example of such a fusion is the old Hebrew term "ʾolam" which, derived from an Ugarith root, denotes already in the Bible both spatial ubiquity and temporal eternity. Similarly, the ancient Chinese word "yü-chou", found for example in the _Huai Nantzu_ of the second century B.C., denotes space spread out in every direction and time "that has passed from antiquity until the present."[2]

The first who explicitly stated that both notions together are a common condition for the ontological possibility of existence was probably the Irish philosopher and theologian Johannes Scotus Eriugena of the ninth century A.D. when he declared in his _De Divisione Naturae_ (3, 19) : "Omnium itaque existentium essentia localis et temporalis est."

The idea that space and time penetrate each other has been emphasized, probably for the first time, by the English empiricist John Locke in his _Essay concerning Human Understanding_ (1690): "Expansion and duration do mutually embrace and comprehend each other; every part of space being in every part of duration, and every part of duration in every part of expansion. Such combination of two distinct ideas is, I suppose, scarce to be found in all that great variety we do or can conceive, and may afford matter to farther speculation."[3]

The first step toward a mathematization of this unification has been made by someone of whom it is only recorded that he was a contemporary of D'Alembert. For the latter wrote in the famous Encyclopédie in 1774 : "Un homme d'esprit de ma connaissance croit qu'on pourrait ... regarder la durée comme une quatrième dimension, et que le produit du temps par la solidité serait en quelque manière un produit de quatre dimensions."[4]

Most likely the "homme d'esprit" was no other than Joseph Louis Lagrange; for in his work on analytic functions, published in 1794, he declared: "On peut regarder la mécanique comme une géometrie à quatre dimensions, et l'analyse mécanique comme une extension de l'analyse géometrique."[5]

About a century later "the British Lagrange", as William Rowan Hamilton was called by C.G.J.Jacobi, merged in his theory of quaternions space and time "into a unified algebraic number expression."[6] Subsequently quite a few mathematicians proposed to adjoin to the three coordinates of space a fourth coordinate representing time, among them an author who signed his article on this subject, published 1885 in the prestigious periodical Nature[7], only with the initial "S", the Darmstadt mathematician R. Mehmke[8] and the British mathematician R. Hargreaves[9], to mention only a few.

Lagrange's program of formulating classical mechanics in terms of a four-dimensional space-time could have been carried out, indeed, long before the advent of the theory of relativity. A formally simple, though perhaps conceptually not unobjectionable[10] proof of this possibility may be found in the fact that by letting the velocity of light c approach infinity, relativistic kinematics and dynamics in their four-dimensional tensor formulation reduce to their pre-relativistic antecedents. A mathematically more rigorous demonstration can be found in Philipp Frank's article[11] on the relation between Newtonian and relativistic formulations of mechanics and electrodynamics, published in 1909 shortly after the appearance of Hermann Minkowski's well-known four-dimensional spacetime representation of special relativity.

In a more recent generally covariant formulation of Newtonian dynamics in flat spacetime Peter Havas rightly remarked: "The formulations of Newtonian mechanics given here are in principle independent of the theory of relativity and could have been developed without it."[12] It is an interesting question why such a development has never been carried out prior to the advent of relativity.

Let us briefly review the relation between space and time as conceived in the different periods of physical and cosmological thought before the advent of relativity.

In Aristotelian physics, as already mentioned, space and time are each <u>sui generis</u> and never brought into a mutual relation. Strictly speaking, in the Aristotelian system there is no theory of space but only a theory of "places" ("topoi", "loci") according to which each element, like earth, water, air or fire, has its "natural place". If we nevertheless call the sum-total of places "space", then Aristotelian space has a spherically symmetric structure, with earth at the center and the other elements forming concentric spheres around it. Obviously, space according to Aristotle is neither homogeneous nor isotropic. Time, on the other hand, is in Aristotelian physics generally regarded as homogeneous in the sense that different temporal intervals of equal duration are physically equivalent. This homogeneity is not violated even by Aristotle's statement that "things ... are in some respect affected by time, just as we are wont to say that time crumbles things."[13] For it is the length of duration and not its epoch (or date) which is claimed "to crumble things."

It would lead us too far to discuss the growing criticisms levelled against these Aristotelian conceptions already in the late antiquity and during the middle ages. At the end of this process, in the middle of the fifteenth century, Nicholas of Cusa argued for the homogeneity and isotropy of space and the homogeneity of time. Less than a century later Galileo Galilei represented time by the geometrical model of a straight line and regarded physical processes as functionally depending upon time as the fundamental independent variable. As is well known, Isaac Newton proclaimed in his Principia Mathematica Philosophiae Naturalis a theory of absolute space and absolute time according to which it is meaningful to say, without referring to any reference frame, that two events occur at the same place at two different times or that two events occur at the same time at two different places. This theory will be called the Newtonian space-time theory, the hyphen between "space" and "time" inserted to indicate that space and time are regarded as separate and unrelated entities. The fundamental symmetry group of Newtonian space-time consists only of the identity group. For any two different events e_1 and e_2, occurring in this space-time at the locations x_1 and x_2 and at the times t_1 and t_2, respectively, the temporal interval $t_{21} = t_2-t_1$ and the spatial distance $s_{21} = x_2-x_1$ have obviously definite, i.e. uniquely determined, values (ignoring of course changes of scale).

However, Newton himself admitted that in all practical applications of mechanics and, in conformance with the mechanical world-picture of his time, of physics quite generally a certain relativity principle is valid which he expressed in these words :

"Corporum dato spatio inclusorum iidem sunt motus inter se, sive spatium illud quiescat, sive moveatur idem uniformiter in directum absque motu circulari."[14] Expressing the Galilei-Newtonian relativity principle in modern terms we say that all (mechanical) processes proceed in the same way independently what inertial reference frame has been chosen for their description.

The fundamental symmetry group, that is the group of transformations under which the laws of mechanics, or according to the Newtonian world-picture of physics quite generally, are covariant (i.e., unchanged) is the the simplest case, namely for standard configuration of the inertial reference frames, given by the equations

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t \tag{1}$$

and is graphically represented in the diagram of Fig. 1.

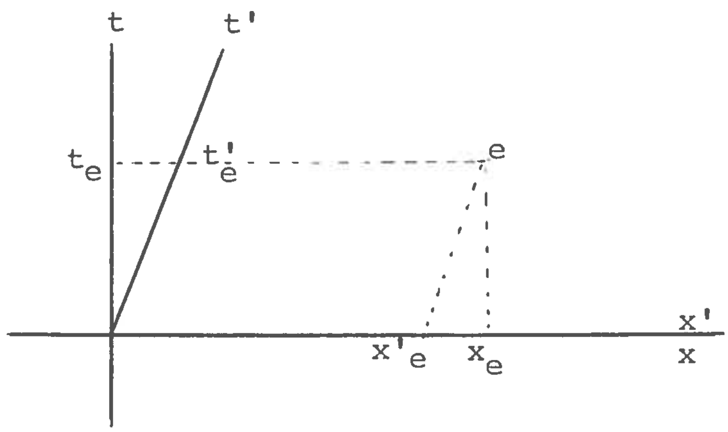

Fig. 1

The theory of space and time whose symmetry group is the Galilei (10 parameter) group will be called the Neo-Newtonian (or simply : Newtonian) spacetime theory (without a hyphen between "space" and "time"). As Fig. 1 shows, in Newtonian spacetime, contrary to Newtonian space-time, the space coordinate of an event e has no definite value but depends upon the direction of the time-axis which direction varies with the velocity of the relevant frame of reference. The temporal interval t_{21} between any two given events has a definite value, just as before; however the spatial distance s_{21} has a definite value only for simultaneous events (i.e. if $t_{21} = 0$) and otherwise can be given any real value whatever; in particular, if $t_{21} \neq 0$, taking the time-axis in the direction from event e_1 to event e_2 assigns to s_{21} the value zero, implying that the two events occur at the same place at different times. It will be understood that "changing the direction of the time-axis, e.g. of the t'-axis," means "changing the velocity of the primed reference frame, defined by the x'-axis and the t'-axis."

To reach a more profound insight into the structure of Newtonian spacetime and to compare it in the sequel with relativistic spacetime let us recall the following definitions pertaining to the theory of relations.

Let S be a set of elements denoted by a (with or without subscripts); and let S_r be a subset of the Cartesian product SxS (i.e. $S_r \subset SxS$). S_r defines a binary (two-place) relation r on S : $a_1 \, r \, a_2$ if and only if $(a_1, a_2) \in S_r$.

The relation r is reflexive if $a \in S$ implies $a \, r \, a$.

r is irreflexive if $a \in S$ excludes $a \, r \, a$.

r is symmetric if a_1ra_2 implies a_2ra_1. r is asymmetric if a_1ra_2 excludes a_2ra_1. Finally, r is transitive if a_1ra_2 and a_2ra_3 implies a_1ra_3.

Now, a reflexive, symmetric and transitive relation r on S is an equivalence relation on S which generates in S equivalence classes which will be denoted by E and which are defined as follows :

E is an equivalence class of r if $a_1, a_2 \in E$ implies a_1ra_2 and if $a_1 \in E$ and a_1ra_2 implies $a_2 \in E$.

Two equivalence classes are either identical ($E_1 = E_2$) or they are disjoint (i.e., their intersection $E_1 \cap E_2$ is empty). It follows that an equivalence relation r on S partitions S into a set of equivalence classes which set is called the quotient of S by r and is denoted by S/r.

On the other hand, an irreflexive, asymmetric and transitive relation r' on S is a partial ordering on S. If a partial ordering on S satisfies the additional condition that for any two given $a_1, a_2 \in S$ either $a_1r'a_2$ or $a_2r'a_1$ or $a_1 = a_2$, where the symbol "=" denotes here the identity, then it is a total ordering on S.

Finally, if r is an equivalence relation on S and r' is a partial ordering on S, r' is said to induce the relation R on S/r where R is defined on S/r as follows :

For $E_1, E_2 \in S/r$ $E_1 R E_2$ if and only if $a_1 \in E_1$ and $a_2 \in E_2$ implies $a_1r'a_2$.

Let us now apply these notions to Newtonian space-time and Newtonian spacetime. In both these theories the time coordinate of a given event e has (up to scale and zero point) a definite value, independent of the reference frame employed. The relation of simultaneity σ between two events e_1 and e_2, defined by the condition that $e_1 \sigma e_2$ if and only if $t_{21} = 0$, is obviously an equivalence relation. For the same reason, in both theories the relation of temporal precedence π , defined by the condition that $e_1 \pi e_2$ if and only if $t_{21} > 0$, is obviously a partial ordering. But π is <u>not</u> a total ordering on the set S of all events, for there are events e_1 and e_2 for which neither $e_1 \pi e_2$ nor $e_2 \pi e_1$ nor $e_1 = e_2$ (identity!), namely distant simultaneous events. However, the relation Π induced by π on S/σ , is a total ordering. The equivalence classes (or elements) of S/σ will be called "instants" and denoted by I (with or without subscripts). The set of all instants which, as we have seen, is totally ordered independently of the reference frame employed, may be defined as being the "Time" of the set of all events S. Furthermore, if S is topologically considered an (affine) four-dimensional manifold, a (continuous) coordinate function τ with a constant value on every instant ($\tau(I) = \text{const}$) and increasing in monotonic order with the total order (i.e. $\tau(I_1) < \tau(I_2)$ iff $I_1 \Pi I_2$) may be defined and serve as the time coordinate for the events in S. Finally, the intersection of S with a given instant I, that is, all the elements or events of S contained in I, constitutes a three-dimensional "space" (at that instant), which possesses a Euclidean metrical structure.

Newtonian space-time or spacetime, or for that matter any other pre-relativistic space-time-scheme, can thus be decomposed into "time" and "space" (or sequence of "spaces"), each space pertaining to a given coordinate value of the time.

The objection may be raised that the preceding analysis of the structure of pre-relativistic theories of space and time contained a vicious circle. For the decomposition of the man ifold into "time" and "space" was obtained by means of the relations of simultaneity and temporal precedence for the definition of which use had been made of the time coordinate. It should therefore be emphasized that these relations can be defined without the use of a time coordinate as shown, for example, by the causal theory of time, as proposed by Robb, Mehlberg or Grünbaum. To show that this, in fact, is possible our analysis of the structure of relativistic spacetime will be carried out in an intrinsic manner which is not based on the use of coordinates.

The special theory of relativity has its ultimate origin in Maxwell's introduction of the displacement current in his famous equations of the electromagnetic field. For without this term no electromagnetic waves would be possible and Maxwell's theory would be covariant under Galilean transformations.[15] This does not mean that the theory of relativity could not have been developed along other lines than it did historically. For it can be shown, for example, that the Lorentz transformations can be derived from the purely mechanical phenomenon of the variation of mass with velocity, combined with the principle of the conservation of linear momentum.

The introduction of the displacement current gave a more symmetric form not only to Maxwell's equations; to preserve the empirically confirmed covariance of the laws of the field, it led to the replacement of the Galilean transformation by the Lorentz transformation and consequently gave a more symmetric form also to the geometric spacetime representation, depicted in Fig.1. For what had to be done, for the physical situation under discussion, was to tilt the x'-axis relative to the x-axis in a position precisely symmetric with that of the t'-axis relative to the t-axis (taking c = 1). All the apparently "paradoxical" innovations of relativistic kinematics, like time dilatation or length contraction, have their ultimate origin in this "symmetrization" procedure, as shown by comparing Fig. 1 with Fig. 2. This "symmetrization" puts "space" and "time" on an equal footing; for not only, as was the case in the Galilean transformation (1), is x' a function of x and t, but now - and this is the meaning of this "tilting" - is t' a function of t and of x. "Expansion and duration do mutually embrace and comprehend each other," as John Locke has been quoted above. Or "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality," as Hermann Minkowski expressed it in his famous talk on "Space and Time".[16]

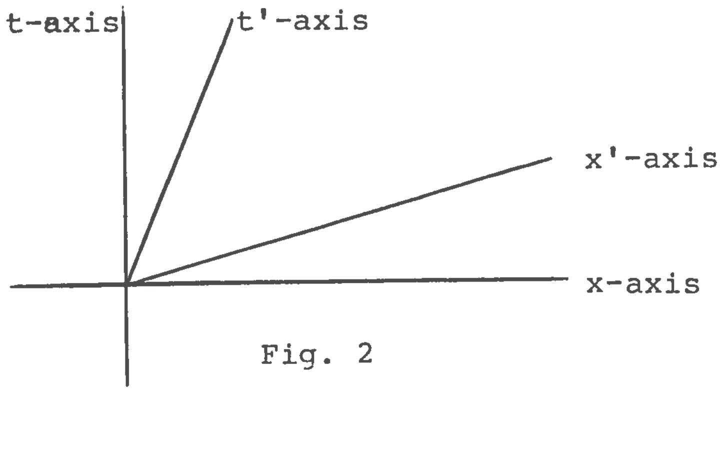

Fig. 2

Minkowski spacetime M is a four-dimensional real affine manifold of elements, called events and denoted by e (with or without subscripts). Being affine, M contains subsets, called straight lines, which satisfy the axioms of incidence, and hence of intersection and parallelism, of four-dimensional Euclidean geometry.

Ordered pairs of events (e_1, e_2) and (e_3, e_4), such that for some e_5, e_6 of M the quadrilaterals $e_1 e_2 e_5 e_6$ and $e_3 e_4 e_5 e_6$ are parallelograms (unless $e_1 = e_2$ and $e_3 = e_4$) are said to belong to the same class, defining the vector $\overrightarrow{e_1 e_2}$.

By introducing the usual vector addition and multiplication of vectors with scalars (reals) the set of all vectors becomes a four-dimensional real vector space on which an inner product can be defined as a real-valued symmetric function of vector pairs, linear in each member of the pair. In case both members are the same vector, the inner product is called the square of the vector. Two vectors are orthogonal if their inner product is zero. The inner product is to be non-degenerate in the sense that only the zero-vector (i.e. the vector formed by identical events)is orthogonal to all vectors. Finally, it is postulated that the sign (positive or negative) of the squares of three out of any four linearly independent and mutually orthogonal vectors is opposite to the sign of the square of the remaining fourth vector (i.e. the index of inertia is 1 or 3).

This completes the postulational definitions of Minkowskian spacetime M. The following definitions, non-postulational in nature, serve to clarify the structure of M.

A vector is <u>lightlike</u> if its square is zero; it is <u>timelike</u> if the sign of its square is opposite to that of the square of any vector orthogonal to it; it is <u>spacelike</u> if it is neither lightlike nor timelike. Consequently, relative to any event e Minkowskian spacetime decomposes into the following four classes of events :

(1) the "Here-Now" $H_e = \{e\}$ (i.e. the set containing only e itself).

(2) the "Light-cone" $L_e = \{e_1 \in M \mid e_1 \neq e$ and $\overrightarrow{ee_1}$ lightlike$\}$ (i.e. the set of all events e_1, other than e, such that the vector $\overrightarrow{ee_1}$ is lightlike).

(3) the "Timelike set" $T_e = \{e_1 \in M \mid e_1 \neq e$ and $\overrightarrow{ee_1}$ timelike$\}$

(4) the "Spacelike set" $S_e = \{e_1 \in M \mid e_1 \neq e$ and $\overrightarrow{ee_1}$ spacelike$\}$.

S_e is topologically connected but L_e is disconnected by e into L_e^+ and L_e^- and T_e is disconnected by e into two open lobes T_e^+ and T_e^-, the former two being, respectively, the topological boundaries of the latter two (with e excluded).

Two lobes $T_{e_1}^+$ and $T_{e_2}^+$ satisfying the condition that there exists an event e such that $T_e^+ \subset T_{e_1}^+ \cap T_{e_2}^+$ (i.e. such that the open lobe T_e^+ is contained in the set-theoretic intersection of $T_{e_1}^+$ and $T_{e_2}^+$), are said to be <u>co-directional</u>. This relation is reflexive, symmetric and transitive and hence an equivalence relation which generates two equivalence classes E^+ and E^-.

For any event e the <u>future</u> of e, denoted by F_e, is the topological closure (with e excluded) of that lobe among T_e^+ and T_e^- which belongs to E^+. Analogously, the <u>past</u> of e, denoted by P_e, is the topological closure (with e excluded) of the other lobe, belonging to E^-. (See Fig.3).

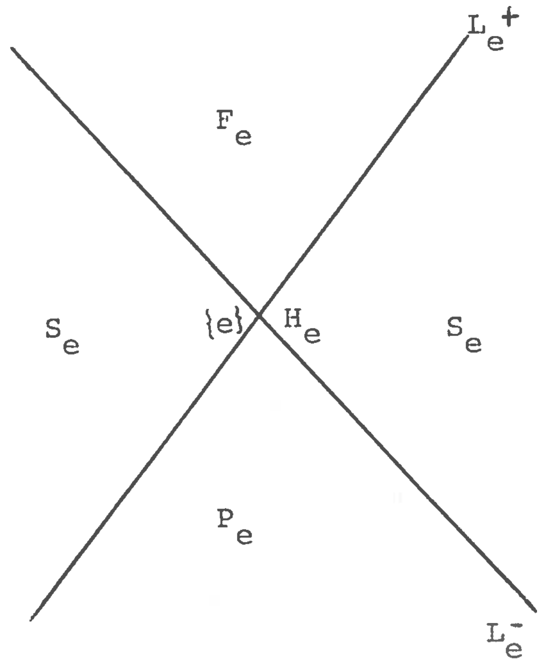

Fig.3

Thus, with respect to any event e Minkowskian spacetime decomposes into the six mutually disjoint components : H_e, L_e^+, L_e^-, F_e, P_e, and S_e. It should be noted that the association of E^+ with "future" and of E^- with "past" is (from our present point of view) merely a matter of pure convention. A physical meaning can be endowed to this association only by considerations transcending the purely spacetime-theoretical analysis, such as those referring to thermodynamics or cosmology.

Now, since $e_1 \in P_{e_2}$ implies $e_2 \in F_{e_1}$, which excludes $e_2 \in P_{e_1}$ and since $e_1 \in P_{e_2}$ and $e_2 \in P_{e_3}$ implies $e_1 \in P_{e_3}$, the relation between e_1 and e_2, expressed by $e_1 \in P_{e_2}$, is irreflexive, asymmetric and transitive and hence a partial ordering , whose domain is obviously confined to P_{e_2}. Furthermore, and most importantly, no simultaneity relation can be established which is an equivalence relation throughout Minkowskian spacetime. This impossibility reflects, of course, only the well-known relativity of simultaneity. It follows that no equivalence classes or "instants" exist and no (universal) "Time" can be defined. Although there are temporal relations, there is no "Time" in relativistic spacetime. This lack of a (total) chronological order lies at the root of the relativistic metrological effects, such as the relativity of temporal or spatial intervals.

In fact, the time interval t_{21} between any two events e_1 and e_2 has no definite value (but depends upon the choice of the time-axis); more precisely, if $e_2 \in F_{e_1}$ the value of t_{21} is any positive number; if $e_2 \in P_{e_1}$ the value of t_{21} is any negative number; if $e_2 \in S_{e_1}$ the value of t_{21} can be any real number (zero included). Nor has the spatial interval s_{21} between any two events e_1 and e_2 any definite value, but can be assigned any non-zero value and, if $e_2 \in F_{e_1}$ or $e_2 \in P_{e_1}$, the value zero as well.

To represent graphically these facts and all the other effects of relativistic kinematics Minkowski introduced, in his above-mentioned paper of 1909, what is known as the Minkowski diagram of relativistic spacetime, with its separate space- and time-axes for each individual inertial reference frame and its calibration curves. The thus established coordinates of events satisfy then the Lorentz transformation equations which for a four-dimensional spacetime (with reference frames in standard configuration) read :

$$x' = \gamma (x - vt) \quad y' = y \quad z' = z \quad t' = \gamma (t - \frac{v}{c^2} x) \quad (2)$$

where $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$ and v is the velocity of the frame with primed coordinates relative to the frame with the coordinates without primes.

It should be noted, however, that all our results remain valid also in geometrical representations that differ from the Minkowski diagram. An early alternative has been proposed in 1920 by the Düsseldorf industrialist Friedrich Paul Liesegang; it has the advantage of not requiring a calibration curve for the scale factor.[17] Another graphical representation of the Lorentz transformation in spacetime has been proposed in 1948 by Loedel[18] and independently rediscovered in 1955 by Amar[19], and still another in 1962 by Brehme.[20] Since these alternative representations gained little acceptance in the literature on relativity[21] I shall confine the following discussion of some philosophical problems concerning relativistic spacetime to the Minkowski diagram, although it would be instructive to employ also another kind of representation.

2. Some Philosophical Considerations

As mentioned already, any theory of space and time involves - in the sense of both originating from and giving rise to - physical as well as philosophical considerations. That Aristotle's theories of space and time constituted an integral part of his natural philosophy is well known to every student of ancient philosophy. Likewise, the Newtonian conceptions of space and time, although usually regarded as constituting the logical foundation of Newtonian physics and philosophy of nature, at least if formalized in an axiomatic system, are nonetheless also logical consequences of the latter, as shown e.g. by Newton's interpretation of his famous pail experiment as an argument for the existence of absolute space. That Newtonian physics, moreover, is strongly influenced by philosophical thought is well illustrated by Newton's belief in the ontological priority of "force", in contrast e.g. to Cartesian physics in which kinematical collision phenomena form the ultimate elements of physical processes.

Minkowskian spacetime is, of course, an outgrowth of Einstein's special theory as developed in 1905 in his famous article "On the Electrodynamics of Moving Bodies,"[22] while the main physical ideas, such as Einstein's novel definition of distant simultaneity, have been greatly influenced by Mach's philosophical positivism.

Space does not allow us to give even only a brief analysis of these interrelations between physics, philosophy and the construction and implications of the various theories of space and time.

The following discussions of philosophical issues of theories of space and time will therefore be confined to relativistic spacetime and its Minkowskian representation and, moreover, to only those aspects which are apt to further clarify its structure and significance.

The first problem of philosophical or methodological import to be raised in this context concerns the very notion of "event." In axiomatic formulations of relativistic spacetime it is taken as a primitive notion and interpreted as an actual or potential occurrence of pointlike character. According to Einstein, moreover, it is the ultimate element of physical reality - at least within the context of relativity. "Nicht der Raumpunkt, in dem etwas geschieht, nicht der Zeitpunkt, in dem etwas geschieht, hat physikalische Realität, sondern nur das Ereignis", he once said.[23]

Now it seems legitimate to ask whether this notion of "event", so basic for the construction of spacetime, does not presuppose the concepts of space and time, especially if it is conceived as an occurrence without spatial and temporal extension. In other words, can eventism, as professed by Einstein and even more emphatically by Alfred North Whitehead, provide consistently a foundation for the construction of relativistic spacetime without committing the error of a vicious circle ?

Two ways are apparently possible to resolve this problem. One may adopt the formal approach and regard the primitive notion of "event" as an uninterpreted concept, just like the concept of "point" in Hilbert's axiomatization of Euclidean geometry. It then has only a syntactical meaning, i.e. a meaning which is determined only by the condition that it has to satisfy the axioms in which it appears. This solution, however, would be unacceptable to those philosophers of science who insist that even a primitive notion in an axiomatized formulation of an empirical theory should be endowed with operational meaning. The other way of solving this problem consists in differentiating between a pre-scientific (qualitative) conception of extension (both in space and in time) and a scientific (quantitative) conception of space and time. This would agree, in fact, with the thesis that the language of physics is after all merely a refinement of everyday language, an idea which has been emphasized, though in a different context, by Niels Bohr. But, just like Bohr's point of view, this second way depends upon the adoption of a particular philosophical platform and hence may not seem acceptable to all philosophers of physics.

Having described, though for the sake of brevity not in a rigorously axiomatic manner,[24] how relativistic spacetime can be constructed, it is important to understand what it is not. It is not a physical entity if such is defined as being located in space and enduring in time. Unfortunately, however, the history of scientific thought abounds with misconceptions of this kind. Zenon of Elea already committed this error when in one of his famous paradoxes he said that space, if it exists, must exist in space, a statement which obviously implies an infinite regression. Temporal analogues are expressions like "time flows" or "time passes fast" etc. which imply the existence of a "hypertime" to measure the velocity of this "flow." In the context of pre-relativistic spacetime such statements may in general be innocuous, used as a *façon de parler* without leading to philosophical misconceptions.Not so, however, if used in relativistic spacetime theories.

In his lecture on "Man and the Universe" Sir James Hopwood Jeans, after having described how in the theory of relativity space and time are so closely interwoven that it is impossible to divide up time into past, present and future in an absolute manner, compares spacetime with a piece of tapestry which "cannot consistently be divided into those parts which are already woven amd those which are still to be woven." Therefore, he concluded, "the shortest cut to logical consistency was to suppose that the tapestry is already woven throughout its full extent, both in space and time, so that the whole picture exists, although we only become conscious of it bit by bit - like separate flies crawling over a tapestry. It is meaningless to speak of the parts which are yet to come - all we can speak of are the parts to which we are yet to come. And it is futile to speak of trying to alter these, because, although they may be yet to come for us, they may already have come for others."[25]

To illustrate the situation discussed by Jeans, let us assume that an observer A, located at the origin of his inertial system S, moves relative to an observer A' and his inertial reference frame S' with a velocity $v = \frac{4}{5} c$ in uniform motion and standard configuration. Let e be an event which in S has the coordinates $x_e = 50\,c$ and $t_e = 10$ (with appropriate units of length and time). A simple calculation by means of the Lorentz transformation $t' = \gamma (t - \frac{v}{c^2} x)$ yields for the time coordinate of the event e in S' the value $t'_e = -50$. The event e, as we see, has "already come" for B although it has "yet to come" for A. (See Fig. 4).

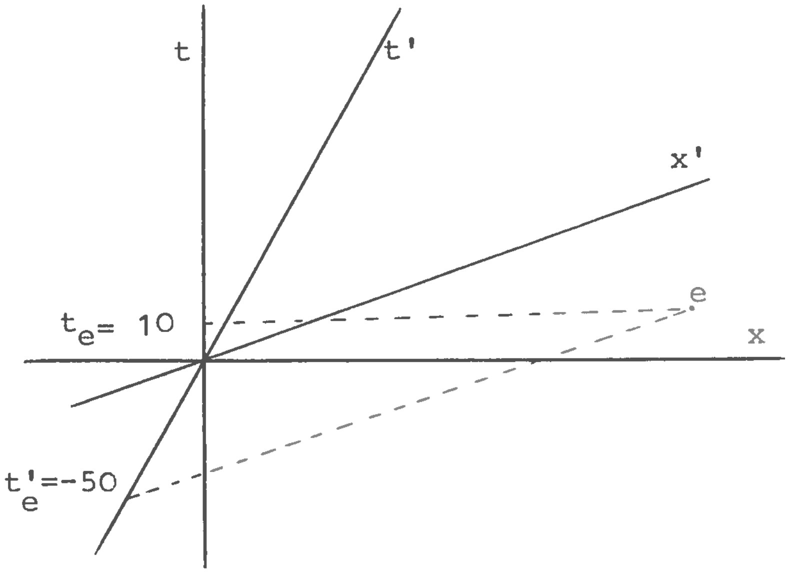

Fig. 4

By comparing spacetime with a piece of tapestry, parts of which "are already woven" while other parts are "still to be woven", Jeans conceived spacetime as a physical object, which led him, in fact, to introduce a "meta-time"; for the temporal adverbs "already" and "still" , as just quoted, cannot be part of the spacetime under discussion for they say something about it, as if from the vantage of an outside observer. Furthermore, Jeans' statement that "it is futile to speak of trying to alter" the event seems to suggest that, if e is for example a car accident involving observer A, it is futile for observer A' to warn A of the impending danger so as to prevent it, for the accident has already occurred for A', and hence can under no circumstances be prevented. This way of reasoning ignores that e cannot lie in the future of A' ($e \notin F_{A'}$) nor in his past ($e \notin P_{A'}$) when the two observers momentarily coincide, as shown in the Figure.

Since the event e lies in the spacelike sets S_A and S_B, neither can B, at the moment of his coincidence with A, have any information about e, nor will A ever experience himself the event e. No "warning" could, or had to, be given.

Let us now generalize the situation and assume that an event e "is yet to come" for an observer A, allowing that e may lie also in the future of A ($e \in F_A$). It can easily be seen that there always exists an observer B for whom, when being in the present of A ($t_B = 0$), the event e has already occurred.(See Fig. 5).

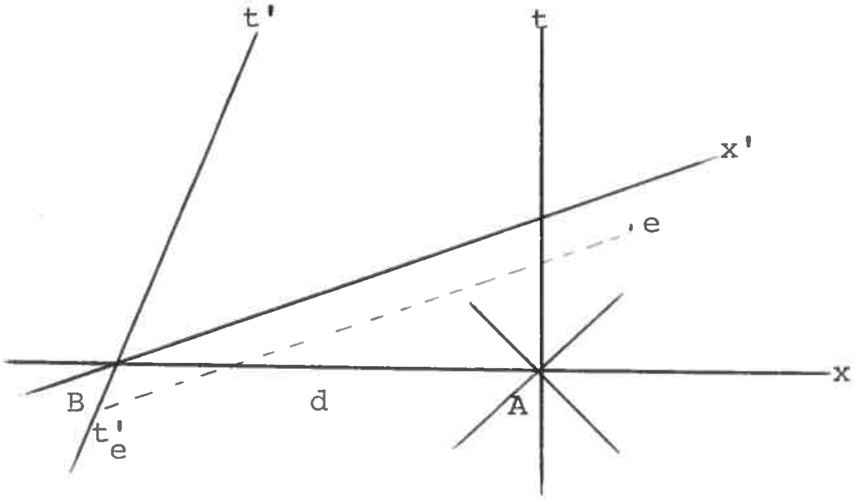

Fig. 5

The Lorentz time transformation for the situation under discussion obviously reads :

$$t'_e = \gamma (t_e - \frac{v}{c^2} [x_e + d]) \qquad (3)$$

where v is the velocity of B relative to A and d is the spatial interval between A and B at the time $t_A = t_B = 0$.

Equation (3) shows that for arbitrary x_e and positive t_e it is always possible to find a value of d and a (positive) value of v such that t'_e is negative.

Considerations of such a physical situation prompted some authors to declare that relativistic spacetime implies strict determinism [26] in the sense that every event, whenever it occurs, has been determined in advance. They argue as follows. Event e has already occurred for observer B when he is in the present of observer A, even if B does not know anything about e. In any case, event e must therefore be determined. Since B is in the present of A, the event e must also be determined for A although it is still to come for him or in his future. Since for every event, where ever and when ever it occurs, such observers A and B must exist, every event must be determined in advance.

This thesis that relativistic spacetime does not admit an "open" or "creative" future has been voiced by philosophers and physicists of world-wide reputation. Ernst Cassirer declared that "the direction into the past and that into the future are distinguished from each other ... by nothing more than are the 'plus' and 'minus' directions in space, which we can determine by arbitrary definition. There remains only the 'absolute world' of Minkowski; the world of physics changes from a process in a three-dimensional world into a being in this four-dimensional world."[27] In the same year Sir Arthur Stanley Eddington wrote : "Events do not happen, they are just there."[28] According to Hermann Weyl "The objective world simply is, it does not happen."[29]

Returning to Fig. 5 and the discussion illustrated by it, we can easily see that, again, the event e does not lie in the past of B ($e \notin P_B$) and hence cannot be known by him at the moment under consideration. Nor can A receive (at $t = 0$) any information about the event from B, even if B would possess knowledge about the event. Hence, even if e were determined, it would not be predictable for A.

In pre-relativistic physics determinism has often been identified with predictability. This is how Laplace defined determinism : "Une intelligence qui, pour un instant donné, connaîtrait toutes les forces dont la nature est animée et la situation respective des etres qui la composent ... rien ne serait incertain pour elle, et l'avenir, comme le passé, serait présent à ses yeux."[30] Translating freely these words we may say that complete knowledge of the initial or boundary conditions on a spacelike hypersurface ("pour un instant donné") makes it possible to predict the state of the universe at any moment in the future as well as to retrodict the state at any moment in the past. Laplacian determinism, though meaningful in pre-relativistic theories of space and time, ceases to be so in relativistic spacetime theories. On the assumption that information can be communicated only by physical means and not, e.g., by extra-sensory perception (ESP), the premiss of Laplace's statement, namely the fact that an intellect or observer could obtain knowledge of what happens on a spacelike hypersurface, cannot be satisfied. But that such knowledge would be required to predict any event in the future of an observer A ($e \in F_A$) follows from the easily provable fact that the intersection between P_e and S_A is not empty and any event in this intersection is apt to affect e.

A claim, similar to that illustrated by Fig. 5, but not referring to determinism but to reality, has been made by Hilary Putnam[31] in 1967. Instead of starting the argument, as Putnam does, with the assertion that for "the man in the street" what is present is real, let me quote Arthur Norman Prior, one of the founders of modern tense logic. At the First Conference of the International Society for the Study of Time, which convened 1969 in Oberwolfach, Germany, Prior spoke about the notion of the "present" and said : "Before discussing the notion of the present, I want to discuss the notion of the real. These two notions are closely connected; indeed on my view they are one and the same concept, and the present simply <u>is</u> the real considered in relation to two particular species of unreality, namely the past and the future."[32] Starting from such an assumption, which may be called a "temporalistic conception of reality", Putnam tries to prove that the structure of relativistic spacetime implies that all events which are yet to come for an observer are already real for him. Let e be such an event for an observer A (see Fig. 6). There exists an observer B who is

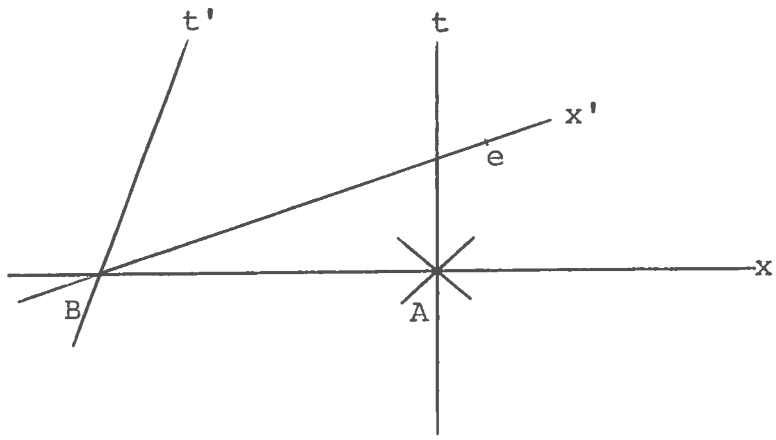

Fig. 6

in the present of A and for whom the event e is in his present. According to the temporalistic conception of reality e is real for B and B is real for A. Now, it seems plausible to assume, says Putnam, that the relation " x is real for y" is a transitive relation, for otherwise "being real" would have to be relativized, like intersystemic simultaneity, an assumption which contradicts the meaning of reality, namely that the reality of a thing, like its existence, is intrinsic to it and hence independent of any observer or his state of motion. But if this is the case, then clearly the event e is real also for A. In the case e lies outside the lightcone L_A of A, the observer B may even be taken as momentarily coinciding with observer A.

It will have been noted that in the preceding arguments for determinism or reality of future events always three events are involved, of which one, say e_1, forms with each of the other two, say e_2 and e_3, a spacelike vector. In fact, it is easy to prove that for any two given events there always exists a third event which stands in a spacelike relation to each of the former, since for any given e_2 and e_3 the intersection $S_{e_2} \cap S_{e_3}$ is not empty. It follows, as we have seen above, that both t_{21} and t_{31} can be assigned arbitrary values. But as emphasized before, in relativistic spacetime there is no universal chronological order so that no inference can be drawn from t_{21} and t_{31} to t_{23}. But this is precisely what has been done in the preceding arguments.

Strict determinism, it is generally claimed, is incompatible with a belief in free will, moral responsibility or what is often, but erroneously, called "man's power to change the future," - erroneously, because the "future" is what factually will be the case and not what counterfactually could have been the case. Most religions, and among them the Judeo-Christian tradition, believe in an "open" future, not predetermined and hence unpredictable. But if there is a genuine becoming in the world, so it is said, there is also a genuine becoming in God's knowledge of the world : at any given moment God knows the past and the present but not what will be the future, for otherwise, it is argued, the future could not be "open." According to this view, often called the doctrine of "temporalistic theism", such a restriction of God's knowledge is not incompatible with the thesis of God's omniscience, for the latter refers only to actualities and not necessarily to all eventualities. Temporal theism has been professed, for example, by Alfred North Whitehead and William A. Christian.[33]

Let us now consider, from this point of view, two events e_1 and e_2, separated by a spacelike interval in relativistic spacetime. As explained above, e_1 and e_2 may be simultaneous in one reference frame, e_1 may temporally precede e_2 in another reference frame, and e_2 may precede e_1 in a third reference frame. God knows e_1 and/or e_2 as soon as they actualize. If we now ask whether God's knowledge of e_1 is simultaneous with His knowledge of e_2, or whether He knows e_1 before He knows e_2 or vice versa, any answer would single out a reference frame which, since employed by God, would be preferential to all other reference frames. Moreover, if as Whitehead contended, "the perfected actuality (of God) passes back into the temporal world, and qualifies this world so that each temporal actuality includes it as an immediate fact of relevant experience"[34], God's immanence in the world would assign to the reference frame which corresponds with the order of God's cognitive processes of getting knowledge of the events a physically distinguished state, in contrast to the theory of relativity. That theologians are indeed concerned with such a problem transpires for instance from the writings of J.T.Wilcox.[35]

A number of solutions of this highly abstract problem have been proposed, but none of them seems satisfactory. It may well be that temporalistic theism and the theory of relativity are incompatible.

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