Committee 5
Non-linear Structures in Natural Science and Economics

Draft – February 1, 2000
For Conference Distribution Only

Change, Organization, and Complexity: A Coherent Paradigm

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The Twenty-second International Conference on the Unity of the Sciences
Seoul, Korea February 9-13, 2000

CHANGE, ORGANIZATION AND COMPLEXITY
A COHERENT PARADIGM

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> **"Something immense is occurring around us. Everywhere you look, things are flowing together, creating new connections, new hybrids, new ways of seeing the world"**
> **"Convergence", Editorial, Forbes ASAP, October 4, 1999**

1. INTRODUCTION

This short monograph deals with a subject, the dynamics of complex systems, that I think is very relevant but has not yet found its way into the general physics courses. Also it should be a subject of great interest to all science teachers and in particular to physicists because, as it has been recognized only recently, it relates closer to the "real" world in which we live in, that is characterized by its "complexity", rather than the somewhat "simplified", and some time disconnected, version of physical systems we usually teach in science courses. However because complexity is a very broad subject, that encompasses a diversity of situations, I will cover only some fundamental aspects, that I hope will be sufficient to place the subject in a new perspective, adequate to science teachers, and stimulate its incorporation in the science curriculum, and in particular in physics courses. Because my own interest is in physics, the analysis will be mostly related to physical situations, touching only briefly other science areas, particularly biology, where it is most important. However the problem of complexity is relevant also in economics, politics, urbanism, sociology, etc, but these cases are more difficult to reduce to general quantitative laws.

Although for simplicity I will use consistently the term "physical world", it should not be considered as restricted to the inanimate world we deal with in the physical sciences but rather as encompassing everything related to matter and energy, not only on Earth but in the Universe, including living systems, because they are all regulated by the same basic laws.

The title, "Change, Organization and Complexity", is similar to that of a book based on a seminar I organized some years ago[1], in which scholars from different disciplines discussed the subject from diverse points of view. Since then a lot has been said and published about the subject and this monograph offers the opportunity of reviewing the subject from a fresh, though brief and restricted, point of view. However I wish to make it clear from the beginning that there is no way of dealing with complexity as a unified comprehensive subject. In fact it has been discussed extensively in a great variety of ways, some times in a clear way that makes it accessible to the educated reader[2] and others rather superficial or even misleading. For that reason, by concentrating on a fundamental approach I believe it will help to understand the subject in the context of our current paradigm of the physical world, by which I mean a strongly interconnected world, whose changes are at times difficult to predict with precision, and in other instances predictable only statistically, because of the many factors involved, some not known precisely, in spite of knowing the underlying laws that regulate it.

2. NEWTON AND MAXWELL

Before I address the subject I wish to put it in the proper context. Physics, the way we understand and teach it today, is an exciting and elegant science that has an internal coherence and unity and is the basis for understanding the world around us, but to a certain extent it is an oversimplification of the real world. Physics is the result of the collective intellectual efforts, through many years, of many brilliant thinkers, too many to mention all of them, that were able to extract or distill the most fundamental aspects of the physical world and identify laws and regularities that helped to carry out many important technological developments. Among these great thinkers there are two that in my opinion are the truly GIANTS responsible for shaping physics: Isaac Newton (1642-1727) and James Clerk Maxwell (1831-1879), whose seminal ideas are still valid in the context in which they were formulated.

In the case of Newton, because he developed the general framework for dealing with the motion of bodies, based on the notion of "force", as expressed in his three celebrated laws of motion, which have been the basis for all future developments of dynamics, or even all of physics, and recognized gravitation as a universal phenomenon, that extends beyond the confines of Earth, expressing it in terms of the distance separating the bodies, thus formulating the first fundamental force of nature to be identified. In the course of time Newton's formalism and the concept of force have been refined by many others[3] ,and to a certain extent became secondary to the conservation and exchange of momentum and energy during an interaction, but it is still valid to describe the motion of bodies at the macroscopic level (planets, falling bodies, machines, etc). At the microscopic level (molecules, atoms, nuclei, etc), where the formalism of quantum mechanics applies, the concept of force becomes "evanescent", to use Newton's terminology, and has been replaced by that of "interactions", which is more general, although Newton's formalism is still applicable in many situations such as the motion of a charged particle in an accelerator, or in a first approximation, the motion of an electron in a hydrogen atom.

In the case of Maxwell, because he formulated quantitatively the second fundamental force of nature, electromagnetism, by unifying electric and magnetic phenomena, as expressed in his famous set of equations, fundamental for all future developments of electromagnetism and its applications, including quantum electrodynamics. But what I consider equally important, Maxwell also provided a general methodology for analyzing physical phenomena in terms of the notions (or models) of "particles" and "fields". This idea is still at the core of physics, and has given rise later on to the notion of "interactions", which as I said before, is more fundamental and general than that of "force", applying both at the macro- and the microscopic levels. Maxwell also developed a formalism for dealing with "connected systems" , that is basic for the study of complexity although he did not touch on the subject, that lead him to formulate a "dynamical" theory for time dependent electromagnetic fields[4] . In fact Maxwell was a firm believer in a "mechanical" universe and one of his goals was to express electromagnetic theory (as well as a statistical theory of gases) in a quantitative form, within the frame of the laws of dynamics, goal in which he succeeded. .

Although the physics built around the ideas of Newton and Maxwell became a most successful science for analyzing many physical phenomena in a precise, deterministic and predictable form, it has some limitations. For example it elevated the concept of "force", as it appears in the second law of motion, to that of an entity that has an existence by itself. If you know the "force" you can determine the motion, if you know the motion you can find the "force". Because of that, physics developed around this methodology for using the concept of "force", without a critical analysis, mixing fundamental "forces" with "forces" of macroscopic and statistical nature , always trying to express the "forces" in terms of the geometry and other parameters of the problem considered.

We may say that Newton-Maxwell physics is "simple" and "linear" in the sense that it treats the dynamics of a particle as if the particle were an independent entity, not part of a system, and the action of the world around the particle could be reduced to a well defined "force". However, many developments during this century have forced to reconsider the universal validity of this paradigm of the physical world based solely on Newton-Maxwell "rigid" framework, in which the world consisted of "particles" and "forces". As a result, since around 1900 many scientists began to explore alternative approaches for dealing with certain dynamical problems, based more on the notion of fields.

Many developments contributed in this direction. In the first place, it became clear that the world at the microscopic level behaves differently from the world at the macroscopic level, and to explain that behavior it has become necessary to develop a new formalism, quantum mechanics, that at that level replaces dynamics based on Newton's laws, by a dynamics based on Schrödinger's equation, that removes "force" from its privileged position or even ignores it, and makes statistical considerations an integral part of the formalism. In the second place the development of information technologies (IT) has enabled us to examine physical phenomena using elaborate mathematical computations, as well as by means of modeling and simulations, allowing us to gain new insights into some of those phenomena, that were intractable using traditional computational methods. In fact models have become an essential component of research, complementing experiment and theory. And in the third place, and this is most important, it has been recognized that, because the coupling or interaction among its components, the physical world is not as simple and linear as the Newton-Maxwell model led us to think, but rather "complex" and in some cases "non-linear", requiring a new approach for analyzing some phenomena in the context in which they occur. Of course Newton-Maxwell model remains a valid formalism for what it was designed, that is the phenomenology of the macroscopic world, that can be analyzed using deterministic laws.

I firmly believe that this more realistic post Newton-Maxwell way of looking at the physical world must be conveyed to the students of the introductory physics course, allowing them to have a more comprehensive understanding of the "real" world in which we live. This is not easy, particularly because physics courses are built around specific or distinct phenomenologies based on our direct sensorial experience (motion, sound, light, heat, etc), a method that does not lend to generate that kind of coherent and flexible understanding. In the next section I elaborate a little bit on this point.

3. A DYNAMICAL DIVERSE WORLD

When we look at the world we live in the first thing that strikes us is its extraordinary diversity. The second thing we notice is that the world is dynamic, displaying a multitude of changes, that we call phenomena and processes. Some changes, such as the succession of days and nights, are amazingly regular and predictable, others, such as the weather, are not. One of the functions of the scientists is to analyze those changes, identify whatever regularities are apparent in those changes, formulate the laws obeyed by the world's dynamics, design models that serve to explain how the changes occur, design methods for predicting possible future outcomes, and try to find some unity in the observed diversity, for which mathematics is an essential tool. In carrying out this task, scientists usually concentrate the analysis on well defined portions of the world which are called "systems". In turn systems can be desaggregated into certain units or components, that may be simple or composite, that is they can be considered as "subsystems". I call this methodology the "science program" (Table 1).

Systems can be analyzed from several points of view. A first step is to determine the "components" of the system, which depends on the desired level of desaggregation of the system, that may go from the microscopic to the macroscopic . A second step is to analyze how the components are arranged and interact, that is the "structure" of the system, and in some systems determine the "functional relations" among the components. A third step is to determine the "dynamical laws" that regulate the interactions among the components. And a fourth step is to analyze how the system interacts with other systems, or rather with its "environment". These four steps jointly constitute the central points of the scientific enquiry of a system and, by extension, of the physical world.

Form the point of view of the interaction of a system with other systems (or environment), we may designate systems as "open" or "closed" (or isolated), depending on whether they exchange momentum, energy and matter with the surroundings (environment) or not (Fig. 1). (In some systems "instructions" and/or "information" may also be exchanged). Most systems of interest to physics, and to science in general, are open but in some cases, as a first approximation, may be assumed closed. Some systems, mostly those that are closed, exhibit only some kind of "structural order" (such as a crystal), while others, mostly open, may exhibit in addition "functional organization", that implies "functional correlations" among its components (such as in a cell), and requires internal as well as external matter and energy fluxes (and some times of instructions and/or information) to perform their functions. These systems are more difficult to analyze because of their critical dependence on the correlations among the components and with the surroundings, as well as for the their time variations, that may result in changes in the components and/or their functions, but they are the most important and interesting to study from the scientific point of view.

An amazing feature of nature is the emergence in many systems of patterns and forms, which may be considered as macroscopic manifestations of the organization of the components of the system[5]. Patterns may appear "spontaneously" by the action of internal agents, or as a result of the "driving" actions of external agents. No general theory of patterns formation exists, and the best we can say is that they are the result of a combination of factors, that differ from case to case. In my opinion the two main factors of pattern formation are the interactions among the basic components of the systems and their internal symmetries. It is interesting to note that nature uses a limited number of basic patterns (cylinders, triangles, hexagons, circles, etc) that reflect the symmetries involved in each case.

4. COMPLEXITY

Only recently it has been recognized that an important feature of a system is its "complexity". There are many ways of considering the notion of complexity, that manifests differently according to the nature of the system. Thus we may talk of physical complexity, biological complexity, social complexity, economic complexity, etc. Each aspect of complexity has to be dealt with in a different way. In other words, there is no generic treatment of complexity. And not all systems are necessarily complex.

In what follows we will consider complexity in the context of physics. For the purpose of physics we may consider complexity from two complementary points of view. One, the "structural", emphasizes the conceptual understanding of the system in terms of its components and interactions, and the other, the "formal", relies on describing quantitatively the behavior of the system in terms of dynamical laws.

From the "structural" point of view a complex system is one composed of a very large number of strongly interacting units, in which structures, processes and changes appear as a result of internal and external actions. This approach considers complexity as an "intrinsic property" of the system, independent from the observer, and emphasizes the role of "interactions" as a source of complexity, that by coupling the components of the system may make it difficult to predict the specific behavior of each component unit or subsystem, and even of the system as a whole, since any modification in the dynamics of one component may affect the whole system.

From the "formal" point of view we may define the "complexity" of a system as the amount of information needed to explain and describe quantitatively the structure, properties and behavior of the system, so that complexity is associated with the perception of the system by the observer. In most instances mathematical relations or rules, that may be rather complex, are required for the quantitative expression of the dynamical laws that underlie the physics of the problem. In that sense a system may have different types of formal complexity: dynamical complexity, functional complexity, mathematical complexity, etc, depending on how we want to analyze the system.

In either case, to be complete, the analysis of a complex system requires the determination of the dynamical laws that determine the time-dependent behavior of the system in response to internal and external actions. In many systems this is relatively simple in principle if we know the fundamental interactions among the components of the system and with the surroundings or environment (precisely this is one of the tasks of thermodynamics and statistical mechanics), though in some systems this is extremely difficult, as we will discuss later on.

A particular case of complexity is "adaptive complexity", that applies to open systems that not only may evolve in time but also "learn", in the sense that their evolution and change is determined by their present exchanges of information and instructions, and/or energy and matter with the surroundings, as well as on their past history, that is on their current and past flow of information and instructions, and/or energy and matter (Fig. 2). Adaptive complex systems (ACS) identify regularities in the inputs received, select and compress them, and adapt to them, may even grow and evolve. For that reason ACS are prone to evolve through irreversible unpredictable changes, called "frozen accidents". In that sense the evolution of some ACS may be mostly the result of frozen accidents. Most ACS are biological and social systems. (Of course the term "adaptive" should no be interpreted in a Darwinian sense). However ACS are very seldom considered in physics, ignoring an important fact of nature, although they are very important in biology.

5. LEVELS OF RESOLUTION OF A SYSTEM

As indicated earlier, the analysis of most physical systems requires the identification of the relevant fundamental components of the system, and the expression, in concise quantitative terms, of their internal and external interactions and the corresponding dynamical laws, as to yield non-ambiguous predictions. As a complex system is observed with increased resolution the nature of its fundamental components and of their interactions may change. The difficulties for carrying out that analysis in a meaningful way at the required level of resolution are also an indication of the complexity of the system. Changing the resolution may change the level of observed complexity of the system. Some systems may become so complex at certain level of resolution that their behavior is not fully predictable at that level. Also some levels of resolution may be meaningless for the analysis of a system.

The levels of aggregation or degrees of resolution at which we may analyze a system, depend on the type and amount of information we need about the system, the size and complexity of the system, the length over which the different properties of the system varies throughout the system, also called "correlation length" [6], and the range of the external actions on the system (Table 2). Each level requires different methodologies and techniques of analysis and different theoretical models. Fig. 3 shows an alloy at different levels of resolution and the technologies used in each case for the analysis. Fig. 4 shows different levels of resolution of a nucleon. At the lowest resolution a nucleon is a particle with certain mass, spin ½, charge +e or 0, and a magnetic moment. At the next level of resolution, according to the standard model, we consider a nucleon as composed of three quarks, two up and one down (proton) or two down and one up (neutron). However the quarks are held together by exchanging gluons. Thus at the third level of resolution a nucleon is a mix of three quarks and several gluons. But according to field theoretic considerations the strong interaction can give rise to "virtual" pairs of quarks-antiquarks and more gluons, so that at the highest level of resolution a nucleon is a "soup" of real and virtual quarks surrounded by gluons. No wonder it is so difficult to calculate some properties of the nucleons, such as the magnetic moments!

Therefore an important judgement that the scientist has to exercise when analyzing a system is first to decide on the level of resolution and then proceed to identify which are the subsystems or components of the system relevant for its purpose (nucleons, atoms, molecules, cells, organs, planets, galaxies, humans, groups of humans, etc), and whether it is necessary to consider their structure or not. Thus it is customary to ignore the structure of the molecules in a gas at low temperatures and densities, and even treat them as structureless balls, but that is not adequate at high temperatures and densities, when molecular collisions may become inelastic, or when subject to electromagnetic radiation, when molecular excitations and ionization are important, as in a laser. Similarly to discuss many atomic properties one may ignore the structure of the nucleus, but that is not the case when discussing for example the hyperfine structure of electronic energy levels.

Also the nature of the components of a system may change with external actions, when new structures or new states of aggregation may appear. For example as the pressure on a volume of water is increased, the molecular arrangement and composition of water changes until at pressures of several G Pa it becomes simply a mix of H and O atoms (Fig. 5). Finally the properties of the components when they are part of a system may be different from those when the components are isolated. In general the more complex a system, the more challenging may be to identify its relevant components.

6. INTERACTIONS

The other important task in the analysis of a system is to identify the "interactions" among its components and with the surroundings. For a system to be of interest its components must interact among themselves and with the surroundings. Without interactions nothing happens in a system, and no structures emerge or processes are possible, except perhaps elastic collisions, as in an ideal gas, so that to a certain extent the system is irrelevant from the scientific point of view. In the case of physical systems, physicists try to determine quantitatively and express mathematically in terms of the parameters of the system the internal interactions (i.e. electric forces among the electrons in atoms and molecules, intermolecular forces in a gas) so that the system can be analyzed quantitatively. The history of physics is the history of the search for interactions. Since most systems are affected by the actions of their surroundings (applied electric and magnetic fields, collisions at the boundary, etc) , it is also important to know how the surroundings interact with the components of the system, which structures may result or disappear, or which processes may occur as a result of such external actions.

As indicated earlier, one aspect of complexity that is important and poses a challenge in the analysis of a system is that, as a result of the interactions, the behavior of a unit or component of a system is not the same if it is isolated or if it is part of the system. That means that a complex system is not strictly the sum of its independent components, but rather a different entity, although the properties of a system depend critically on the arrangement and number of its components. For example the properties of a carbon atom, such as the electronic configurations, are not the same if a carbon atom is isolated or if it is part of a molecule, a polymer, a fullerene, or a cluster of fullerenes, including nanotubes, which are aggregates of the order of a few nm, whose collective state and electric and magnetic properties depend on the size and geometry of the cluster. Same applies to the DNA molecule that does not behave in the same way in isolation or in the environment of the cell, and for the DNA molecule to be able to express the instructions contained in its structure the molecule must be in the environment of the cell, where it interacts with many other molecules (water, proteins, etc), something that has important biological implications and is a good example of bio-complexity (Actually, as it is well known, the whole genetic program involves more than DNA and depends on the organized context of the cell).

As I said earlier, the larger the number of components of a system and the stronger the interactions among them, the more difficult it is to carry out the analysis of a system, identify its relevant components and predict its behavior; in other words the more "complex" is the system. One reason is that the larger the number of particles, the larger the number of variables or of degrees of freedom needed to meaningfully describe the system, and this complicates the mathematical relations needed to analyze the system. Also strong interactions may give rise to several effects. One is that the interactions couple the dynamics of the components so strongly that the nature of the components may change with the strength of the interactions or it may be very difficult to consider each component as a separate entity (recall Fig 5). Another is that a disturbance in the dynamics of one of the components, even if it is small, may produce a "domino effect" quickly propagating through the system in a way that the final outcome is difficult to predict, and may even give rise to irreversible changes, which is what were called earlier "frozen accidents". A third possibility is that even a small input or change in the organization or configuration of the system may result in profound changes in the properties of the system. This is the case of nanotubes, that when twisted (Fig. 6) may change from conductors to semiconductors depending on the (chiral) angle they are twisted. Or a small change in the chromosomes of an embryo may a result in an organism with different properties. To these effects we must add the action of external agents that change with time, that may be so complex as to make impossible to predict the exact effects on the system (think of a storm), as we discuss in connection with Eqs. (4) and (4') below.

In some cases the behavior of the system as a whole is predictable, at least in a statistical sense, but that of the individual components is not. For example if we have a sample of a radioactive substance, we know that after a certain time interval, called "half life", statistically half of the radioactive nuclei will have disintegrated, but we can not predict which will be the next nucleus to disintegrate at a particular time.

Ideally the analysis of a physical system should start at the fundamental or microscopic level, identifying its ultimate components and their properties (nuclei, atoms, molecules, ions, etc, whose sizes are of the order of 10^{-9} m or less), for which quantum mechanics is required, determining the interactions among the components, how they are grouped into the basic structures of the system, and how the properties vary throughout the system at that scale. From that analysis one should proceed to explain the overall properties of the system at different scales or levels of resolution as the number of components is increased. This method is not always feasible or even desirable. At the other extreme or macroscopic level a system may be considered as a continuum, and it is assumed that the properties do not vary appreciable over distances of the order of 1000 times the size of the components (or in the case of condensed matter, when the correlation length of the property considered is about 10^{-6} m or larger), so that the basic components may be ignored and a phenomenological description of the system, in terms of empirical parameters and experimental relations, is used (i.e. gas law, Navier-Stokes equation, Ohm's law, etc). Clearly, for a comprehensive understanding and analysis of a system eventually one has to establish the connection between all levels of analysis of resolution[7].

It has become customary to consider a mesoscopic level of analysis[8], intermediate between the micro and macro extremes, at which one considers the fine structure of the system in less detail by lumping together groups of components into subsystems and combines the techniques of the two extremes. This method is particularly useful when the correlation length of the properties studied is between 10 and 1000 times the size of the basic components of the system (recall Table 2), although it may be necessary to take into account the fluctuations in the properties of the system over regions of the order of the correlation length. It is widely used to deal with some optical, electrical and mechanical properties of certain systems (electronic devices, micro-robots, turbulence, quantum computers, etc). Clusters, which are relatively small systems consisting of a few hundred atoms, may be considered just at the mesoscopic level. Medical practice combines the three levels of analysis: some parts of the human body can be analyzed at the macro-level, others at the meso-level, and others at the micro-level. I do not think it is necessary to illustrate this point.

7. THE STRUCTURAL APPROACH. INTERNAL ENERGY

As I said before, physics is a relatively simple science because in general it deals only with systems with structural complexity, while biology is not so simple because it is concerned with systems with functional and adaptive complexity in addition to structural complexity, that require a more elaborate analysis. The two most interesting and traditional problems in physics are determining the structures in a system and the analysis of its evolution and changes in time under its internal interactions and those of external agents or environment. Several conceptual frameworks have been developed to tackle these two problems. In my opinion the more effective conceptual framework in physics for the structural approach to the analysis of a system is the one that uses the concept of energy and in particular of "internal energy" U. Although this is well known I will briefly elaborate on it because its direct relation to the subject of this monograph.

It is an implicit assumption in physics that the internal energy of a physical system is composed of three terms: the kinetic energy of each of the particles or subsystems composing the system, their interaction energy, that is assumed to couple only pairs of subsystems, and the proper energy of each of the particles or subsystems, that is

$$U = \sum E_k(i) + \sum E_{int}(ij) + \sum E_p(i) \qquad \text{or} \qquad U = E(K) + E(I) + E(P) \qquad (1)$$

I call this expression the "equation for everything" because it applies to a multitude of situations[9]. The particular form of each term depends on the specific situation. For example in the case of a system composed of polyatomic molecules one assumes that E(P) is the sum of the electronic, vibrational and rotational energies, that at the first order of approximation can be considered independent of each other. It is important to note (1) that because the interaction energy term couples pairs of subsystems one can not talk about the individual energy of each subsystem, (2) that the assumption that the interaction energy involves only pairs of subsystems is only an approximation and the possibility exists that it may involve more subsystems, E_{int} (ijk...), and (3) that whatever changes or processes occur in the system involve exchanges between the three types of energy, including changes in the particles P (excitations, dissociations, chemical reactions, etc)[10]..

If the system is closed the only processes or changes that can occur are those that maintain U constant, that is

$$\Delta U = 0, \quad \text{or} \quad \Delta E(K) + \Delta E(I) + \Delta E(P) = 0. \qquad \text{(closed system)} \qquad (2)$$

An implicit assumption in physics, supported by experimental evidence, is that the system, as a result of the internal exchanges of energy and the dynamical laws, evolves until it eventually reaches a state of statistical equilibrium, at which the average value of the macroscopic parameters and of each of the components of the internal energy of the system remain constant: $<E(K)>_{ave} \approx$ const., $<E(I)>_{ave} \approx$ const., $<E(P)>_{ave} \approx$ const. As a consequence the average of the fluctuations in each of the energy terms are zero, that is,

$$<\Delta E(K)>_{ave} = 0, \quad <\Delta E(I)>_{ave} = 0, \quad <\Delta E(P)>_{ave} = 0 \qquad \text{(equilibrium)} \qquad (3)$$

Once the system reaches the state of equilibrium no more large scale changes or processes occur in the system as a whole and certain structures and /or patterns may appear at the mesoscopic and/or macroscopic levels, depending on the balance among the components of the internal energy. It is an important problem in physics to determine those structures and patterns, although the details may be difficult to predict. An elementary example at a mesoscopic scale, is that of a ferromagnetic material in statistical equilibrium, when the magnetic dipoles align in parallel in small clusters or "domains", under the action of the spin-spin interaction. Each cluster or domain is magnetized but the size, the shape and the orientation of each cluster, that is their pattern, are different and impossible to predict. At the macroscopic level the system may not show any magnetization except at certain spots called magnetic poles, as any student of physics knows. (Recall that this is the way magnetism was first recognized). Another example of mesoscopic statistical equilibrium related to spin-spin interactions are spin glasses, which are systems in which the molecular magnetic moments are oriented randomly, that exhibit an additional property called "frustration", because the system can not reach the lowest possible energy state.

Because the equilibrium of a system is statistical, some systems may fluctuate between several very close configurations or microstates (this fluctuation is also called "microstate democratic egalitarism") so that the macroscopic state of the system is a sort of average or superposition of microstates. The possibility also exists that a system may experience a large fluctuation and jumps to an entirely different state of equilibrium, though the possibility of this "fatal accident" is extremely small in most systems with only structural complexity. There is also the possibility that, if the system is very complex, far from equilibrium, and the interaction between its components is very strong, the system may have the option of following different routes toward equilibrium, making it difficult to predict the final state, and similar systems under similar circumstances may end up in different states. This is sufficient to recognize that the predictability of the properties, structures, and evolution toward equilibrium of a closed complex system may differ according to the level of analysis and of resolution[11].

Summarizing, it is a well accepted assumption in statistical physics (and the basis for the second law) that closed or isolated systems evolve until they reach a macrostate of equilibrium which is a mix of microstates compatible with its total energy, the nature of its components and their interactions. This is not a state of disorder because some structures and organization may appear. It is rather the state in which the closed system attains its optimum macroscopic configuration or state, that may be assumed is the one associated with the largest number of microstates compatible with the above parameters (largest multiplicity)[12].

Though the study of the evolution toward statistical equilibrium of a closed complex system is very important in physics, it is even more important to study the behavior of open systems, whose phenomenology is rather rich. When the system is open the evolution of the system depends on the external agents. Suppose for simplicity that the system exchanges energy, E_{exch}, but not matter, with the surroundings. Then

$$\Delta U = E_{exch} \qquad \text{or} \qquad \Delta E(K) + \Delta E(I) + \Delta E(P) = E_{exch} \quad \text{(open system)} \qquad (4)$$

The most important interactions between the system and the surroundings are exchanges of kinetic energy by collisions, that is "heat" Q, by absorption and/or emission of photons, that is "radiation" R, and in some cases by changes in the structures and/or configuration of the system by the action of macroscopic forces, such as pressure, and external fields, (gravitational, electric, magnetic, et.), that is "work" W. Thus Eq. (4) can be written as

$$\Delta U = E_{exch} = Q + R + W$$

These three mechanisms of energy exchanges affect differently each of the components of the internal energy of the system and determine the processes or changes that occur in the system and what new structures may emerge or disappear. These effects may be explained or even predicted if we know in detail the internal and external interactions and the dynamical laws of the system. Referring to the previous example of a ferromagnetic material, under the action of an external magnetic field, the clusters or domains may be reoriented and forced to grow in the direction of the field, so that the material exhibits a forced order, that may be a stable state or not, and becomes a magnet at the macroscopic level; the system may or may not revert to the original state if the magnetic field disappears. Same applies to the electric polarization of a material.

In general the energy exchanges disturb the state of the system, regardless of whether it was originally in statistical equilibrium or not, and the system either may be carried to a new state of equilibrium, that may be stable or not, may never reach a new state of equilibrium, or may change in an irreversible way. This is the phenomenology we observe continuously around us, and that students should understand well.

There is the possibility that the average net energy exchanges of a system with its surroundings are zero, that is the system absorbs as much energy as it releases, $\langle E_{exch}\rangle_{av} = 0$, so that $\langle \Delta U\rangle_{av} = 0$, and the system is "isoergic". (A simple example is a thermally insulated metallic rod that is kept hot at one end and cold at the other by steady supply of thermal energy, that is molecular kinetic energy, at the hot end, and removing the same amount of energy at the cold one). In this case the system may reach a steady state that is a new type of statistical equilibrium in which each small volume is an open subsystem in statistical equilibrium, different from that of its neighbors, so that there are gradients in the macroscopic parameters of the system and perhaps some dynamical structures or configurations appear, that are maintained as long as the external agents continue supplying and removing energy at a steady rate.

Sometimes it is the "rate" of energy transfer, dE_{exch}/dt, that is more critical for determining the changes in the system. In this case Eq. (4) becomes

$$dU/dt = dE_{exch}/dt = dQ/dt + dR/dt + dW/dt \qquad (4')$$

or expressing dU/dt in terms of its components,

$$dE(K)/dt + dE(I)/dt + dE(P)/dt = dQ/dt + dR/dt + dW/dt. \qquad (4'')$$

The behavior of the system may vary dramatically depending on the value of dE_{exch}/dt and how it affects each component of dU/dt, as indicated in equation (4"). If dE_{exch}/dt is small and constant the rates of change in the components of U, that is the change of internal interactions in the system and of the molecular kinetic energy by collisions, and the adjustment in the states of each particle, are also small and can carry the system smoothly to new well defined states (that is what in thermodynamics are called reversible transformations). But if dE_{exch}/dt is large or experiences abrupt variations the components of U can not adjust quickly enough, particularly if the strength and range of internal interactions are not sufficient to produce the necessary adjustments. Consequently irreversible changes may take place and the system may become unstable or even evolve unpredictably, in spite of obeying deterministic dynamical laws, that is the system has a "chaotic" behavior.

The situations just described are typical of transport processes and fluid dynamics. For example as the gradients increase or change very rapidly over the correlation length of some properties, the motion of a fluid may change from laminar to convective to turbulent (Fig. 7). Even in this case some kind of organization and patterns may appear, and it may be assumed that Navier-Stokes equation still applies even if we can not explain precisely how the large gradients produce the chaotic behavior that, however, shows some macroscopic regularity. A well known example are the Benard cells that are convective structures in a fluid in which a small temperature gradient is maintained between two metal plates (Fig. 8), but may disappear if the temperature gradient is very large and the fluid motion becomes turbulent or chaotic.

It should be clarified that the term chaos (from the Greek "chaskein", that in the Greek literature and in Genesis 1 is used to refer to the emptiness or state of the world at its creation, and usually means high disorder or the absence of order), refers in physics to systems whose space-time evolution is unpredictable and even erratic, even in some instances they may show some kind of regularity at the macroscopic level. Chaotic behavior is closely related to the complexity of systems whose internal structure requires non linear equations for its mathematical description, making the state of the system highly sensitive to its internal and external parameters. That means that from the structural point of view chaotic behavior may be a combination of microscopic and macroscopic factors, some of which can not be known with sufficient precision to be able to make long term predictions about the system. In some instances a system may show macro-predictability combined with micro-unpredictability. (Many efforts have been made to deal with non equilibrium processes at the macroscopic level, that is irreversibly thermodynamics; see e.g. Ref. 18)

The situation is more complex when the exchanges of energy and matter give rise to changes in the composition of the system (chemical reactions, dissociations, ionizations, etc) and the system may grow or shrink. Although the preceding analysis applies also in this case, it is more difficult to describe the processes in precise mathematical terms and usually one uses the empirical formalism of thermodynamics and statistical mechanics to connect the initial and final states.

The above considerations serve to illustrate the importance of the interactions, the composition and the internal energy in determining the changes and organization in complex systems. The diversity of structures and patterns that may appear in physical systems is extraordinary but they all have in common that they are due to the interactions, both internal and with external agents, and the nature of the components, including their internal symmetries, that may be traced in last instance to molecular and/or atomic symmetries. To emphasize this point I will mention two coefficients, called "structure factors", one related to the interactions and the other to the composition, that are useful to explain what changes may take place in a system when it is disturbed or when it evolves toward equilibrium:

$$F_{int} = <E_{int}(ij)>_{av} / <E_k(i)>_{av}, \quad \text{and} \quad F_{part} = <\delta E_{part}(i)>_{min} / <E_k(i)>_{av} \qquad (5)$$

where δE_{part} refers to the minimum excitation energy of the particles.

Since the difficulty of the task of determining the interactions and structure of the components, which is essential for the analysis of the system at the micro- and meso-levels and its evolution in time, depends on the complexity of the system, in many cases it may be necessary to make some approximations or develop a simpler model[14] of the system, a common procedure in physics.

The preceding ideas are within the level of understanding of the introductory physics students. Also it should be emphasized that in principle the behavior of a system at the meso- and macro- levels can be explained if we know at the micro-level the components of its internal energy and the dynamical laws. A different matter is whether we can have and handle the necessary information to predict with some level of precision the meso- and macro- behavior of the system, and this is where computational techniques can be very useful. The main challenge at the micro-level, beside identifying the right components, is that in many instances at the micro-level we have to take into account the quantum effects, making it more difficult to predict the behavior of the system (quantum complexity).

8. COMPLEX ADAPTIVE SYSTEMS

The structural analysis is more complex in open ACS with functional organization, as a cell, because for the "survival" of the system the energy and matter exchanges, as well as possible exchanges of instructions and information, with the surroundings must assist rather than hinder the functioning of the system. The system must be capable of adaptation, that is modifying some components, structures or functions, and even replacing some of them, for a better performance in response to the actions of external agents, perhaps making it more complex. This type of regime is essential in biological systems, which are composites of coupled open adaptive subsystems, and determines whether the system (organism) survives, grows or dies. These situations do not usually appear in physics, although students should be made aware of them.

However active research is going on to design adaptable programmable hardware, that is chips known as field programmable gate arrays (FPGA) that can be reconfigured by software.

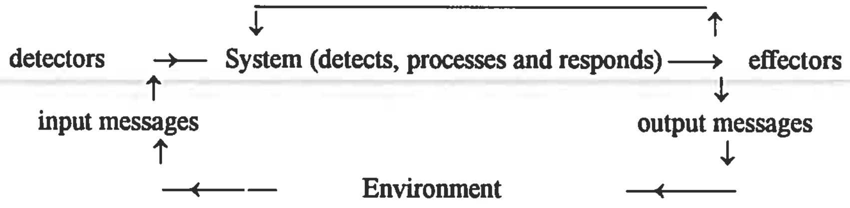
The first difficulty in analyzing an ACS is that the system may follow different paths or adaptation options under the action of an external agent, and reach one of several possible states. Second, it seems that in an ACS there is a competition among its different components to determine the optimum aggregate behavior in response to external agents, and the best performance to achieve the new state. Third, assessing the role of the memory, that is the retained part of the past inputs and the development history of the system, is important because the memory may modify the response or adaptation of the system to the same inputs at different stages of its history, thus in a sense influencing the future of the system. An additional difficulty is assessing the response of the system to the simultaneous action of several agents, that is not simply additive because the possible non-linearity (epistasis) of the response. That means it is difficult to determine in precise deterministic terms the time evolution of an ACS, that can not be expressed just in terms of the properties of its isolated components, but nevertheless depends on the internal and external interactions. However it is possible to develop a formal conceptual theory of how to deal with ACS[15].

In a simplified form, let us designate by τ_E the "adaptive possibilities" of an ACS in response to a given environment E, and by $\Omega_{E\,(o)}$ one of the set of "operators" representing the action of the environment. If $A_{n(i)}$ is the initial state and $A_{m(f)(\tau E)}$ is one of the possible final states of the ACS allowed by the adaptive possibilities τ_E, we may write

$$\Omega_{E(o)}\, A_{n(i)}\,(\tau_E) \longrightarrow \Sigma_m\, \alpha_{nmE(o)}\,(\tau_E)\, A_{m(f)}\,(\tau_E)$$

where the $\alpha_{nmE(o)}\,(\tau_E)$ are coefficients that measure the performance or "fitness" of the system in response to the actions of the environment and to the adaptive possibilities, that is determine the likelihood of a given response to a particular environmental action or operator according to the adaptive possibilities and the internal variables of the system. Of course, for the above equation to be of practical value one must express each term in quantitative form in terms of the composition of the system, a task that is not simple.

One way of looking at how some ACS function (particularly biological and social, but it applies also to several mechanisms and electronic devices) is to say that they use "detectors" (sensors or transducers) to receive input actions or "messages" from the environment, an "internal mechanism" (brain or nervous network in living systems) to process them and elaborate response actions, and "effectors" to send output "messages" to other parts of the system or back to the environment. That is



Several interesting features may appear in ACS as a result of internal actions in response to external actions. One is "emergence", that is the appearance of new features and/or behavior induced by relatively small input "messages". Another feature is that when the rate of input "messages" is very large the system may find it difficult to respond and adapt to them; this situation is called "fatigue". (In the case of the nervous system when the rate of arrival of pulses at the neurons is very large, say several hundred pulses per second, the result is called an "epileptic fit").

The biological and social ACS have an additional complexity and it is the capability of making "autonomous" decisions, supposedly based on the information available to or stored in the system (perhaps we may say based on their level of cognition and awareness) and the influence of internal and external factors. These autonomous decisions are unpredictable except in an statistical sense. As a physicist, I have no explanation for this capability in terms of physical laws, although I am sure there must be one. For example I can visualize that in the case of relatively simple ACS, such as bacteria, they may react predictably to electric fields, concentration and temperature gradients, etc., though perhaps they are not true autonomous decisions (see e.g. H. C. Berg, Phys. Today, **53**, 1 (2000). In any case I think it is a fascinating capability that we may associate with the emergence of intelligence in ACS, and certainly has an important bearing on the survival of life on Earth. I leave the explanation to the neuro-biologists.

9. DYNAMICAL ANALYSIS: NEWTONIAN APPROACH

For certain time dependent complex systems, for which their dynamical laws, based on Newton's formalism, are known or can be reasonably guessed, a methodology, of empirical and mathematical nature, may be appropriate for analyzing their space-time evolution, that allows to concentrate the analysis on specific features of the dynamical complexity of the system. This methodology was developed initially by H. Poincare and others at the turn of the 19th century for dealing with systems with few degrees of freedom and whose parameters are coupled by non-linear relations, and thus exhibit some type of mathematical complexity, such as the motion of three bodies subject to their gravitational interaction. The main challenge to this approach is to determine with sufficient precision the coefficients that appear in the dynamical equations since any small variations in those coefficients may alter substantially the results of the analysis.

It is simple to illustrate this point with systems with one degree of freedom to which the Newton-Maxwell formalism applies. The simplest of these systems is a body of mass m whose structure can be ignored (it is what in dynamics is called a "particle"), at the position x, and subject to the actions of a "force", that may depend on time and position, that is $F(x,t)$, to take into account the changes in the force over time and position as a result of the motion of the particle. The only possible process or change that may occur is the motion of the particle, which is described using Newton's second law,

$$m\, d^2x / dt^2 = F(x,t). \qquad (6)$$

The classic problem in dynamics is to determine the motion of the particle, that is $x(t)$, if we know F, or conversely determine F if we know the motion. (This is where Newton-Maxwell physics enters into the picture). In either case F has to be determined experimentally or derived from a more general theory or principles. As it is well known, Eq. (6) has been used successfully to describe the motion of a particle under the action of gravitational, electric and magnetic fields. However in many cases F is not known exactly and has to be estimated empirically, as is the case with elastic and viscous forces, that are relatively easy to estimate, and when the particle is subject to the actions of an environment that is changing with time in a way difficult to predict exactly. In this case it is the environment that is the source of the dynamical complexity of the problem. What physicists usually do in this case is express empirically the part of the "force" that is not well known, in terms of the displacement x and the velocity dx/dt of the particle. This is the case when discussing oscillatory motion (harmonic and damped), motion in a fluid, etc. (It is interesting to note that Newton was aware of this problem and, in Book II of the *Principia*, discussed the motion of bodies in a fluid using his geometric methodology).

Therefore to deal with dynamical complexity in a first approximation we may express part of the "force" as a power series of x with coefficients that may depend on time and some features of the particle, such as size, mass, charge, etc, to be determined experimentally or guessed empirically. Therefore, dropping the coefficient m, we may introduce the dynamical equation

$$d^2x / dt^2 = \Sigma\, a_n(t)\, x^n + F(x,t), \qquad (7)$$

where $F(x,t)$ corresponds now to the part of the "force" that can be expressed explicitly as a function of position and/or time, that is a Newton-Maxwell force such as the gravitational force, an oscillating mechanical or electric field, $A_0 \cos \omega t$, or the weight mg of the particle. When the coefficients a_n are independent of time the summation corresponds to a "conservative" force with the potential energy $-\Sigma\, a_n x^{n+1}/(n+1)$. The case $n = 1$ corresponds to a simple harmonic force .

In some instances the action of the environment on the particle is more complex and may depend also on the velocity of the particle, as is the case when the particle moves in a fluid. Then we must try a more general empirical nonlinear expression for describing the motion,

$$d^2x / dt^2 = \Sigma\, a_n(t)\, x^n + \Sigma\, b_n(t)\, [dx/dt]^n + \Sigma\, c_{nm}(t)\, x^n\, [dx/dt]^m + F(x.t) \qquad (8)$$

The first summation corresponds again to a "conservative" force, although the associated "potential energy" can not be treated in the same way as in the traditional conservative systems because the effects of the other terms. The second and third summations are associated with "dissipative" forces, that is forces that do not conserve the energy[16]. The last term corresponds to a "forced" motion of the system by a sort of macroscopic "Newton-Maxwell" force. The resulting motion depends critically on the values and signs of the a,b,c coefficients. The nonlinear equation (8) encompasses most situations found in the dynamics of complex systems that can be analyzed using Newton's formalism, though in general it is difficult to solve exactly because its non linearity. Therefore it may be necessary to make some simplifications to analyze the dynamics of each particular system, that may be rather complex or unstable. A reasonable assumption is that the value of the coefficients decreases as n increases. The simplest case is to keep only the terms with n = 1 in the first two summations, and assume a_1 and b_1 are constant, making equation (8) linear,

$$d^2x / dt^2 = a_1 x + b_1 [dx/dt] + F(t) \qquad (9)$$

This is the well known equation for discussing the forced motion of a linear damped oscillator and the charge flow in some electric circuits, in which case a_1 and b_1 are negative. The conservative part corresponds to the potential energy $-\frac{1}{2} a_1 x^2$. The trajectory of the system in the phase space given by x and dx/dt is shown in Fig. 9 for different oscillatory motions. However in many cases this approximation is not sufficient and it is necessary to consider terms of higher order resulting in a more complex equation and the trajectories in phase space, become very elaborate, but the actual situation depends on the values of the a & b coefficients. That means that the motion may even be unpredictable and/or show some instabilities. This situation is designated with the generic term "dynamical chaos". It should be made clear that not all nonlinear systems exhibit dynamical chaotic behavior, although the converse seems to be true.

A second example of Eq. (8) is the equation introduced by Helmholtz in 1870 to discuss the vibration of the ear's membrane when the vibrations are so large that the square of the displacement affect the motion; it includes the term a_2:

$$d^2x/dt^2 = a_1 x + a_2 x^2 + b_1 [dx/dt] + F(t) \qquad \text{(Helmholtz equation)} \qquad (10)$$

The conservative part corresponds to the potential energy $-\frac{1}{2}a_1 x^2 - {}^1/_3\, a_2 x^3$ of an asymmetric oscillator It has been represented in Fig. 10 for some particular values of the a&b parameters, showing that in this case, depending on the energy and the initial conditions, the motion is oscillatory or unbound. However the presence of the dissipative and forced terms may alter radically the dynamical behavior of the system, that may move back and forth on both sides of the potential barrier. More recently, Eq. (9) has been used in 1990 by J.M.Thompson to discus ship's stability to waves in windy situations.

A third example of Eq. (8), introduced by Duffing in 1918, contains the terms a_1 and a_3,

$$d^2x/dt^2 = a_1 x + a_3 x^3 + b_1 [dx/dt] + F(t) \qquad \text{(Duffin equation)} \qquad (11)$$

This equation describes a forced damped symmetric oscillator that, depending on the values and signs of the coefficients, may move in a single or a double potential well, the latter being the case of the inverse motion of the N atom in the N H_3 molecule. The potential energy associated with Eq. (11) is $-\frac{1}{2} a_1 x^2 - \frac{3}{4} a_3 x^4$ and has been represented in Fig, 11. The resulting motion depends critically on the last two terms. Again the dissipative and forced terms may move the particle over the potential barrier from one well to the other. (I should mention in passing that if the system is at the microscopic level these two cases may show quantum tunneling effects).

A fourth nonlinear Newtonian equation of physical interest contains the term c_{21} in addition to a_1 and b_1,

$$d^2x / dt^2 = a_1 x + b_1 [dx/dt] + c_{21} x^2 [dx/dt] + F(t) \qquad \text{(Rayleigh/van der Pol eq.)} \qquad (12)$$

This type of equation, with the proper choice of coefficients, applies to several physical problems[17]. It was first used by L. Rayleigh in 1896. Later on, in 1926, it was used by B. van der Pol to discuss an electric circuit in which the damping coefficient can change from positive to negative depending on the value of the electric charge, and in 1928, with J. van der Mark, was applied to discuss the aspect of heartbeats known as arrhythmia, Since then it has been applied to many other problems. Eq. (12) is relatively easy to analyze mathematically and the trajectory in phase space depends critically on the values of the three coefficients. Fig. 12 illustrates the trajectory in phase space for $a_1 = -1$, $b_1 = 0.3$, $c_{21} = -1$, and $F = 0$, that is

$$d^2x/dt^2 = -x + (0.3 - x^2)\, dx/dt .$$

Another situation that may appear in complex systems is when an external action on a coordinate or degree of freedom of a system affects other coordinates or degrees of freedom in the system. This is not surprising if the degrees of freedom are coupled by the internal interactions. The empirical equation using Newton's formalism for analyzing this type of complex system when the external action on each degree of freedom or "coordinate" x_i can be expressed in terms of a few parameters $y_j\ (x_j)$, each with a different probability w_{ij} of action on the degree of freedom or coordinate x_{i}, is obtained by adding a term $\Sigma\, w_{ij}\, y_j$ to Eq. (8). For example, in the case of Eq. (9) it has the generic form

$$d^2x_i/dt^2 = a_i\, x_i + b_i\ [dx_i/dt] + \Sigma_j\, w_{ij}\, y_j\,(x_j) + F_i(t) \qquad (13)$$

Terms with higher powers of the coordinates and the velocities may be added to Eq. (13). The coefficients a, b, w and F have to be determined experimentally or guessed empirically in each case. The non linearity is the result of the internal coupling of the degrees of freedom so that any external action on one degree of freedom has some probability of affecting others. The time behavior of the system depends on the values of the coefficients and in many instances the system develops instabilities and chaotic behavior. Eq. (13) appears in many physical and biological systems. For example, Eq. (13) applies to a chain of coupled damped harmonic oscillators, and it has been used by W. Freeman for his model of the olfactory system[18]. (In fact Freeman uses a set of 14 equations like Eq. (13), each with a different set of parameters)

The preceding examples (which are the standard given in most books dealing with nonlinear systems) serve to illustrate the variety of physical situations observed experimentally, that can be analyzed using empirical non-linear dynamical equations, based on an extension of Newton's formalism, and that in some instances give rise to an unpredictability in the motion and/or "dynamical chaos". As I said before, the main difficulty in dealing with those systems is not just the physics but the mathematical complexity, that requires knowing the coefficients and the initial conditions with great precision. Since the coefficients and/or initial conditions are not always known with absolute precision, the prediction of the path of the system in space-time may spread as the system evolves, some times exponentially, a situation opposite to that of linear systems (Fig. 13).

10. DYNAMICAL ANALYSIS: HAMILTONIAN APPROACH

Some complex systems with a few degrees of freedom, coupled by the internal interactions in the system, can be analyzed more conveniently in phase space using Hamilton's formalism. By a proper choice of the variables or "coordinates" x_i associated with each degree of freedom, the dynamical laws of the system may be simplified in some cases by expressing them in canonical form (by similarity with Hamilton's canonical formulation of the equations of motion, $dq/dt = \partial H/\partial p$, $dp/dt = - \partial H/\partial q$, where $H(p,q)$ is the hamiltonian of the system) by a set of (nonlinear) equations of first order in the time derivatives or "velocities" dx_i/dt,

$$dx_i/dt = f_i(x_1, x_2, \ldots, x_n, t) \qquad (14)$$

where $f_i(x_1, x_2, \ldots)$ has to be determined from first principles or guessed based on the model chosen for the system, and varies from one problem to another. Actually the Newtonian Eq. (6)can be written in the canonical form, Eq. (14), as follows:

$$dx/dt = y, \quad dy/dt = F(x,y,t) \qquad (6')$$

For example, in the case of a body falling in a viscous fluid, the Newtonian equation of motion $d^2x/dt^2 = - g - k\,(dx/dt)$, can be written in canonical form as $dx/dt = v$, $dv/dt = - g - k\,v$, that can be found in many introductory physics texts. For a damped harmonic oscillator, such as a pendulum of small amplitude, the Newtonian equation of motion is $d^2x/dt^2 = - k\,x - k'\,(dx/dt)$, that can be expressed in the canonical form $dx/dt = v$, $dv/dt = - k\,x - k'v$. These equations have already been represented in Fig. 9, that shows the possible "trajectories" in phase space of a damped harmonic oscillator. Similarly Eqs. (7), (8) and (12) can be written in canonical from as

$$dx/dt = y, \quad dy/dt = \Sigma\, a_n\,(t)\, x^n + F(t) \qquad (7')$$
$$dx/dt = y, \quad dy/dt = \Sigma\, a_n\,(t)\, x^n + \Sigma\, b_n\,(t)\, y^n + \Sigma\, c_{nm}\,(t)\, x^n\, y^m + F(t) \qquad (8')$$
$$dx/dt = y, \quad dy/dt = a_1\, x + b_1\, y + c_{21}\, x^2\, y + F(t), \qquad (12')$$

Eq. (12') corresponds to Rayleigh/van der Pol equation and was represented in phase space in Fig. 12, as indicated before. This equation has been used by M. G. Velarde et al. in a more general form to discuss neuron dynamics (See Table 3).

In most cases of practical interest it is sufficient to express the function $f_i\,(x_1, x_2, x_3, \ldots)$ as a power series, $dx_i/dt = \Sigma\, a_{i,jkl..;mnr\ldots}\, x_j^m\, x_k^m\, x_j^r$,,,,. Or equivalently

$$dx_i/dt = \Sigma\, a_{ij}(t)\, x_j + \Sigma\, a_{ijk}(t)\, x_j x_k + \Sigma\, a_{ijkm}(t)\, x_j x_k x_m + \ldots\ldots, \qquad (15)$$

The "a" coefficients are determined empirically in terms of the parameters (mass, electric charge, etc) associated with the components, and may depend on time if the system and/or the environment are changing. Ideally one should be able to derive the coefficients from first principles if we know the micro- and/or meso- descriptions of the system and how it is affected by the environment, but that is not always possible or even necessary. The larger the number of terms that have to be considered in Eq. (15), the more complex is the analysis of the system, making it more difficult to represent its time evolution in phase space, and the more unpredictable or chaotic may be its behavior.

The simplest approximation to Eq. (15) is to assume that the "a" coefficients are constant and linearize the equations by keeping only the first summation, that is $dx_i/dt = \Sigma\, a_{ij}\, x_j$, in which case the analysis of the system is relatively simple[19]. This applies for example to the "normal" atomic vibrations in a crystal lattice. The next approximation is to consider only the first two summations, so that the equations are not linear, that is

$$dx_i/dt = \Sigma\, a_{ij}\, x_j + \Sigma\, a_{ijk} x_j\, x_k\,. \qquad (16)$$

This is the type of equation most commonly used in discussing non-linear systems in which there is a coupling between pairs of "coordinates" or degrees of freedom. Amazingly it applies to several quite different systems. The systems to which this approximation applies usually display unpredictable or chaotic behavior.

The best known, and perhaps the first, example of this approximation are the Lorenz equations, that are a set of three equations introduced in 1963 by the meteorologist E. Lorenz for modeling convection in the atmosphere using three variables related to convection velocities and temperature gradients, but their derivation is rather elaborate and not very relevant for an introductory physics course. Another similar set of three equations are the Maxwell-Bloch equations, that relate the electric field, the mean polarization of atoms, and the amount of population inversion in laser operation, as rearranged by H. Haken in 1975 to discuss laser instabilities. A third well known example of three non linear equations are those formulated in 1972 to analyze the time evolution of the concentration of the chemicals involved in some complex chemical reactions, such as the Belousov-Zhabotinskii auto-catalytic reaction. These three sets of equations can be found in most books dealing with chaos and have been studied extensively[20]. They are shown in Table 3. Lorenz equations are represented in Fig. 14 for some particular values of the coefficients. These equations may show chaotic dynamical behavior depending on the values of the coefficients. (It is interesting to note that systems that are so different can be described by the same type of equations).

Needless to say, I have not exhausted all the mathematical possibilities for dealing with the dynamics of nonlinear systems and some systems may require empirical equations more elaborate than the preceding equations or different mathematical methods[21]. The fact is that, though there is not a single prescription for treating mathematically a nonlinear system, Eqs. 8,13&15 provide a general framework, and each system has to be dealt with almost on an ad hoc basis to identify the right set of equations and obtain the appropriate solutions. Also in my analysis I have deliberately ignored the rich terminology and concepts that have been developed for dealing with nonlinear systems[22], (Poincare sections, Lyapunov exponents, chaotic attractors, bifurcation diagrams, K-entropy, etc), because my intention has been to show that the basic issue for dealing with complex systems in an introductory physics course is that students become aware of the need to find the right and accurate set of dynamical equations for the system, based on some model, and solve them using modern computational and modeling techniques, and that this need is even more critical when the system is nonlinear.

11. CONCLUSION

I have tried to present the analysis of changes and organization in complex systems from different but complementary approaches, that I believe are appropriate for introductory physics courses. I hope I have made clear that the study of complex systems is very important because indeed they are the most common and interesting and because it is necessary to recognize that they do not represent a new physics or a new science, as some have said (there are no specific laws of complexity or of chaos). Complex systems obey well established laws of physics (Newton's law for the motion of particles, Navier-Stokes equation in fluid dynamics, Fick's law for transport phenomena, Ohm's law and Faraday-Henry's law in electric circuits, Schrodinger's equation for the motion of electrons in atoms and molecules, etc) that are general and relatively simple.

The only thing new to point out to the students is the recognition, or perhaps the revelation, that, because of their complexity, the behavior of some systems can not be predicted with precision, even if we know the dynamical laws, and the analysis of their evolution in time requires much more precise information and measurements and elaborate mathematical and computational techniques than those required for the analysis of the quasi-linear systems that have been dealt with traditionally in physics, such as planetary motion and electronic structure of atoms, for which more simple approximations, such as the technique of perturbations, are sufficient.

My second purpose has been to emphasize that chaotic behavior is nothing mysterious or unusual, but directly related to the interactions and structure of a system, something physics teachers should bring to the attention of the students. A third point, I wanted to make clear is the importance of the fine tuning of the coefficients for properly describing the deterministic, though may be unpredictable or chaotic, behavior of a system when its dynamical laws are expressed mathematically by empirical non-linear equations. A fourth point is that the behavior, some times chaotic, of many complex systems encountered in chemistry, biology, economics, sociology, etc, can not be expressed quantitatively by equations of the type (4), (8) and (15) that apply to most physical systems, and that poses a challenge to those trying to formulate a general theory of complex systems.

In summary the students must recognize that the challenge in the study of changes and organization of complex systems is to identify the relevant components of the system, determine quantitatively their external and internal interactions, explore the role of interactions that lead some systems to become unpredictable and even chaotic, identify the dynamical laws, and play with a computer to obtain the solutions of the dynamical equations. This is the "Nintendo" game of the physicists.

Acknowledgement. I wish to thank all those with whom I have had interesting exchanges of ideas, particularly Manuel G. Velarde, that helped me clarify some of the topics I have discussed.

NOTES

1. See Ref. 1
2. See Ref. 14
3. See Ref. 16
4. See Ref. 15, Vol. II, Part IV, Chs. IV, VI and IX
5. P. Ball, Ref. 5; D'A Thompson, Ref. 12; A. Turing, The Chemical Basis of Morphogenesis, Phil. Trans. Roy. Soc. **B237**, 37, 1952 (pioneer research on the origin of forms and patterns}

6. To study how a property varies throughout the system the notions of "correlation length" and "correlation function" have been introduced. There are different ways of defining these two notions. I will use a simple one. Let us designate by $f(x_I)$ and by $f(x_I + \lambda)$ the values of a property at two points separated the distance λ Then the space correlation function $C(\lambda)$ is defined by

$$C(\lambda) = (1/N)\,[\Sigma_i\, f(x_i)\, f(x_i + \lambda)] = \langle f(x_i)\, f(x_i + \lambda)\rangle, \qquad (i = 1 \ldots\ldots N)$$

where the summation is carried over all the space cells that are occupied by the system. Obviously $C(0)$ is positive and presumably $C(\lambda)$ decreases as λ increases. We may assume that $C(\lambda) \approx \exp(-\lambda/\lambda_o)$ and call λ_o the "correlation length". When $\lambda_o = 0$ there is no correlation of the property in the system and the correlation function is a δ-function. As $\lambda_o \to \infty$ the correlation of the property is stronger. One can define in the same way a time correlation function $C(\tau)$, to study the correlation between a series of signals emitted at times t_i. It is useful, for example, for detecting anomalies in the heart beats.

7. This is the usual methodology in physics, although some times the complexity of the system makes it difficult to follow it. Historically the first serious effort in this direction was the kinetic theory of gases, in whose development Maxwell played an important role. Classical thermodynamics is an example of the use of a methodology for the analysis of a system at the macro-level, while statistical mechanics is a methodology for the analysis of the system at the micro-level; same applies to fluid dynamics and to condensed matter. In general the two methodologies have to be blended together for a comprehensive analyze a system.

8. See Y. Imry, Ref. 9, The term mesoscopic was introduced by N.G.van Kempen in 1981, Ref. 11.

9. See M. Alonso and E. J. Finn, An Integrated Approach to Thermodynamics, The Physics Teacher, **33**, p. 296 (1995)
10. Incidentally the evolution of the universe since the initial singularity or BB shows the continuous emergence of new levels of structures and organization, some only with structural complexity and others with functional complexity. Which structures and organizations appear in each portion of the universe depends on the dominant interaction and the kinetic energy and nature of the particles in that particular portion.. However it seems that there is a trend toward uniformity in the grand scale.
11. To illustrate the versatility of Eq. (1), suppose the system is a social group. In this case the subsystems are individuals, the term E(K) corresponds to the activity of each individual (economic, cultural, etc), the term E(I) corresponds to the diverse modalities by which individuals can interact (communication, exchange of ideas, services and goods, etc), and the term E(P) corresponds to the characteristics of each individual (education, character, etc). Of course in this case it may be very difficult to express the three terms quantitatively.
12. This is the real meaning of the second law. See Ref. 17. By the same token, entropy is not a measure of disorder (this is nonsense) but a measure of how close an isolated system is to its optimum configuration in phase space and for that reason the entropy of an isolated system increases until it attains its maximum value when the system reaches equilibrium. This is another way of stating the second law. Another matter is how to measure entropy. Unfortunately the concept of entropy has been misused and abused by many people and applied in other contexts. For example when Shannon uses the term entropy in connection with the information content of a message he is talking of a different concept that he should have called info-entropy, even if it is expressed in a form that bears some algebraic resemblance to statistical entropy, but with a different meaning.
13. See Refs. 3, 7 and 19.
14. One well known approximation is the "single particle approximation", in which $\Sigma_{ij} E_{int}(ij)$ is replaced by $\Sigma_i E_{int,av}(i)$ (Fig. 15) but it is inadequate for explaining many physical situations because it smoothes out the interactions by linearizing the system. However this approximation is very useful in statistical mechanics and in atomic and nuclear structure.
15. For a more extensive treatment of ACS see J.H.Holland, Ref. 8
16. For a definition of conservative and dissipative systems in phase space see Ref. 7, Secs. 3. 5 and 3.9 and Ref. 4, Ch. 3. The one given is more appropriate for the introductory physics course.
17. See R. Hilborn, Ref. 7
18. H-J Chang, W. J. Freeman & B. C. Burke, Neural Networks, **11**, p. 449, 1998; Y. Yao & W. J. Freeman, Neural Networks, **3**, p. 153, 1990
19. In fact this set of equations can be represented symbolically by $dX / dt = \mathbf{A} . X$, where $\mathbf{A}$ is a linear operator acting on the vector X..
20. See R. Hilborn, Ref. 7, and G. L. Baker, Ref. 3
21. An interesting idea, that has been considered by Rodolfo Llinas, is to express the interaction of a system with the surroundings by an operator H (we call it in physics an interaction hamiltonian) and express it by X = H Y, where X is the set of internal variables and Y the set of external variables, that is H establishes a correlation or mapping between the external variables Y and the internal variables X. (see e.g. R. Llinas, Neuroscience, 7, p.2949, 198, and Lecture Notes in Biomathematics (1982). I have not studied well enough Llinas' ideas to assess to what extent the model is general and quantitative (it depends critically on H, X and Y), but certainly it is very attractive from the point of view of a physicist.

Similarrly W. Kauffman, Ref. **8**, ha tried to represent a complex system by a network with N components, each coupled to K neighbors and for that reason are called NK-networks. Though the model is interesting, I consider this approach too simplistic because in the first place the model is not quantitative in the sense that the interaction is not expressed mathematically and second K is kept small, so that long range effects are not considered, although they are very important when the interactions are very strong. Beside the model does not take into consideration in an explicit way the interaction with the surroundings.

22. It is interesting to note that H. Poincare developed at the turn of the 19th century the graphical methods for analyzing nonlinear systems in phase space and introduced many of the concepts used today in non linear dynamics.

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FIGURE CAPTIONS

TABLE 1. THE SCIENCE PROGRAM

Basically all scientists in their research follow what may be called the "science" program, that consists of the following elements

.

1. Research depends on observation, experimentation, theoretical analysis and computation.

2. The subjects of study are "systems", that in physics may be galaxies, stars, condensed matter, gases, molecules, atoms, nuclei, fundamental particles, etc

3. Systems are dissected into components or "subsystems"

4. To systematize observations scientists formulate empirical relations and general laws (which of course are subject to change) and formulate new concepts (momentum, energy, etc)

5. Scientists look for structures and processes in the systems analyzed

6. The next step is to look for the "interactions" among the components to explain the structures and processes in the systems

7. Scientists build a "model" of the systems, based on the preceding ideas. Models are refined or discarded as more information about the systems is gathered

TABLE 2

LEVELS OF ANALYSIS OF PHYSICAL SYSTEMS

MICRO (quantum)	MESO	MACRO (continuum)	
$< 10^{-9}$ m		$> 10^{-6}$ m	(size of system)
$N \approx 10$	10^{2} -- 10^{9}	$> 10^{9}$	(number of particles)
$\Omega \approx 10^{-27}\ \mathrm{m}^3$	$10^{-26}\ \mathrm{m}^3$ -- $10^{-18}\ \mathrm{m}^3$	$> 10^{-18}\ \mathrm{m}^3$	(volume per particle)

For continuum physics: $N \to \infty$, $\Omega \to \infty$

$n = N / \Omega$ = particles per unit volume $\approx 10^{27}\ \mathrm{m}^{-3}$ (For most systems)

For comparison: metals $n \approx 10^{28}$ electrons/m^3
semi-metals $n \approx 10^{26}$ electrons/m^3
semi-conductors $n \approx 10^{21}$ conduction electrons or holes/m^3

$L = n^{-1/3}$ = basic length per particle $\approx 10^{-9}$ m (for most systems)

λ_0 : correlation length of a property; length over which a property varies

$\approx L$: requires microscopic description (QM)

$\gg L$: macroscopic description (SM, no QM)
(continuum if $L \to \infty$)

$\approx$ 10 L to 10^3 L : mesoscopic description of some electric, optical, mechanical, etc properties (applicable to electronic devices, micro-robots, quantum computers, etc)

TABLE 3

EXAMPLES OF EQ. (16)

$$dx_i / dt = \sum a_{ij} x_j + \sum a_{ijk} x_j x_k$$

1. Lorenz equations

$$dx/dt = -a(x + y)$$

$$dy/dt = -xz + bx - y$$

$$dz/dt = xy - cz$$

2. Bloch-Maxwell-Haken equations

$$dx/dt = a(-x + y)$$

$$dy/dt = b(xz - y)$$

$$dz/dt = c(k + 1 - z - kxy)$$

3. Belousov-Zhabotinskii equations

$$dx/dt = -kxy + k'z - k''x$$

$$dy/dy = -kxy + k'z - k''y$$

$$dz/dt = -kxy - k'z - k''z$$

4. M. G. Velarde et al. (to be published)

$$dx/dt = y$$

$$dy/dt = -(k'' + k''' u)x + (k + k'u)y - kx^2y$$

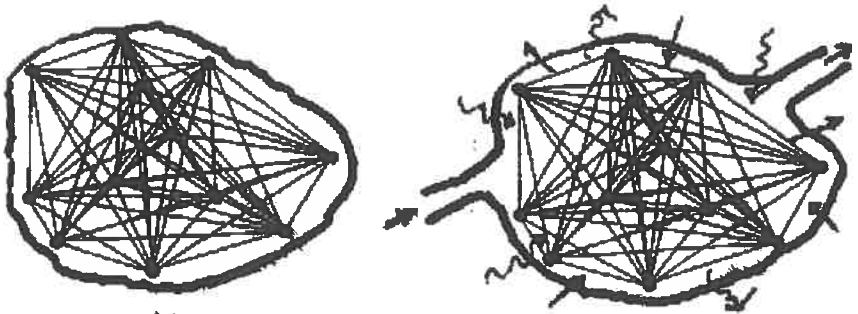
$$dw/dt = g(w) - z - x$$

$$dz/dt = 0.5(w - a)(w^2 + 0.1)$$

$$du/dt = f(u) - v + hw$$

$$dv/dt = 0.05(u - b)(u^2 + 0.5)$$

(Note the similarity of the first two equations with Eq. (12'))



(a)
closed

(b)
open

Fig. 1

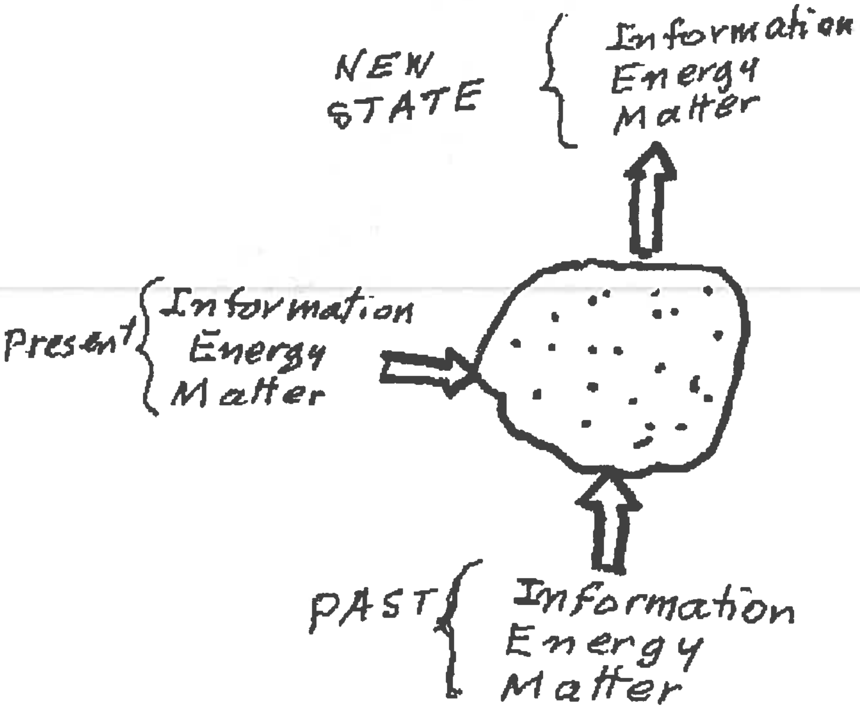


Fig. 2

HEWLETT PACKARD

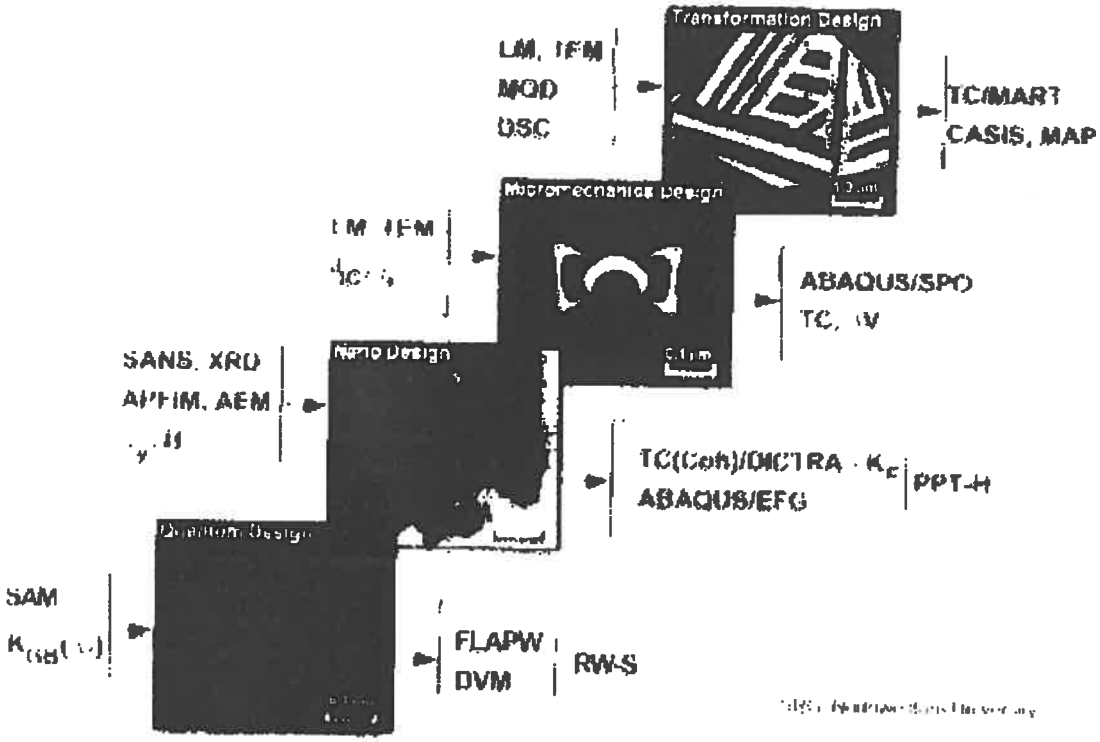
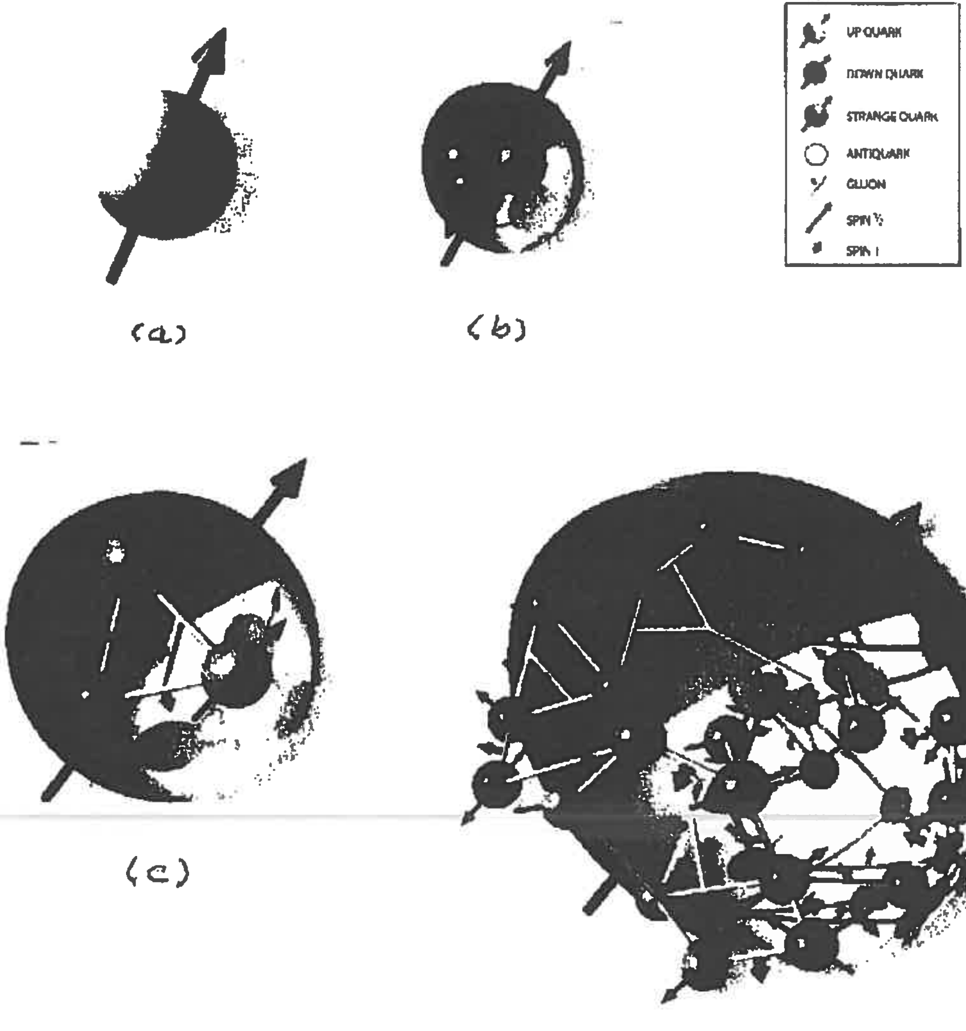

Transformation Design
LM, TEM
MOD
DSC
TC/MART
CASIS, MAP
Micromechanics Design
LM, TEM
ABAQUS/SPO
TC, IV
Nano Design
SANS, XRD
APFIM, AEM
TC(Coh)/DICTRA · K_c
ABAQUS/EFG
PPT-H
Quantum Design
SAM
K_{GB}
FLAPW
DVM
RW-S

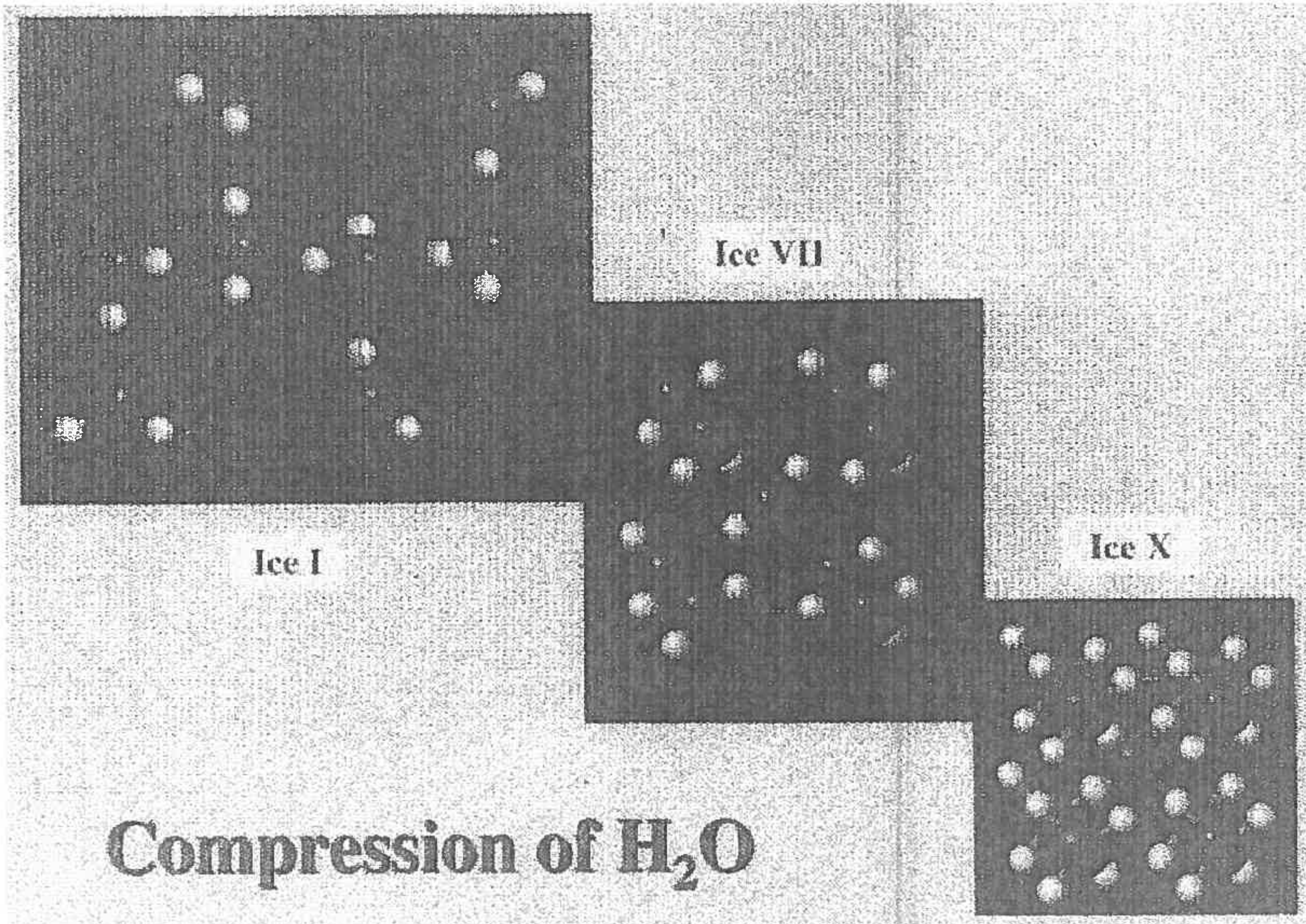
Fig. 3

Levels of Resolution of a NUCLEON



(a) (b) (c) (d)

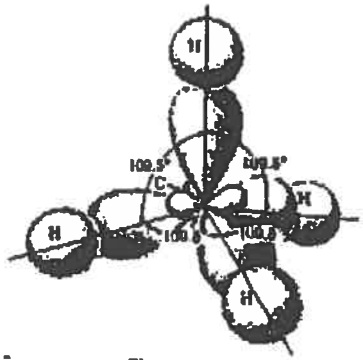
Fig. 4



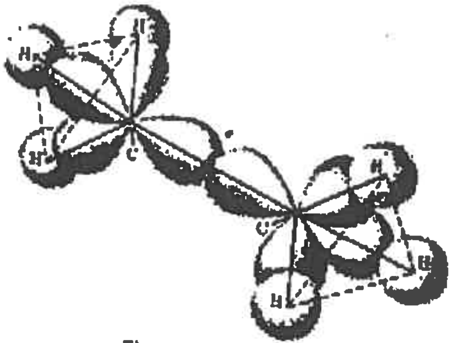
Compression of H_2O

Fig. 5

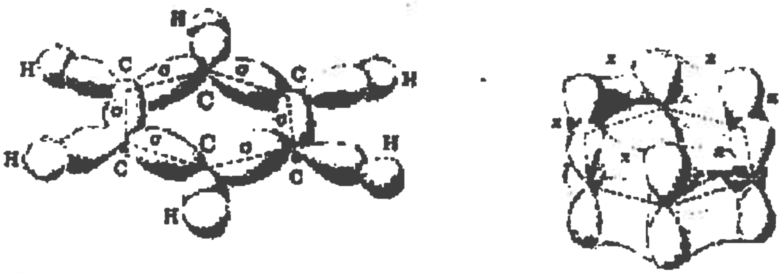
Electronic configuration of simple C compounds.



Electronic distribution in the CH_4 molecule, using sp^3 orbitals.



Electronic distribution in the C_2H_6 molecule, using sp^3 orbitals.



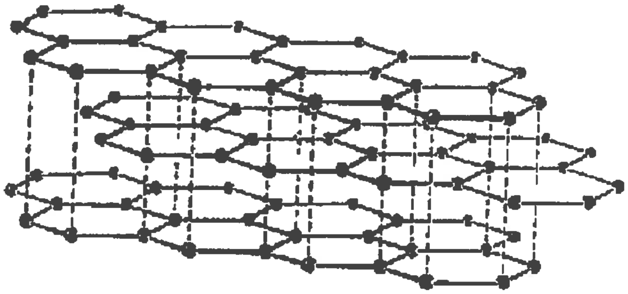
(a)

(b)

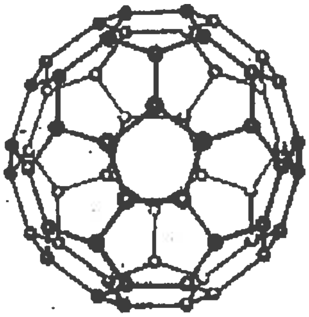
Benzene molecular orbitals. (a) Localized sp^2 bonds; (b) unlocalized p_z bonds.

Electron distribution in butadiene corresponding to non-localized p_z electrons.

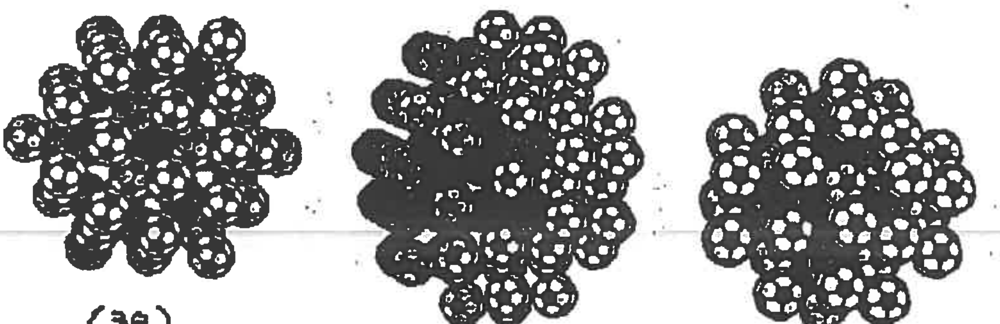
Fig. 6a



Graphite lattice. The carbon atoms are arranged hexagonally in parallel layers bonded by electrons localized along the solid lines. Electrons move freely between planes



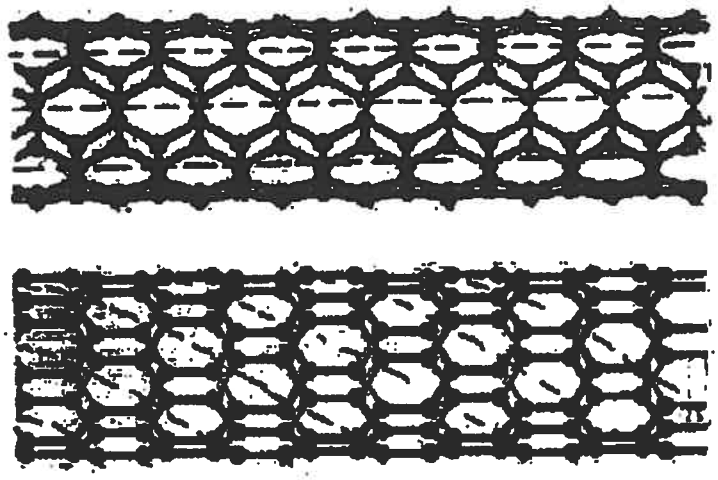
C_{60} molecule



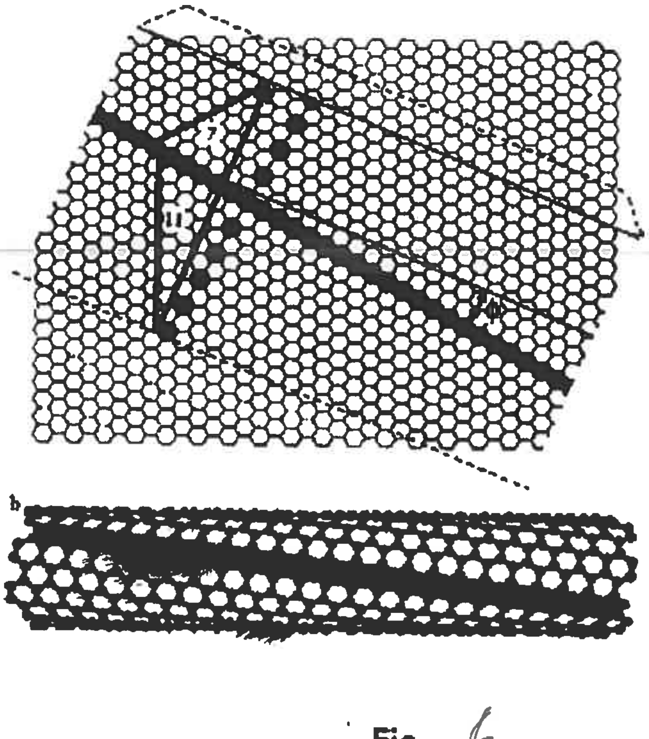
(38) (75) (55)

Fig. 6b

NANOTUBES

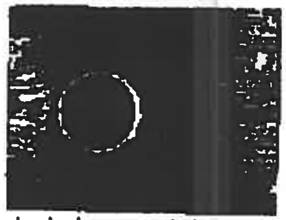


Twister. In-line carbon hexagons *(top)* conduct easily, while spiraling hexagons make a semiconductor.



HOW TO WRAP A NANOTUBE from a single sheet of graphite. First, copy the top part (a) onto a transparent overhead sheet. (This panel can also be downloaded from http://vortex.tn.tudelft.nl/~pt on the web.) Second, cut out the part of the sheet indicated by the dashed lines. Third, fold up the cut-out part so that the blue arrow's tail coincides with its head. Match the hexagons on the overlap tab carefully to those on the tube body. Finally, fasten the tube together with transparent tape. A nanotube with indices (11,7) now has been formed. The indices describe the projection of the circumferential vector (the blue arrow) onto the basis vectors of the graphite lattice, as shown. The red row illustrates the winding of hexagons along the tube direction. It forms the angle Φ (the chiral angle) with respect to the tube axis. Wrapping the sheet along the yellow or green hexagon rows instead of along the blue vector leads to $(n,0)$ "zigzag" tubes (so named because of the rows of alternating carbon bonds around the circumference) or (n,n) "armchair" tubes with hexagon rows parallel to the tube axis, respectively. The indices (n,m) are crucially important for the electronic properties of nanotubes: Tubes for which $n - m = 3i$, with i an integer, are metallic; all others are semiconducting. (b) Atomic structure of the (11,7) nanotube that results from the folding above. (Image courtesy of Philippe Lambin, University of Namur.)

Fig. 6



[illegible]

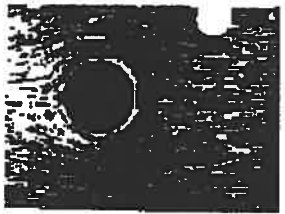


Fig. 2. [illegible] cylinder [illegible]

Fig. 3.—Formation [illegible] trailing [illegible]

Fig. 4. [illegible]

Fig. 5. The picture [illegible] after starting.

Fig. 6. The eddies grow still more; [illegible]

[illegible]

Fig. 7 a

[illegible]

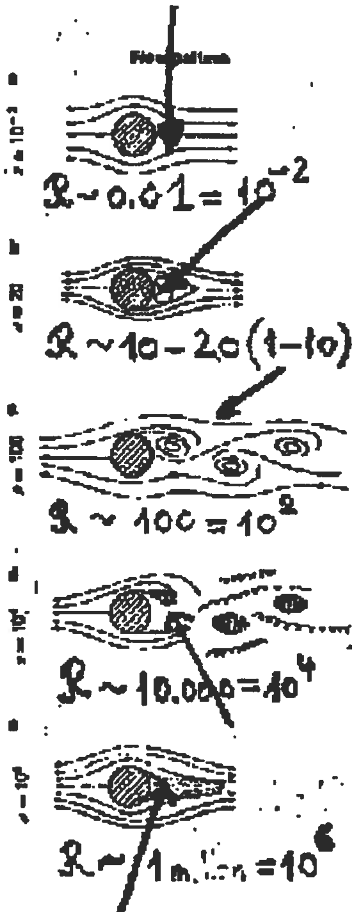
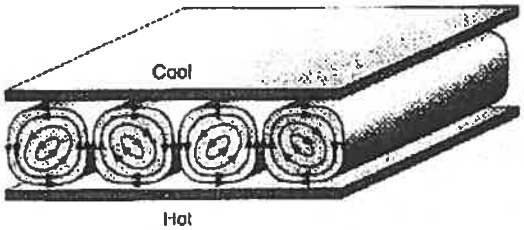
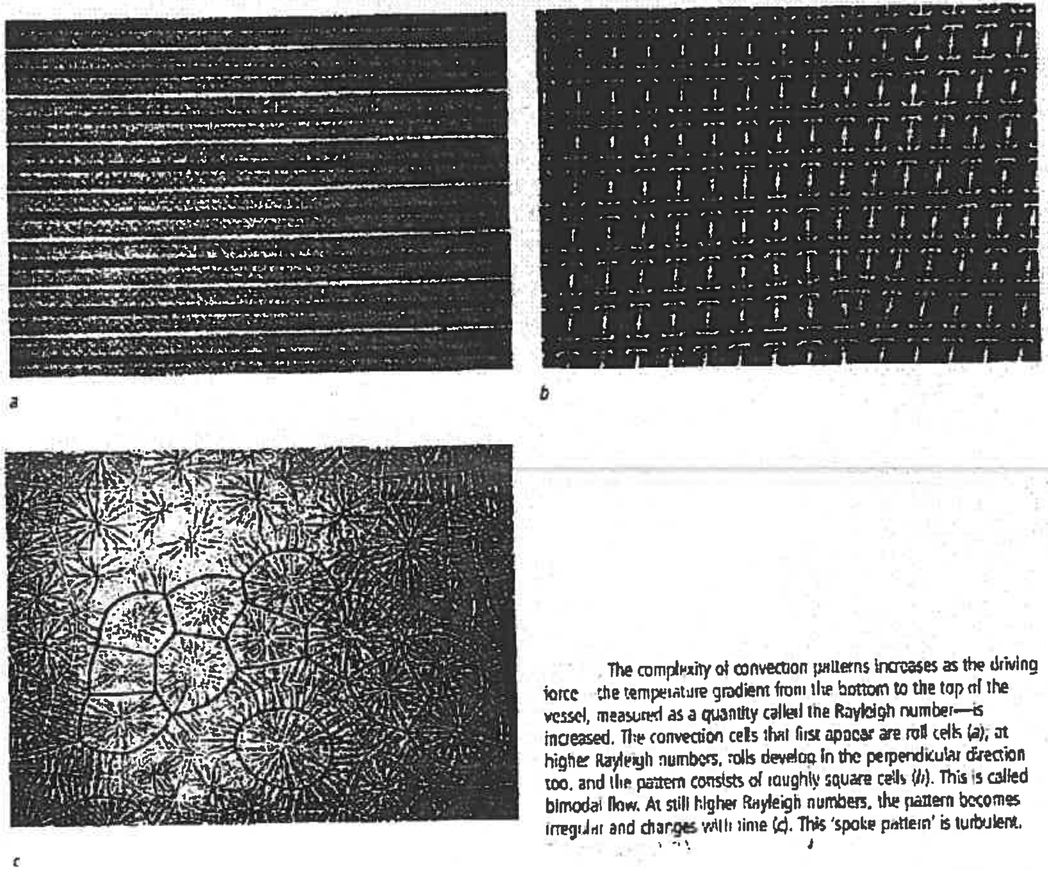


Fig. 7 b

Benard Cells



. Convection roll cells, which appear in a fluid confined between a hot bottom plate and a cooler top plate. The cells are roughly square in cross-section, and adjacent cells rotate in opposite directions.



The complexity of convection patterns increases as the driving force the temperature gradient from the bottom to the top of the vessel, measured as a quantity called the Rayleigh number—is increased. The convection cells that first appear are roll cells (*a*); at higher Rayleigh numbers, rolls develop in the perpendicular direction too, and the pattern consists of roughly square cells (*b*). This is called bimodal flow. At still higher Rayleigh numbers, the pattern becomes irregular and changes with time (*c*). This 'spoke pattern' is turbulent.

Fig.8

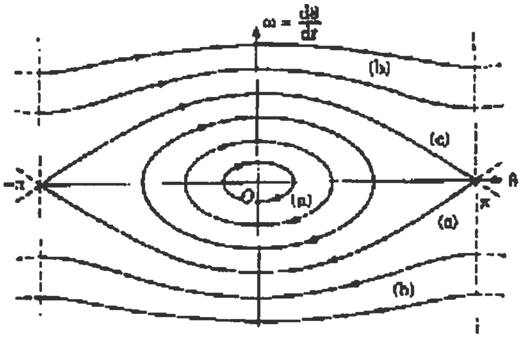


Figure 9 a Phase space diagram of pendular motion. (a) Oscillatory motion $E < E_0$, (b) modulated rotation $E > E_0$, (c) $E = E_0$.

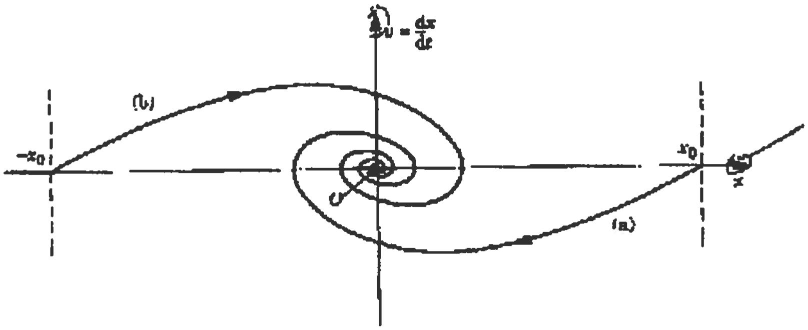


Figure 9 b Phase space diagram of a weakly damped oscillator. The attractor is the origin.

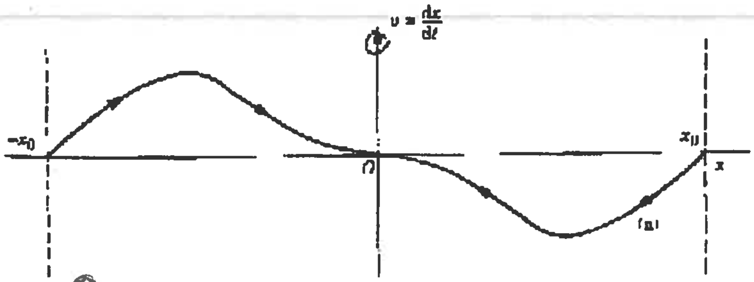


Figure 9 c Phase space diagram of a strongly damped oscillator.

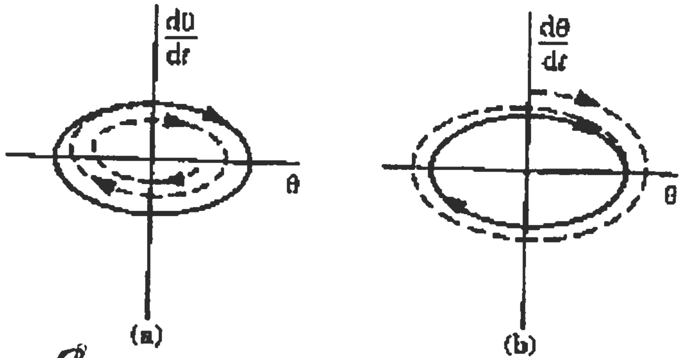


Figure 9 **d** Phase space diagram of a forced pendulum.

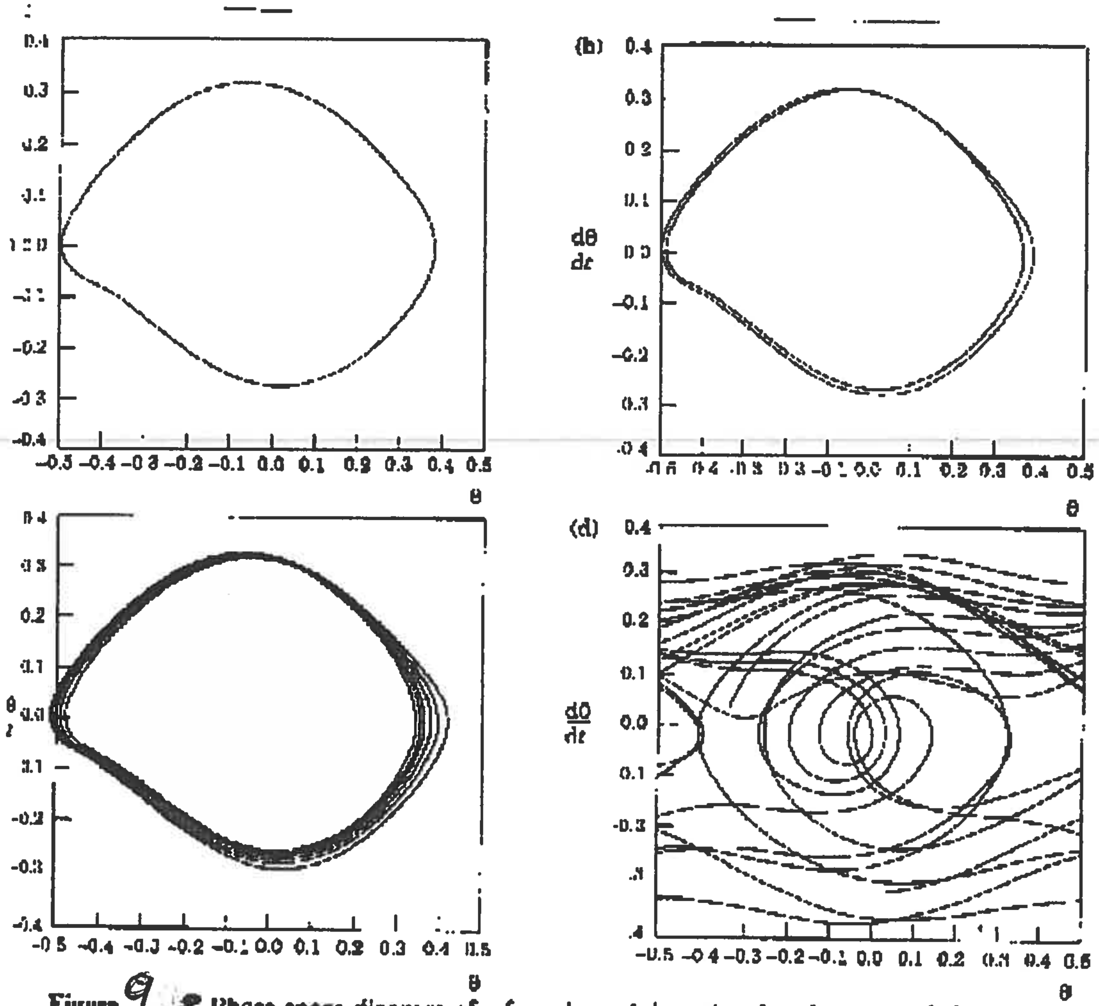


Figure 9 **e** Phase space diagram of a forced pendulum showing the onset of chaos for $\omega_f = 2\omega_0$ and different dampings. (Courtesy of T. Bohr and F. Christiansen, Niels Bohr Institute.)

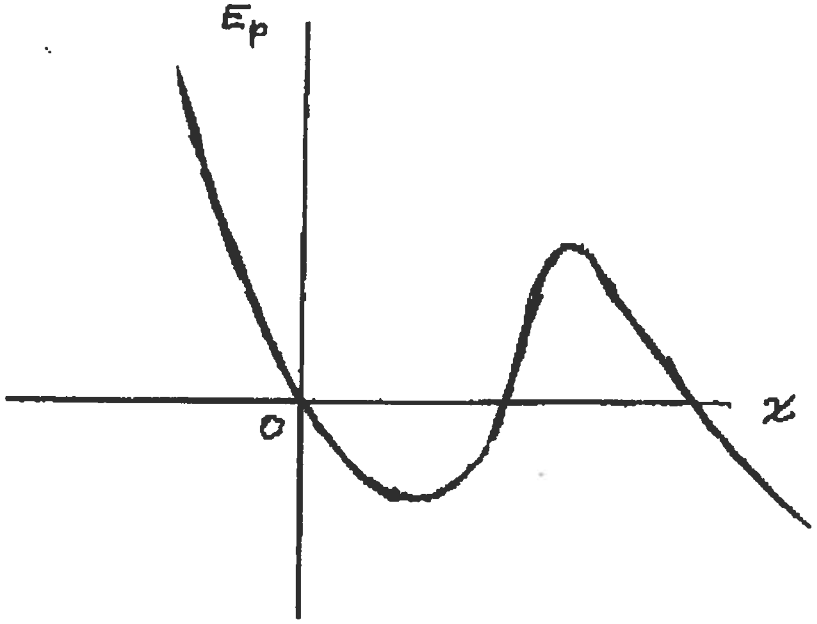
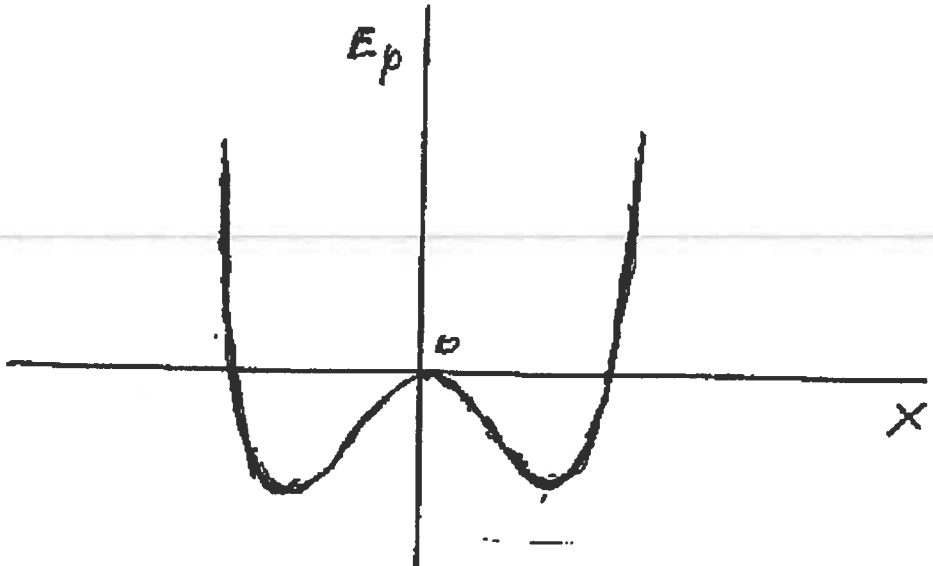

E_p
O
x

Fig. 10


E_p
O
X

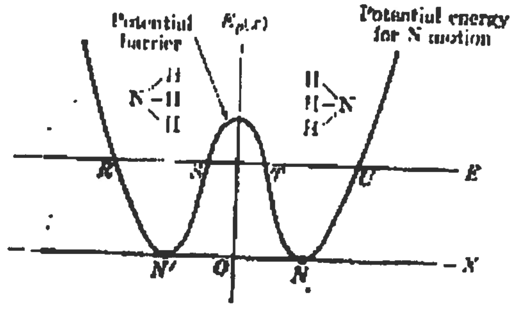
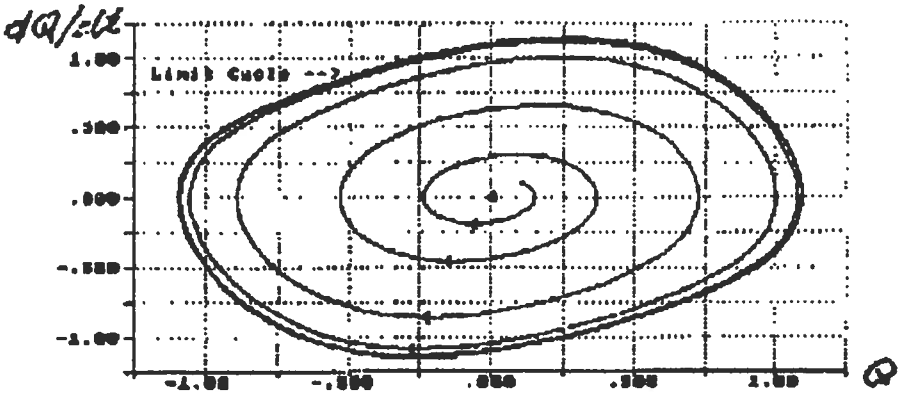

Potential barrier
$E_p(x)$
Potential energy for N motion
R
S
T
U
E
N'
O
N
X

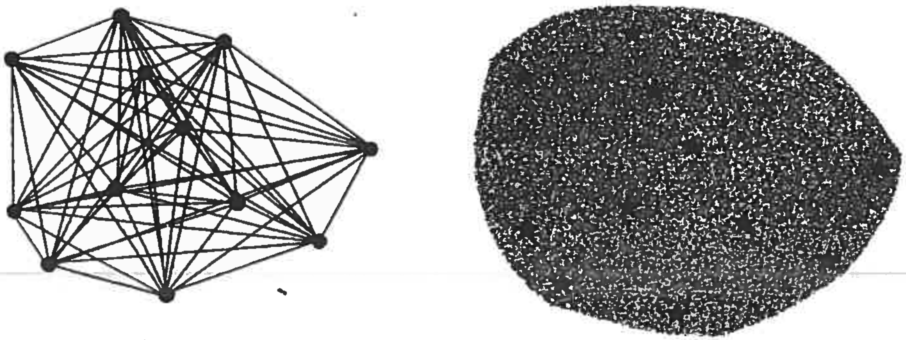
Fig. 11



A state space trajectory starting near $U = 0.1$, $Q = 0.1$ for the van der Pol oscillator with $R = 0.3$. The fixed point at $U = 0$, $Q = 0$ is clearly a spiral repellor. Trajectories approach the limit cycle as $t \to \infty$.

(R. C. Hilborn, Ref. 7) (u = dQ/dt)

Fig. 12



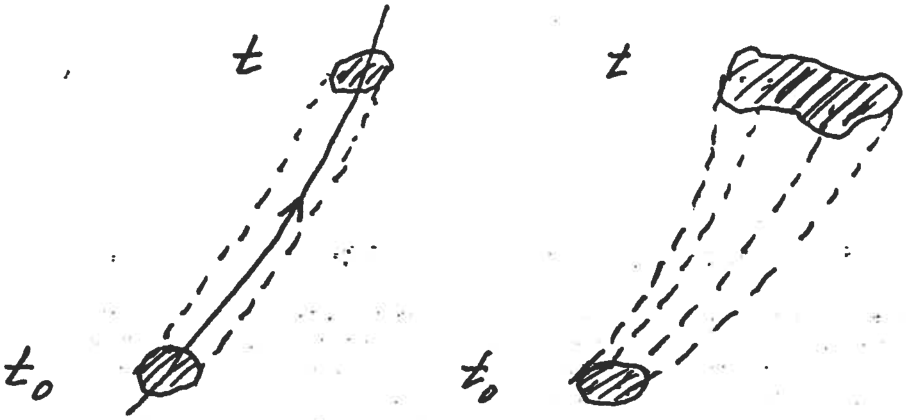
(a)

interacting particles

(b)

independent particle approx.

Fig. 15



(a) linear (b) non-linear

Prediction of the evolution in time of a system. The shaded areas show the possible states associated with the system at times t_0 and t.

Fig. 13

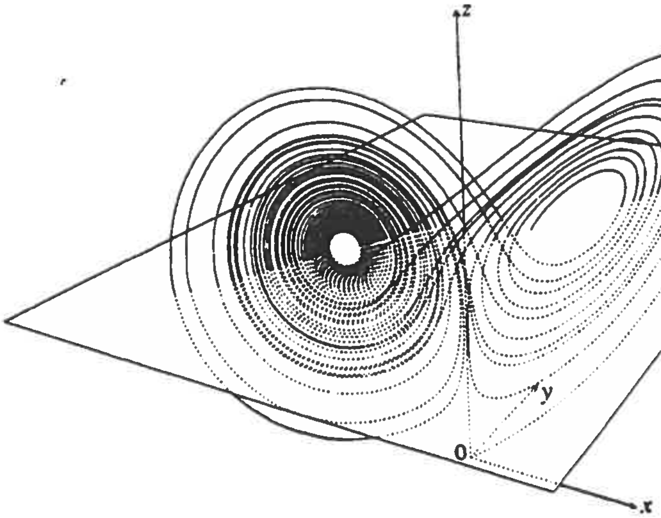


Fig. 14

Lorenz trajectories in phase space