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Nonlinear Dynamics of Galactic Disks

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Nonlinear Dynamics of Galactic Disks

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ABSTRACT

It is convenient to split the nonlinear dynamics of galactic disks into the wave and vortex dynamics, which could be splitted, in turn, into structures and turbulence.

Nonlinear stationary wave structures — envelope solitons — have a form of spirals and described by the nonlinear stationary Schrödinger equation.

The wave turbulence is presented by the theory as a turbulence of the Rossby waves, and it is described by the Charney-Obukhov (or Hasegawa-Mima) equation with a vector nonlinearity. Its spectrum differs from Kolmogorov's one, because the medium is anisotropic — rotating disk — and inhomogeneous. The theoretical turbulent spectrum of the Rossby waves coincides with observational one for the interstellar medium.

There are two kind of vortex structures — linear and nonlinear. Linear vortices jointly with spiral arms compose unique spiral-vortex structure. Recently linear vortices were discovered at 6m telescope in Russia. Nonlinear vortices have not been yet observed in galactic disks, although the theory predicts their existence in the form both of solitary vortices (cyclones and anticyclones) and double vortices (modons). Dynamics of vortices is described by the Charney-Obukhov (or Hasegawa-Mima) equation with a scalar and vector nonlinearities.

Vortex turbulence in galactic disks must be strong, as a rule.

Also we consider the interaction of a medium with vortices and waves. The vortex-medium interaction leads to the decay of a vortex and the excitation of waves and wave turbulence, the nature of which has not been investigated so far.

The interactions of nonlinear spiral density wave with a dissipativeless medium causes a large-scale 3D convection in the form of four vortices separated by vertical cylindrical surface $r = r_{cor}$ and the central $z = 0$-plane of a disk. For trailing spiral waves the radial velocity in the central plane of the disk is negative inside the corotation circle and is positive outside. For leading spirals the situation is vice versa. The large-scale convection can play a significant role in the redistribution of the chemical composition in the disk.

The interaction of nonlinear trailing spiral density wave with a dissipative medium of the disk causes accretion radial mass flux inside the corotation circle, which feeds the central part of a galaxy.

INTRODUCTION

Among different astrophysical objects disks have the most various dynamical structures and different kind of turbulence. So far the origin of many observed structures in disks is puzzle as well as turbulence mechanisms of different kinds of disks. The problem of the galactic spiral structure is waiting for own solution more than one and a half century. The origin of the Cassini Division with its complex inner structure, the cause of the turbulent viscosity many orders greater than the molecular one in accretion disks, non-Kolmogorov turbulence spectrum of the interstellar gas in the solar vicinity of the Milky Way and the problem of double galactic nuclei not connected with merging are still unsolved problems. It is convenient to split the non-linear dynamics of astrophysical disks onto the wave and vortex dynamics, which in turn could be subdivided onto structures and turbulence. The main topics of the paper are summarized in the Table 1.

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NONLINEAR DYNAMICS OF MARGINALLY UNSTABLE SELF-GRAVITATING DISK

Special investigations[1] have shown that the gaseous disks of galaxies are near the boundary of own gravitational instability. This fact is an expected one as the instability increases the velocity dispertion and the disk is coming to the boundary of the instability.

If a rotation velocity curve of the gaseous disk has rather large jump or kink that is observed in more than one half spiral galaxies[2], then hydrodynamical instability can be developed[3,4]. In this case disk also lies near the boundary of own hydrodynamical instability[5]. The reason is similar. As a result of the instability the smearing of the jump begins to grow until the disk reaches the marginal stability.

The nonlinear dynamics of the marginally unstable self-gravitating disk was analyzed in works of Mikhailovskii, Petviashvili and Fridman[6,7,8] (see also Fridman and Polyachenko[9]). As the eigen frequency of marginally unstable disk in co-rotating frame of reference $\hat{\omega}$ is small ($\hat{\omega} << \Omega$), the consideration of the problem in 2D approximation is valid[10]. Under condition that only the small region of wave vectors is unstable $\Delta k << k_0$, where k_0 is the wave vector of the most unstable perturbations, the non-linear dynamical equation was derived:

$$\frac{\partial^2 \epsilon}{\partial \tau^2} = -\nu_0^2 \epsilon + \frac{3}{2}(2-\gamma_S)\left(\gamma_S - \frac{5}{3}\right)|\epsilon|^2 \epsilon. \quad (1)$$

Here ϵ is the dimensionless amplitude of the azimuthal velocity, τ is non-dimensional time,

$$\nu_0^2 = \frac{(\pi G \sigma_0 / c)^2 - \kappa^2}{\Omega_0^2} << 1 \quad (2)$$

determines the dimensionless grow rate of the most unstable perturbations, G - gravitational constant, σ_0 - unperturbed surface density of the disk, κ - epicyclic frequency, Ω_0 - unperturbed angular velocity of the disk, γ_S is a "surface" polytropic index. Eq. (1) leads to the non-linear dispertion relation in the form

$$\nu^2 = \nu_0^2 + \frac{3}{2}(2-\gamma_S)\left(\gamma_S - \frac{5}{3}\right)|\epsilon|^2. \quad (3)$$

It is easyly seen that this relation describes either a solution propagation or explosive instability depending on the value of the "surface" polytropic index γ_S.

According to Hunter[11] the surface polytropic index for self-gravitating disk can be expressed through the real 3D polytropic index γ_V as $\gamma_S = 3 - 2/\gamma_V$. Thus in the region

$$5/3 < \gamma_S < 2, \tag{4}$$

which is equivalent to $3/2 < \gamma_V < 2$, the nonlinear stabilization of the instability is possible at the certain level of the azimuthal velocity amplitude

$$\epsilon^2 = \frac{2\nu_0^2}{3(2-\gamma_S)(\gamma_S - 5/3)}. \tag{5}$$

Under condition of marginally unstable disk ($\nu_0^2 << 1$) the stabilization is achieved at low perturbed amplitude. The soliton solution is possible in the region (4) in the form of an envelope soliton (FIGURE 1). In the absence of the viscosity the soliton has a classical symmetrical form likes a normal distribution function (FIGURE 1a). But in the presence of the small viscosity a soliton transforms into the shock wave with oscillating front[12] (FIGURE 1b). The shock wave which was predicted in a rotating stellar disk — collisionless shock wave — has a similar form[13].

If $\gamma_S < 5/3 < 3/2$ — then the explosive instability occurs:

$$\frac{\epsilon}{\epsilon(\tau = 0)} = \frac{1}{\tau - 1/(A\epsilon(0))}, \tag{6}$$

where $A^2 = 3(2-\gamma_S)(5/3 - \gamma_S)/4$.

SPECTRUM OF TURBULENCE OF CLOUDY POPULATION OF THE MILKY WAY

Up to there are numerous investigations devoted to measurements of the turbulent spectrum of cloudy population of the Milky Way both in the every gaseous cloud and in the ensemble of clouds. In 1955 Kaplan found the correlation function $B_{rr} \sim r^{0.71}$ what is very close to the Kolmogorov spectrum. The systematic observations and the construction of correlation functions began from 1964. Larson result[14] was close to that by Kaplan: $\Delta v \sim l^{0.38}$. But later more accurate investigations resulted in more steeper spectrum. Mayers[15], Henrikson & Turner[16], and Vereschagin & Solov'ev[17] obtained the spectrum $\Delta v \sim l^{0.5}$. Sanders, Scovill and Solomon[18] obtained $\Delta v \sim l^{0.62}$. These spectrums are different from the Kolmogorov one and to explain them we should take into account the anisotropy.

The attempt to explain the observed spectrum as a result of the turbulence of the Rossby waves in galactic gaseous disk was made in the work by Dolotin and Fridman[19] (see also[20]).

In accordance with observational data it was adopted: 1) correlation for the velocity fluctuations (see above) $\Delta v \sim l^{0.5}$; 2) correlation for the density fluctuations[15] $\rho \sim L^{-1}$. These correlations mutually agree if we adopt the assumption of virial equilibrium on all scales.

The following general nonlinear equation can be derived in the low-frequency approximation

$$\omega << \Omega_0, \tag{7}$$

for the solid body rotating selfgravitating cloud[19]:

$$\left(\frac{\partial}{\partial t} + \frac{1}{2\Omega_0}[\nabla_\perp \hat{\chi}, \nabla_\perp]_z\right)\left(\nabla_\perp \hat{\Psi} - \frac{\omega_0^2}{4\Omega_0^2}\nabla_\perp \hat{\chi}\right) - \frac{(\omega_0^2)'}{2\Omega_0}\frac{1}{r}\frac{\partial \hat{\chi}}{\partial \varphi} = 0. \tag{8}$$

Here $\omega_0^2 \equiv 4\pi G \rho_0$, $(\omega_0^2)' \equiv d(\omega_0^2)/dr$; $\chi \equiv \wp + \Psi$, Ψ - is the gravitational potential; $\wp$ is the "pressure" function determined by relation $\wp \equiv \int dP/\rho$, where P is the usual "volume" pressure, ρ is the volume density.

For the small-scale perturbations, corresponding to the case $\hat{\Psi} << \hat{\wp}$, Eq.(8) can be reduced to

$$\left(\frac{\partial}{\partial t} + \frac{1}{2\Omega_0}\left[\nabla_\perp \hat{\wp}, \nabla_\perp\right]_z\right)\nabla_\perp \hat{\wp} - 2\Omega_0 \frac{\rho_0'}{\rho_0}\frac{1}{r}\frac{\partial \hat{\wp}}{\partial \varphi} = 0. \tag{9}$$

The latter equation is similar to well known in hydrodynamics Charney-Obukhov's equation[21,22,23] and in plasma physics Hasegawa-Mima equation[24]. This analogy allows to use results from the well elaborated fields to describe the dynamics of the perturbations in gravitating gaseous medium. Particularly in accordance with Sazontov[25] and Mikhailovskii *et al.*[26] the nonstationary solution of Eq.(9) describes Rossby waves turbulence with energy spectrums $\omega_k^{(1)} \sim k_y^{-3/2}k_x^{-2}$, and $\omega_k^{(2)} \sim k_y^{-3/2}k_x^{-3}$.

At the limit of the theory application, $k_x \simeq k_y \simeq k_\perp$, and taking into account characteristic property of the Rossby waves $k_\perp \simeq k$ we have $\omega_k^{(1)} \sim k^{-3.5}$, and $\omega_k^{(2)} \sim k^{-4.5}$. According to Hasegawa *et al.* numerical result[27] $\omega_k \sim k^{-4}$. The last relation gives[28] $v_k^2 \sim \int_k^\infty E_k dk = \int_k^\infty \omega_k k^2 dk \sim k^{-1} \sim \lambda$, that is 1) $v_\lambda \sim \lambda^{0.5}$. On the other hand $v\nabla v \simeq \nabla\Psi$ that is $v_\lambda^2/\lambda \simeq \Psi/\lambda \simeq \lambda 4\pi G\rho_\lambda$. This gives for the density spectrum 2) $\rho_\lambda \sim \lambda^{-1}$. We see that the obtained turbulent spectrum[19] corresponds to the observed spectrum (see also [29]) that is the evidence of a week turbulence of the Rossby waves in cloudy population of the Milky Way.

SOLITARY VORTICES IN ASTROPHYSICAL DISKS

For a disk in outer gravitational field for slow-frequency perturbations (7) we derive[28] from initial 3D hydrodynamical equations the following nonlinear dynamical equation for a solid-body rotating part of a disk, $\Omega = const$,

$$\frac{\partial}{\partial t}\left(\hat{\Pi} - a_R^2 \Delta \hat{\Pi}\right) + U_R \frac{\partial \hat{\Pi}}{\partial y} - \frac{c_s^2}{8\Omega^3} J(\hat{\Pi}, \Delta\hat{\Pi}) + \frac{(\ln C)'_x}{4\Omega}\frac{\partial \hat{\Pi}^2}{\partial y} = 0. \qquad (10)$$

Here $\hat{\Pi}$ is perturbation of Π — the pressure function $\wp$ or enthalpy; $a_R \equiv c_s/2\Omega$ is Rossby radius; $U_R \equiv -2a_R^2\Omega \cdot (\ln\sigma)'_x$ is Rossby velocity; c_s is the sound speed determined by the "flat" functions

$$c_s^2 \equiv \sigma \left(\frac{\partial \Pi}{\partial \sigma}\right)_0 ; \qquad (11)$$

$$J(A,B) \equiv \frac{\partial A}{\partial x}\frac{\partial B}{\partial y} - \frac{\partial A}{\partial y}\frac{\partial B}{\partial x} \qquad (12)$$

is Jacobian; x, y are Cartesian coordinates with x along radius and y along azimuth; the function C is determined by the equation of state for gas ($P = A\rho^\gamma$, where A and γ are constants, γ is a polytropic index), and by the second vertical derivative of the external gravitational potential, Ψ, in the disk mid-plane:

$$C \equiv \left[\frac{\Psi''}{2\pi}\frac{\Gamma^2\left(\frac{\gamma}{\gamma-1}+\frac{1}{2}\right)}{\Gamma^2\left(\frac{\gamma}{\gamma-1}\right)}\right]^\lambda \left(\frac{A\gamma}{\gamma-1}\right)^{2/(\gamma+1)}, \qquad (13)$$

where $\lambda \equiv (\gamma-1)/(\gamma+1)$, Γ is gamma-function.

Equation (10) contains both the vector (third term) and scalar (last term) nonlinearities. It should be emphasized that the scalar nonlinearity is a consequence of the dependence of the "surface" equation of state from the external parameter (through the dependence on external gravitational potential Ψ_e). If we start from the usual pure 2D hydrodynamical equations this term would be overlooked.

Derived for astrophysical disks nonlinear dynamical equation (10) is similar to well-known in hydrodynamics the Charney-Obukhov equation[21,22,23]. In plasma physics the similar equation was derived by Hasegawa and Mima[24]. The use of well-worked-out theory and laboratory modelling of these equations in hydrodynamics and plasma physics leads to the following results[30,31,32].

Equation (10) has two kind of ***stationary solutions,*** which describes, respectively, two types of solitary vortices: single and double vortices. The sizes, a, of these structures are restricted: $1 < a/H < (R/H)^{1/3}$, where H is the disk semi-thickness, R is the typical scale of the density inhomogeneity.

Single vortices: cyclones and anticyclones.

At a certain distance from the centre of the disk it is possible to form only one kind of solitary vortices: cyclones or anticyclones. The type depends on the profile of the external potential and is determined by the sign of $(\ln C)'_x$. A cyclone ($(\ln C)'_x < 0$) is characterized by a minimum of the surface density and an anticyclone ($(\ln C)'_x > 0$) by a maximum.

Double vortices: modons. Modon sea. Vortex turbulence.

A double vortex (or modon) represents a cyclone-anticyclone pair and has one minimum and one maximum of perturbed surface density[20].

Generation of several modons (a modon sea) results in their interaction and in the formation of vortex turbulence[33]. The vortex turbulence is distinct from the wave one in principle. Wave turbulence is strong if the perturbed amplitude $\hat{A}$ is closed to, or larger than, its stationary value A_0. Vortex turbulence can be strong under the condition $\hat{A} << A_0$, as the duration of vortex-vortex interactions is much longer than that for wave-wave interactions.

NONLINEAR RADIAL LAMINATED FLOW AND LARGE-SCALE CONVECTION AS A MANIFESTATION OF 3D DYNAMICS OF ASTROPHYSICAL DISKS

In this section we will show that in presence of spiral density wave there is an observable manifestation of the three dimensional nature of astrophysical disks in the form of large-scale quasi-stationary radial flow laminated in z direction. The flow velocity has an opposite direction in the central plane of the disk, $z = 0$, and on the disk periphery, near the planes $z = \pm H$. The streamlines are closed by the vertical motions of less magnitude, $v_z \simeq v_r H/r$. As a whole, the flow has a form of four vortices separated by vertical surface $r = r_c$, where r_c is a corotation radius, and the central plane of the disk (FIGURE 2). Therefore, the observation of this flow could provide a direct indication of the position of corotation circle.

The characteristic velocity in the flow is about the velocity in the density wave. For the galactic disks it can be as much as several tens kilometers per second. The characteristic scale of the flow is about the disks radius. Therefore the existence of such kind of the convection can play a significant role in the overall dynamics of a disk.

By its nature the flow discussed above is a special kind of an acoustic streaming. It is caused by the quasi-stationary component of nonlinear Reynolds stresses induced by the density wave. Classical acoustic streaming is caused by the Reynolds stresses in strong acoustic waves (for a review see *e.g.*[34].). This phenomenon has been observed in hundreds of different laboratory experiments starting with Faraday's discovery[35] and described in numerous theoretical papers, starting with the pioneering work by Rayleigh[36].

A rotating disk differs from the classical case by the drift nature of the quasistationary flow, which is a consequence of the dominant role of the Coriolis forces, and implies a flow direction perpendicular to the applied force.

A radial drift caused by separate azimuthal forces such as $\langle div(\rho v_\varphi \vec{v}) \rangle$, $\langle \rho \partial\Phi / r\partial\varphi \rangle$, etc.

Unlike an azimuthal drift, a radial drift leads to the redistribution of the surface density of a disk and it is different in dissipative and dissipativeless disks.

In a dissipative medium a radial mass flux integrated over disk thickness is not equal to zero and has the same direction as the flow in the plane $z = 0$. The radial mass can form some gaps in planetary rings[37,38].

In a dissipativeless medium a radial mass flux averaged over disk thickness is zero, and the flow has a form of four vortices separated by the vertical cylindrical surface $r = r_c$ and the central plane of the disk $z = 0$[31,38] (FIGURE 2).

Note that in both cases — in dissipative and dissipativeless disks — the mass flux velocity changes its sign on corotation circle: insight the latter the flow velocity is directed to the disk center, outside the corotation — to the disk periphery. This must lead to a depression in the surface density distribution just in the corotation circle, that was observed in the gaseous disk of the Milky Way[39].

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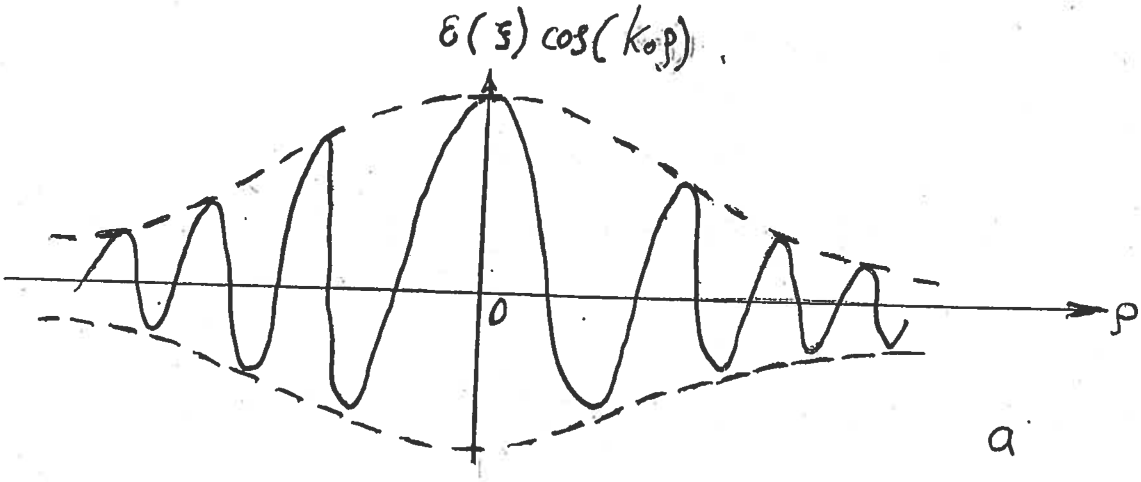
FIGURE CAPTIONS

FIG. 1 Schematic view of the envelope solitons generating in a marginally unstable disks.

(a) The case of dissipativeless disk; (b) Disk with small viscosity.

FIG. 2 Schematic view of nonlinear radial flow induced by a quasi-stationary trailing density wave. The vertical cut of the disk along the radius is presented. The radial scale is squeezed. The flow is azimuthally symmetrical and has a form of four tori.

(a) Structure of streamlines in the flow; (b) The vertical profile of the radial velocity.



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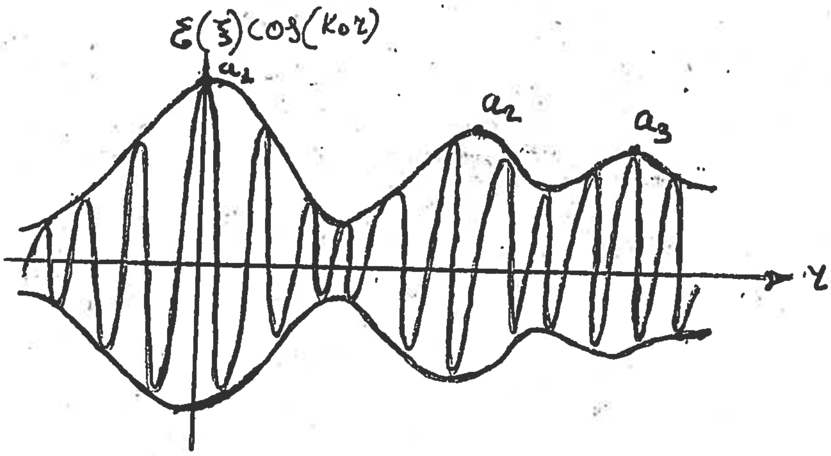

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Fig 1

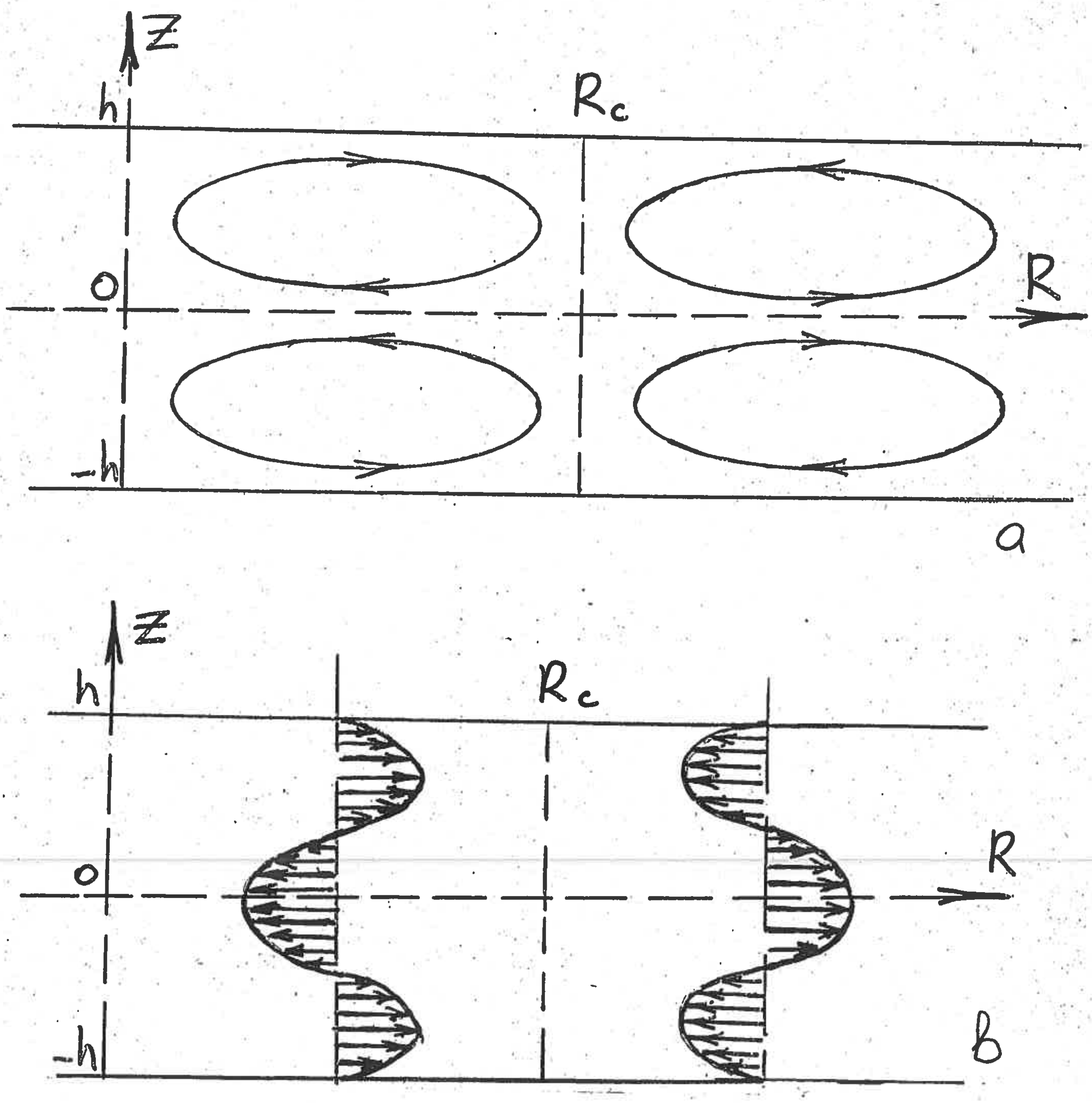

Fig 2