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SELF-ORGANIZATION OF TRADE NETWORKS IN AN ECONOMY WITH IMPERFECT INFRASTRUCTURE

by

Sergei Guriev
Visiting Scholar
Department of Economics
Massachusetts Institute of Technology
Cambridge, Massachusetts USA

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Self-Organization of Trade Networks in an Economy with Imperfect Infrastructure

Sergei Guriev* and Margarita Shakhova
Computing Center, Russian Academy of Science

Abstract

A multiagent model is proposed for analysis of self-organization of trade networks in a transition economy. It is shown that while in the case of almost perfect infrastructure the system quickly converges to a near-competitive equilibrium, more imperfect infrastructure results in significant oscillations of prices and of the structure of the trade networks, bursts of shortages and longer chains of traders; the system may converge to multiple equilibria including the suboptimal ones. Emergence of trader's market strategies such as stabilizing wholesale traders and destabilizing speculators is discovered. The model is studied both analytically and via computer simulations.

1 Introduction

The problem of self-organization of trade structures is of a special interest for Russian economy. It is known that the trade sector plays a special part in an economy during transition to market economy. The trade sector adsorbs both significant capital investments and skilled human resources. Some economists believe that the accelerated development of the trade sector is favorable for the economy while some others argue that the trade is growing at the expense of producers distracting the economy's resources. Anyway, both acknowledge that the intermediaries are outdoing the rest in a decentralized transition economy. Probably, due to high transaction costs, poor infrastructure and high uncertainty, the trade profits are very high, and this attracts new resources and brings about further development of the trade sector. Note that unlike other transition economies, in Russian economy the private trade sector is being built from the scratch in the absence of developed trade infrastructure.

In Section 2 the general setting and models of behavior of individual agents are introduced, in Section 3 results of analytical study and simulations are discussed. Section 4 contains some concluding remarks and directions of further studies. Detailed survey of related literature is provided in [1].

2 The model

2.1 General setting

We consider interaction of economic agents of three types: consumers, producers and traders in a distributed market of homogeneous good. Each type is described by a set of parameters and rules of behavior in the market. Conventionally for the emergent computations, we tend to simplify the rules of decision-making for individual agents, and pay most attention to emergent properties of the whole system.

Denote sets of consumers, producers and traders by $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$, correspondingly. Assume that $\mathcal{P}$ and $\mathcal{T}$ are finite. Producers are pure sellers and their behavior is exogenous to the system. Consumers are pure buyers. Traders can either buy or sell. Buyers are indexed by $i \in \mathcal{C} \cup \mathcal{T}$, and sellers — by $j \in \mathcal{P} \cup \mathcal{T}$. Buyers can buy only one unit of good per transaction. For every pair (i,j) from $\mathcal{C} \cup \mathcal{T} \times \mathcal{P} \cup \mathcal{T}$ a nonnegative number r_{ij} is defined, which we will refer to as *trade distance*. The trade distance is average time that buyer i has to spend to buy a unit of good from seller j. In an economy with developed infrastructure the time spent on purchasing is small and usually is not taken into account. However, in an economy with imperfect infrastructure such as contemporary Russian economy, the time for gathering current information about the seller, reaching the seller and physically transporting the good is significant, which, as shown below, may be crucial for macroscopic dynamical properties of trade networks.

Generally speaking, time spent on transaction is a realization of Poisson stochastic process with mean equal to trade distance; the processes are independent for different buyer-seller pairs. However, simulations have proved that we may use mean flows of good instead of stochastic ones.

Every buyer has buying preferences α_{ij} i. e. if buyer i wants to buy a unit of good, he will go to seller j with probability α_{ij}. Naturally, we require $\alpha_{ij} \geq 0$, $\sum_j \alpha_{ij} = 1$ and $\alpha_{ii} = 0$, $i \in \mathcal{T}$.

Sellers are described by their selling prices p_j and probabilities of availability of the good β_j. We assume that seller does not distinguish buyers. So to any buyer who comes to buy a unit of good the trader sells the good at the price p_i with probability β_i, and refuses to sell with probability $1-\beta_i$. β_i is the trader's control variable and essentially determines the flow of sales.

Producers are passive suppliers of good. Their prices p_j and probabilities of absence of shortage β_j, $j \in \mathcal{P}$ are constant parameters in the model.

In order to describe the process of trade we apply the framework of Bertrand competition:

- under given prices buyers decide from which sellers and how much to buy, so the trade links are established;

- foreseeing buyers' response, sellers set prices in order to maximize their profits.

In this framework, equilibrium prices are given by Nash equilibrium. The corresponding game is defined in a normal form in Section 3.

Our model is a dynamical implementation of Bertrand competition with trade distances. Agents' behavior is described by *fast* and *slow* variables. Fast variables are set by agent at every moment of time in order to maintain his material or financial balance and may change discontinuously over time. Slow variables are continuously adjusted by agent to current optimizers of his objective function. For buyers and sellers fast variables determine how much to buy and how much to sell, correspondingly, and slow variables determine from whom to buy (buying preferences) and at which price to sell, correspondingly. We assume that hierarchy of time is as follows: fast variables (quantities) adjust immediately, buyers' slow variables (buying preferences) adjust more slowly, sellers' slow variables (prices) change even more slowly. So the whole setting may be referred to as Bertrand-Nash one: the sellers set prices and observe buyers' response, change prices and observe response to new prices etc. As sellers do not cooperate, their attempts to maximize profit by changing price represent Nash-style tatonnement. Due to instantaneous adjustment of fast variables all required balances are maintained at every moment of time.

Agents make decisions on the basis of information available to them. Every buyer knows trade distances between all sellers and himself as well as prices and levels of shortage for all sellers. Every seller knows demand for his good at recent moments of time.

As the set of buyers is in general discrete, the variations of the sellers' demand over time may be too large. In order to emulate continuity we consider the adaptation of the buying preferences with some finite rate rather than the instantaneous switching. In this case the agent adjusts his buying preferences trying to attain the desired ones, with the adjustment rate μ_i. So at every moment t current buying preferences $\alpha_{ij}(t)$ may be different from the desired ones $\alpha^*_{ij}(t)$:

$$\alpha_{ij}(t+\Delta) = \alpha_{ij}(t) + \mu_i \Delta(\alpha^*_{ij}(t) - \alpha_{ij}(t)) \ . \tag{1}$$

Here Δ is time step. We will assume Δ to be sufficiently small in comparison with $1/\mu_i$ so that $\mu_i\Delta \leq 1$. The value $1/(\mu_i\Delta)$ shows the number of steps required for the consumer to adapt to the external changes.

Now let us consider the behavior of consumers and traders. Consumer is described by following individual parameters: wage per unit of time s_i and average time of consumption of a unit of good τ_i. We assume that he sets the fast variable in order to maintain financial balance. In [1] it is shown that expected flow of consumption in this case is:[1]

$$U_i = \frac{\sum \alpha_{ij}\beta_j}{\sum \alpha_{ij}(\beta_j\tau_i + \beta_j p_j/s_i + r_{ij})} \ . \tag{2}$$

In [1], we find α^*_{ij} that maximizes this functional over the simplex $\{\vec{\alpha}_i : \sum \alpha_{ij} = 1, \alpha_{ij} \geq 0\}$. In the generic case when $p_j/(s_i) + r_{ij}/\beta_j$ are all different for different j, the consumer will tend to select only one seller: $\alpha^*_i = \vec{e}_{j^*}$, where $\vec{e}_{j^*}$ is j^*-th unit coordinate vector (so that $\alpha^*_{ij} = 0$ for $j \neq j^*$ and $\alpha^*_{ij^*} = 1$), and

$$j^* = \arg\min_j \frac{p_j}{s_i} + \frac{r_{ij}}{\beta_j} \ . \tag{3}$$

In the non-generic case, when there exist several such j^*, we will assume that the consumer shares his demand between these evenly $\vec{\alpha}^*_i = \sum_{J^*} \vec{e}_{j^*}/|J^*|$.

[1] In this and further sections, the summation index is j if it is omitted: $j \in \mathcal{P} \cup \mathcal{T}$, $j \neq i$.

Unlike consumers who maintain the financial balance and maximize the inflow of good, traders tend to maintain the material balance (expected difference of sales and purchases is equal to zero) and to maximize profit (expected financial surplus per unit of time).As a seller, the trader receives the Poisson flow of buyers with the rate λ_i. In [1] we show that the condition of material balance is equivalent to the following relationship between trader's fast variables Λ_i (the flow of purchases) and β_i:

$$1/\Lambda_i = \sum \alpha_{ij}(\beta_j/(\beta_i\lambda_i) - r_{ij}) \ . \tag{4}$$

Note that $1/\Lambda_i$ must be nonnegative.

As shown in [1] the trader's expected profit equals

$$\Pi_i = \beta_i\lambda_i \left(p_i - \frac{\sum \alpha_{ij}(\beta_j p_j)}{\sum \alpha_{ij}\beta_j} \right) \ . \tag{5}$$

To find the fast controls first we shall maximize the functional (5) choosing $\beta_i \in [0,1]$ that satisfies the condition of non-negativity of (4).

We shall assume that the price p_i is such that the profitability condition holds:

$$p_i - \sum \alpha_{ij}(\beta_j p_j)/\sum \alpha_{ij}\beta_j \geq 0 \ . \tag{6}$$

Hence trader wants to increase β_i as much as allowed by conditions $\beta_i \leq 1$ and that of non-negativity of (4). There can be three cases.

1. *Shortage.* The demand is too high: $\lambda_i > \sum \alpha_{ij}\beta_j/\sum \alpha_{ij}r_{ij}$. In this case the trader can not serve all his demand and has to refuse to sell to some of his buyers $\beta < 1$. The share of non-satisfied demand is determined by making (4) equal to zero (i.e. trader spends zero time in the state i): $\beta_i = \frac{1}{\lambda_i}\sum \alpha_{ij}\beta_j/\sum \alpha_{ij}r_{ij} < 1$ and $1/\Lambda_i = 0$.

2. *No shortage.* The demand is sufficiently low: $\lambda_i < \sum \alpha_{ij}\beta_j/\sum \alpha_{ij}r_{ij}$, so that the trader can satisfy it completely, $\beta_i = 1$ and spends some time in the free state i: $\beta = 1$ and $1/\Lambda_i = \sum \alpha_{ij}(\beta_j/\lambda_i - r_{ij}) > 0$.

3. *Edge of shortage.* The demand is exactly equal to maximum possible supply under given slow variables: $\lambda_i = \sum \alpha_{ij}\beta_j/\sum \alpha_{ij}r_{ij}$, so that both $\beta_i = 1$ and $1/\Lambda_i = 0$.

Now we shall describe the adjustment of the buying preferences to the desired ones. To find the latter the trader solves the problem of maximization of the functional

$$\Pi_i = \frac{\sum \alpha_{ij}\beta_j(p_i - p_j)}{\max\{\sum \alpha_{ij}\beta_j/\lambda_i, \sum \alpha_{ij}r_{ij}\}} \tag{7}$$

by choosing $\alpha_{ij} \geq 0$, $\sum \alpha_{ij} = 1$.

This optimization problem is solved in [1]. The solution depends significantly on the magnitude of demand λ_i. If it is low enough (i. e. in the first case) then the trader maximizes profit per unit of good $j^* = \arg\max_j \lambda_i(p_i - p_j)$ and buys from a remote seller with the lowest selling price. In the second case he maximizes profit per unit of time $j^* = \arg\max_j p_i - p_j - r_{ij}/\beta_j$ and buys from some closer seller with a higher price. The intermediate situation is also possible in which the seller with the lowest price is too far away and buying only from him the trader would not be able to satisfy his demand, and vice versa the seller that provides maximum profit per unit of time is too close and buying from him the trader would be able to serve more buyers than he has. In this situation the trader diversifies his purchases and buys some amount from a remote seller at a lower price and the rest from a closer seller at a higher price in order to satisfy his demand exactly. In [1] it is shown that *all three situations are generic.*

To achieve higher profit, the trader can also change his selling price p_i. If the trader knew all internal parameters of his buyers and had unlimited computation capacities, he would be able to calculate the dependence of $\lambda_i(p_i)$ exactly. However, a more realistic assumption is that the trader's capabilities to obtain, store and process information are limited, and in forecasting his demand function the trader uses only his observations of the demand in the past. We assume that after every adjustment of price the trader keeps the price constant for some time Δ_i and observes what happens. He believes that average demand per unit of time between two subsequent adjustments of price $\bar{\lambda}_i(t) = \Delta_i^{-1} \int_t^{t+\Delta_i} \lambda_i(\xi)d\xi$ is function of his price $p_i(t)$ during this period of time $[t, t+\Delta_i]$. Hence, using historical data on demand, the trader can estimate (locally) the derivative of demand by price

$$\frac{\partial \lambda_i}{\partial p_i}(t+\Delta_i) = F_i\left(p_i(t), \bar{\lambda}_i(t), p_i(t-\Delta_i), \bar{\lambda}_i(t-\Delta_i), \ldots\right) .$$

Here F_i is a function that gives forecast for demand sensitivity to price by past values of demand and price. E.g. $F_i = (p_i(t)-p_i(t-\Delta_i))/(\bar{\lambda}_i(t)-\bar{\lambda}_i(t-\Delta_i))$.

We require that the relative change of price be bounded by $-\varepsilon_-^i\Delta_i$ and $\varepsilon_+^i\Delta_i$ to provide the continuity of price over time at sufficiently small Δ_i (ε_-^i and ε_+^i are internal parameters of the i-th trader's). The matter is that trader's demand and therefore profit depend upon not only his behavior but also upon other traders'. This is why if we allow to change the price discontinuously in order to achieve the desired maximizer of profit instantaneously, the system would have oscillations of high magnitude or, in generic case, chaotic behavior caused by interrelationships and imperfect information of traders. If the trader has shortage at this moment of time $\beta_i < 1$, he increases price to $(1 + \varepsilon_+^i\Delta_i)p_i(t)$, and if he has zero demand, he decreases price to $(1 - \varepsilon_-^i\Delta_i)p_i(t)$.

Also, to make the system more robust we let the moments for re-evaluation of price be stochastic rather than deterministic. We assume that re-evaluation moments arrive according to Poisson process with rate $1/\Delta_i$ (as in [3]). This removes artificial coordination of traders' decision-making and eliminates the oscillations that are caused by specifics of simulation methods rather than by the properties of the system itself.

3 Dynamics of trade networks

3.1 Equilibria and oscillations

We have defined the dynamical system, the current state in which is given by the set of the slow variables for all agents: α_{ij}, p_j. In this section we shall study the properties of the whole trade network. We will consider the state of the system to be an equilibrium if all buying preferences are optimal and all traders' prices are local maximizers of their profit functions. The profit functions $\Pi_i(p_i)$ are obtained under given other sellers' prices from (7) with demands and buying preferences determined by buyers. Note that by definition, in equilibrium $\beta_i = 1$ for all $i \in \mathcal{T}$, as if a trader has shortage, his profit function $\Pi_i(p_i)$ is increasing.

Assume that distribution of consumers in space and by wages is such that every trader's profit function is concave for all prices of other traders given. Then a state is an equilibrium if and only if it is a Nash equilibrium in the following game: the set of players is $\mathcal{T}$, their strategies are prices p_i and their payoff functions are the profit functions $\Pi_i(p_i)$ defined above. In this game

Figure 1: Non-concave profit function in case of clustered distribution of consumers.

Nash equilibrium exists if the wages and trade distances are bounded and profit functions are concave (see proof in [1]).

If a trader i has shortage $\beta_i < 1$, his profit function $\Pi_i(p_i)$ is linearly increasing and concave. But if a trader has no shortage $\beta_i = 1$ then profit functions need not to be concave. It depends upon distribution of consumers. In this case the profit received from selling to a single consumer is an increasing fraction-linear function until the consumer moves another seller and the profit function discontinuously falls down to zero. If a trader increases price some of his previous consumers will tend to buy from different sellers. The trader makes more profit on consumers who still buy from him but he loses all profit from the consumers gone. If distribution of consumers is uniform (approximately same number of consumers at every level of wage in every point of the metric space) then with increase of price the profit first grows slowly and then falls slowly and may be concave. But if the distribution is clustered like in Fig.1 then after the trader loses a whole cluster of consumers due to infinitesimal price increase, his profit falls by finite quantity as increase in profit from remaining customers is infinitesimal. Further price increase contributes continuous increase in profit until the trader loses next cluster. In this case the profit function has several local maxima and is not concave so that the Nash equilibrium may not exist.

However, the equilibrium in the dynamical system considered may still exist in the absence of Nash equilibrium, moreover, there may be several equilibria. This can lead to persistent oscillations in the system. As the local maxima of a trader's profit function depend upon other traders' prices, the change of price of trader i may make trader j to switch from seeking one local maximum to another one, consequently change of price of trader j will influence profit function of i and make the latter to change his price again etc.

The other source of instability contributed by singularities in distribution of consumers is caused by sudden shortages. This danger is significant when consumers change their buying preferences too fast (high μ_i). If a large group of consumers has the same location and wage then a small change of trader i's price may force them to go to another trader j. If the group is large enough, trader j that used to have no shortage before, will have shortage now, so his attractiveness to consumers will fall abruptly by finite quantity. Then the whole group of consumers will go back to trader i and create shortage there etc. Note that if consumers' μ_i is small enough the shortage occurred will not be large and the trader will have time to overcome it by increasing his price so the trader considers his profit function to be continuous. The situation becomes more dramatic with the worsening of the infrastructure as the small changes in prices now generate comparatively high levels of shortage.

3.2 Impact of imperfect infrastructure

Thus the average time spent on buying a unit of good $q_i = \sum \alpha_{ij} r_{ij} / \sum \alpha_{ij} \beta_j$ is very important for both consumers and traders. The expression contains both β_j that are determined as a result of interaction of agents and trade distances r_{ij} that are parameters. The greater r_{ij}, the more time buyers spend on buying, so it is reasonable to consider r_{ij} as a measure of imperfection of infrastructure. In order to study impact of imperfection of infrastructure, we will compare systems in which all trade distances differ in ρ times, i.e. we suppose that the trade distance matrix is proportional to some given matrix $r_{ij} = \rho R_{ij}$ and will study dependence of dynamical properties on the coefficient ρ with all other parameters fixed.

If $\rho \to 0$, then there is near perfect infrastructure, price differentials and trade profits are also small and there are no shortages. Indeed, if $\sup s_i < \infty$ then demand is bounded although increasing at $\rho \to 0$, therefore β_j are at least separated from zero and $q_i \to 0$. Hence, $q_i \lambda_i \to 0$, and there are

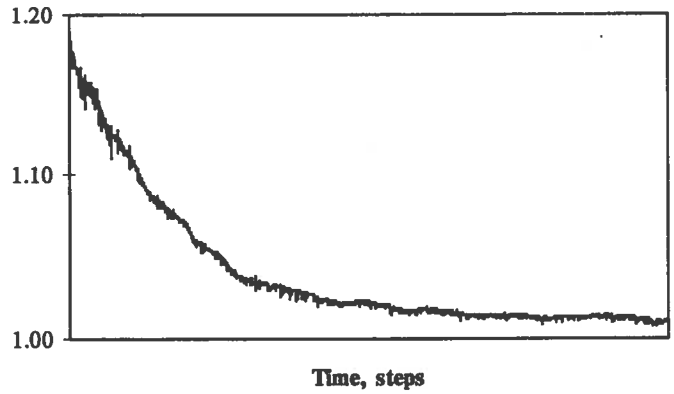

Figure 2: Convergence to equilibrium under near-perfect infrastructure. The graph shows evolution of average traders' price over time. All producers' prices are equal to 1.

no shortages $\beta_i = 1$. In this case system quickly converges to equilibrium without oscillations.

Therefore if ρ is sufficiently small, either consumer will buy directly from producer or consumer will buy from a trader who will buy from producer, and there can not be any chain of traders serving consumers. When ρ increases, traders' prices grow no faster than linearly with ρ. Indeed, for any j if $p_j > p_k + (s_i)(r_{ij} - r_{ik})$, $k \in \mathcal{P}$ then consumer i will buy from producer k. If $\sup s_i < \infty$ then traders will lose all their demand when prices grow faster than linearly with ρ. Therefore in case of non-trivial trade network prices grow not faster than linearly with ρ and consumers' demand falls as $a/(\rho + b)$. Hence quantities $q_i\lambda_i \sim a\rho/(\rho + b)$ increase with ρ. The coefficients a, b are determined by relative trade distances R_{ij} (the network structure), real wages s_i/p_j, $i \in \mathcal{C}$, $j \in \mathcal{P}$ and the consumption rate $1/\tau_i$.Thus, worsening infrastructure results in qualitative change in the self-organization processes. We can see that the trade profit depends on ρ non-smoothly.

When ρ is small all traders have no shortage and no long chains of traders exist. The system quickly converges to near-perfect equilibrium and traders' profits are small (Figure 2). However when ρ grows large enough the phase transition takes place: as quantities $q_i\lambda_i$ grow the shortages become more likely. There is no shortage in equilibrium, but one should distinguish equi-

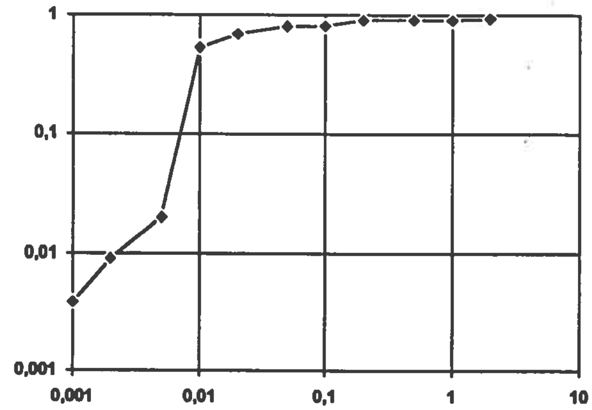

Figure 3: Phase transition because of worsening infrastructure. The graph shows dependence of all traders' profit Π on imperfection of infrastructure ρ in logarithm scale.

libria with $\beta_i = 1$, $1/\Lambda_i > 0$ and $\beta_i = 1$, $1/\Lambda_i = 0$. The former is more likely to happen at smaller ρ and the latter is more typical for greater ρ. The latter corresponds to the case when a trader buys from two sellers; unlike the former, it is an equilibrium at the edge of shortage. A small change of other traders' behavior may make the trader fall into shortage. One should mention that once caught in the shortage trap, the system can not rapidly get out: every trader tries to get rid of shortage but if all his counterparts have shortages and producers are too far, he simply does not have time to satisfy all his demand. So what happens is the bursts of shortages that traders slowly take over (Figure 4). Note that in the phase transition $q_i\lambda_i \geq 1$ so that traders buy from each other and longer chains of traders do exist. This leads to an abrupt increase in trade profits as can be seen in Fig.3. In this case trade hierarchies emerge due to imperfect infrastructure rather than due to economy of scale that wholesale traders possess if the triangle inequality is violated.

Further worsening of infrastructure gives rise to inability of traders to serve all consumers, so consumers with low s_i (poor consumers) will prefer to buy directly from producers,[2] and traders can only satisfy the demand of

[2]In Russia, about USD 11 bln. worth consumer goods are imported annually by individuals which accounts for tens percents of overall Russian import.

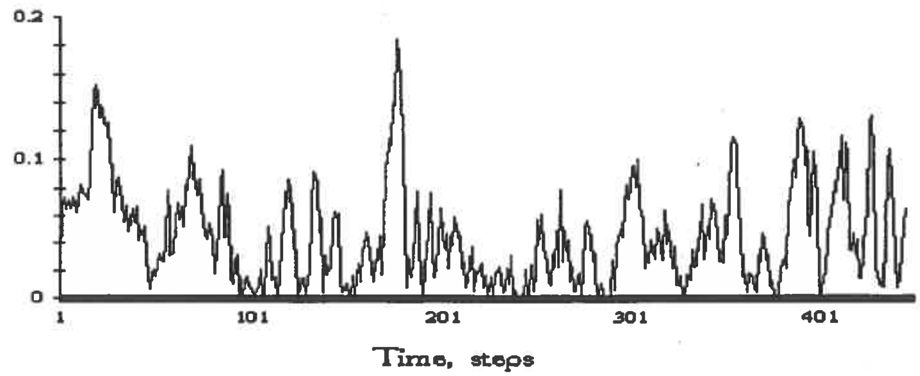

Figure 4: Bursts of shortages at the critical value of parameter ρ. The graph shows time series of average $1 - \beta_i$, $i \in \mathcal{T}$.

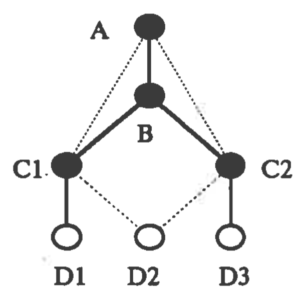

Figure 5: A Multi-level structure.

upper segment of consumers that buy at prices much higher than original prices of producers.

3.3 Emergence of trade structures

The two most interesting types of the behavior emerging may be referred to as hierarchical stabilizing and destabilizing speculation. The former corresponds for supercritical values of ρ (i. e. after the phase transition), the latter is typical for the critical situation.

The former case corresponds to the multilevel system as in Figure 5. Consumers are located at the lowest level D. They buy from the sellers at the

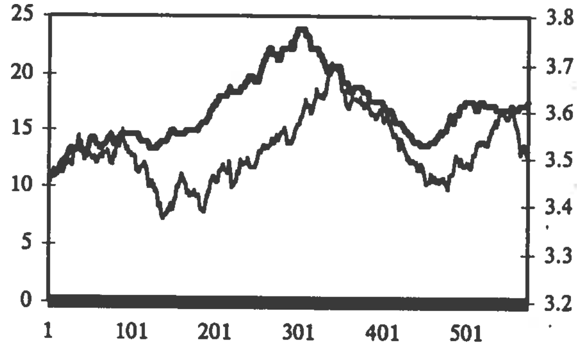

Figure 6: Price destabilizig speculator: a fragment of long-run price oscillations. The graph shows time series of the speculator's price (bold line, left scale) and average price of other traders (right scale). Average time of price re-evaluation is 5 units of time.

level C while they prefer to buy from the wholesale seller B because of having shortage. The seller B is buying from the producer A. For μ_i sufficiently high it will be a persistent oscillations in the demand for the traders' C1 and C2 good, as we have shown above. However, the demand for the seller B's good is not oscillating, or at least is not oscillating as much as the demand at the lower levels, because at every moment of time either C1 or C2 has shortage so he is likely to buy from B.

The latter case requires the distribution of consumers to be singular and an inefficient trader in the system to be present. This trader potentially can't obtain non-zero profit in the static case. Initially this trader can't prevent loosing all his buyers. Then he is decreasing his price in order to have non-zero demand. After he gains some buyers his demand is increasing because of low price and not very high level of shortage. Eventually his attractiveness for buyers falls because of the high level of shortage and rather high price set in order to overcome it. He loses all the demand initially obtained. All this lost demand now is switched to the rest of the traders rather suddenly hence new shortages are generated. Than the inefficient trader begins to decrease price in order to gain some buyers etc. The price oscillations are shown in the Figure 6.

4 Conclusion

We have considered a model that can be applied for study of self-organization processes in trade networks in an economy with imperfect infrastructure. Such model may be applied for analysis of market for imported consumer goods in Russia with international suppliers being denoted as producers. The behavior of world market does not depend upon processes in Russian economy, so producers' prices are given exogenously.

The main result of both analytical and computational study of the model is that evolution of trade networks depends significantly on the degree of imperfection of trade infrastructure ρ. If the infrastructure is almost perfect the system quickly converges to a near-competitive equilibrium without shortages and long chains of traders. Worsening of infrastructure brings about phase transition: large scale price oscillations and regular shortage bursts are observed, long chains of traders emerge and disappear spontaneously. In case of even more imperfect infrastructure the trade system turns into monopoly-like equilibrium: the prices and trade profits are high, and poorer consumers have to buy directly from producers.

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