

Committee 5
Non-linear Structures in Natural Science and Economics

Draft – January 1, 2000
For Conference Distribution Only



Discrete Chaotic Dynamics and Time and Space in Complex Systems

Vladimir Gontar
Professor, Director of the International Group for Chaos Studies
Ben-Gurion University of the Negev
Beer-Sheva, Israel

The Twenty-second International Conference on the Unity of the Sciences
Seoul, Korea February 9-13, 2000

1. Introduction

What do we know about time? Our first thought is that time is some historical means to put the events happening around us in order [1]. Some of these events are deterministic and periodic, e.g., movement of planets and atomic oscillation. For such types of movement, Newton formulated the laws of dynamics together with the concept of time and space and introduced a scientific method for calculation of periodic orbits—calculus of the infinitesimal and differential equations—i.e., the mathematical language for modeling the behavior of systems in time and space. Later, it became clear that most of the systems and processes surrounding us [2,3] are not periodic and therefore never occur twice under exactly the same conditions. For such systems, the idea of absolute continuous time and space, as expressed in continuous integrable differential equations, is faced with an obvious contradiction. This fact was pointed out by Poincare, who demonstrated that even the three-body problem is mathematically non-integrable and is therefore not predictable in terms of Newton's dynamics. Poincare can be considered as a pioneer in starting the modern concept of deterministic chaos. Another ongoing problem that is still under investigation is the relationship between reversible time in Newton's equations and one-way time (arrow of time) in statistical physics and the second law of thermodynamics.

The modern break with the classical approach to the simulation of complex systems came with the appearance of computer chaotic solutions of an ordinary set of differential equations by Lorenz [4]. The contradiction between continuous deterministic differential equations derived from first principles and those of Lorenz resulted in an upheaval in the scientific community, and the number of publications concerning chaotic dynamics is increasing dramatically.

The main problem that we seek to understand is how it became possible that differential equations—being deterministic by definition—could generate unpredictable chaotic solutions. What is the role of computer calculations? Are they just a reflection of the fact that differential equations have a limited ability for simulation of complex systems dynamics as a result of the contradiction between the continuous time and space of differential equations and the discrete character of numerical calculations of computers, with limited accuracy? It was thus inevitable that workers such as Schrödinger [5] and Brillouin

[6] should again raise the questions of the necessity for new physical principles to describe the dynamics of living and complex systems (free of the above-mentioned contradictions) and to serve as a basis for the creation of a new calculus, different from the calculus of infinitesimal. The main problem here is to obtain a clear understanding of the meaning of time and space of classical physics and that in the new physics, which will strive to describe and explain emerging behavior, the process of creation and birth, and bifurcations and death in living and social systems.

2. The Problem

Leaving aside philosophical aspects of time and space, we will concentrate on the meaning of time and space in mathematical equations. We will start by looking at the values of time and space in the definition of velocity. By definition, velocity v is the limit of the ratio Δs to Δt , as $\Delta t \rightarrow 0$. Here, s is a space coordinate and t is astronomic absolute time. The fact that we are postulating that Δt can have infinitely small values implies that Δs must also have infinitely small distances if we are to acquire a final value for v . This mathematical assumption about the existence of a final $v = \lim \Delta s / \Delta t$ when $\Delta t \rightarrow 0$ brings us to the conclusion that time and space are continuous and therefore that it is possible to differentiate the variables t and s , i.e., $v = ds/dt$. Continuous trajectories $s = s(t)$ for moving physical objects become a natural expression of mathematical modeling for the dynamics of systems. Newton's equations and the related principle of a minimum of action demonstrate the way of calculating trajectories by the integration of the differential equations of the system in question. Here, we remember that the above-described way of constructing the basic equations and principle of the continuity of s and t has always been under criticism. The ancient Greeks paradox concerning the logical contradiction between postulating the existence of limits and the infinite number of steps (not in the practical sense) that have to be made to achieve these limits, finds expression in modern times in the contraction between our lack of ability to measure infinitesimally small intervals of Δs and Δt and quantum mechanics, where the continuity of trajectories is replaced by a discrete set of probable states. Nevertheless, until the publications of Lorenz, the critics of conventional calculus did not propose any alternatives that could dislodge the principle of continuity and differential equations from their monopoly in describing all types of dynamics. (In case of quantum mechanics, why do we need to use the differential equation of Schrödinger and then to choose the appropriate solutions according to the additional principle? What is the

physical meaning of probability? Is it another way of escaping from probabilities in natural sciences or is G-d still playing dice?)

Ideas about the possibility of creating a new calculus first appeared in the second part of this century when Lorenz performed his computer simulations and obtained unexpected solutions for a set of ordinary differential equations. Deterministic chaos, strange attractors, fractals and bifurcations are but a short list of the new scientific categories that have appeared and provided explanations for particular events. The postulates behind deterministic chaos made a real break with the classical dynamics paradigm: deterministic chaos showed that infinitely small changes in the initial conditions or parameters could lead to drastic changes in the solutions to differential equations and in the exponential divergence of trajectories. How does this idea fit with the principle of continuity of differential equations? The fact that this type of solution, known as a chaotic solution, can be obtained only by computer brings us to certain conclusions about the role of numerical calculations: The accuracy of numerical calculations is always limited, and they will therefore always have a final value. If a particular system of differential equations generates a chaotic solution, this solution would depend strongly on the accuracy of computation (according to the definition of chaos), and this implies that the final results of mathematical modeling would therefore include computer errors, which could obviously have extensive influence on the results. At this point, we cannot isolate our theoretical assumptions about mathematical modeling of real physical systems but need to include in our consideration the accuracy of numerical algorithms and computer-generated solutions. As is well known, computers use a discrete scheme of operations, which is in obvious contradiction to the general assumption about the continuity of differential equations. The determinism of differential equations with some level of complexity becomes dependent on the method of calculation.

For cases for which we can obtain analytical solutions to differential equations, these numerical solutions are obtained with great accuracy, but the majority of solutions cannot be obtained analytically, and therefore some of them can demonstrate chaotic trajectories. In this case, i.e, when the results are chaotic, we are faced with the obvious contradiction between our definition of continuous differentiable time and space and the obtained results. To resolve this situation, we have to seek different assumptions about time and space—which need not necessarily be continuous—and try to formulate our basic equations in a different way. Some results in this direction appeared in 1976 in the works of May [7] and Feigenbaum [8]. They showed that even simple difference equations (iterations) $X_{n+1} = F(X_n)$ can generate

various types of "discrete" oscillations, which were previously the prerogative of differential equations. Knowing the essential role of oscillations in wave mechanics, electrodynamics, optics and elementary particle physics, we can understand the importance of new mathematical models for the simulation of oscillatory behavior. The main problem in the practical application of difference equations at the time they appeared as a models was the absence of any physical meaning to these equations and their variables, particularly for the meaning of discrete time and space. For example, the simplest mathematical model that generates periodic behavior consists of two conventional differential equations, with clear involvement of time t and a space coordinates:

$$ds/dt = y ; dy/dt = -k^2s$$

The solution to these equations is $s = \sin kt$. This model has been applied for modeling the dynamics of a pendulum. The simplest difference equation that may pertain to oscillatory behavior is:

$$Z_{n+1} = - Z_n$$

Starting from $Z_0 = a$ (an arbitrary real number), we obtain an unlimited series of 1, -1, when $n \rightarrow \infty$.

It hard to expect from this trivial iteration any connection to real physical systems, but some aspects of this form of "oscillations" modeling certainly merit discussion (as may be seen from Figure 1, more complicated difference equations can generate a variety of solutions, which are very similar to experimentally observed patterns). Firstly, it is obvious that these "oscillations" appear in discrete form and that no derivatives could be taken from such a discrete trajectory. Another characteristic of the difference equation is that classical time t is not represented formally in the equation, and therefore some researchers consider "n" as the discrete analog of continuous time. We intend to demonstrate that this assumption is incorrect. Finally, to make difference equations or iterations of some practical use, we need to formulate physical principles that relate to the laws of nature (energy, charges, mass conservation laws, etc.).

To illustrate the possibility of finding a way to construct a set of difference equations, with clear physical sense, from first principles, let us consider physicochemical reaction equilibrium and dynamics in relation to the theory of dimensionality.

3. Physicochemical equilibrium and the theory of dimensionality

Mathematical modeling of chemical equilibrium is an interesting application of classical thermodynamics, providing a way for the construction of a mathematical model for the calculation of a mixture of N ideal gases based on first principles (the first and second laws of thermodynamics). According to the Gibbs equation, $dG = VdP - SdT$, for one component of a system, we obtain for one mole of pure ideal gas:

$$(dG/dP)_T = V = RT/P, \text{ or after integration } G = RT \ln P + G^0,$$

where R – the gas constant, and G^0 - Gibbs free energy for one mole of gas at temperature T and pressure of one atmosphere. To define the total free energy for a mixture of N gases, Dalton's law may be applied:

$$P_i/P = X_i/\sum X_i = \varepsilon_i \quad \text{or} \quad P_i = P X_i/\sum X_i$$

where P_i are the partial pressures of the i -th constituents and X_i - the number of moles of the i -th constituents. The total Gibbs free energy is given by:

$$G/RT = \sum X_i (\ln \varepsilon_i + \ln P)$$

or the Helmholtz free energy by:

$$F/RT = \sum X_i (\ln X_i + f_i - 1), \quad (1)$$

where $f_i = G^0/RT + \ln RT/V$.

To calculate the distribution of the equilibrium mixture X_i , we need to minimize G or F [depending on the conditions of the investigated system, $G(P, T$ -constants), $F(V, T$ -constants)] under the constraints of the law of conservation of mass:

$$\sum_{i=1,2,\dots,N} a_{ij} X_i = b_j \quad i=1,2,\dots,N; j=1,2,\dots,M \quad (2)$$

where a_{ij} - “molecular” matrix, defining the number of gas components of type “j” in the i-th constituent and b_j – the total amount of the j-th component. The minimum of the free energy is expressed in the second law of thermodynamics (maximum entropy). Mathematically, minimization of equation (1) is equivalent to the solution of the following set of non-linear algebraic equations (thermodynamics law of mass action):

$$\prod_{i=1}^N X_i^{v_{li}} = K_l, \quad l=1,2,\dots,L=N-M \quad (3)$$

where K_l - equilibrium constant of the l-th reaction and v_{li} - stoichiometric coefficients, which correspond to the elements of molecular matrix a_{ij} in accordance with the following matrix equation [3]:

$$\sum_{i=1}^N v_{li} a_{ij} = 0, \quad (4)$$

The set of non-linear algebraic equations (2,3) constitutes the mathematical model for calculation of complex chemical equilibrium. The system of N chemical constituents A_i interconnected by L chemical reactions may be represented as:

$$\sum_{i=1}^N v_{li} A_i = 0.$$

We have thus considered an impressive example, in which first principles of thermodynamics give us a clear mathematical model for calculation of multicomponent chemical equilibrium. Our goal, however, is to construct a mathematical model for the dynamic behavior of complex systems—systems that are far from equilibrium. Principles of

thermodynamics can therefore not be applied to such systems, and other principles should thus be used. For this reason, we have chosen to use the theory of dimensionality, which is also based on first principles—but different first principles. We intend to extend principles of thermodynamics and to introduce new principles for multicomponent systems that are far from equilibrium.

4. Theory of dimensionality and stoichiometry of physicochemical reactions

Now let us turn to the foundation of the theory of dimensionality, which operates without revisions of the categories and postulates. According to the claims of this theory, all variables used to identify physical, chemical, social, or any other systems should have dimensions expressed in the so-called main units, e.g., meters, seconds, and grams. Variables such as velocity, energy, and pressure have well-defined dimensions, which obviously depend on the choice of the main units. To overcome the dependence of such variables on arbitrarily chosen main units, we recommended the construction of invariants (dimensionless variables). Invariants are constructed by use of the π -theorem of the theory of dimensionality and are conveniently expressed in the general form of matrix operation [6].

Suppose we have N variables q_i $i = 1, 2, \dots, N$, with dimensions defined by the matrix Q_{ij} , with M main units $j = 1, 2, \dots, M$; for example, $q_1 = s$ - distance, $q_2 = t$ - time, $q_3 = v$ - velocity, $q_4 = a$ - acceleration; here, $N = 4$, and q_i variables define the kinematics of an arbitrary mechanical system. The main units in this example $M = 2$, with u_1 - sec (s) and u_2 - meter (m). In this case, matrix Q can be written in the form:

$$\begin{array}{rcc}
 & m & s \\
 s & 1 & 0 \\
 t & 0 & 1 \\
 v & 1 & -1 \\
 a & 1 & -2
 \end{array} \quad (5)$$

Now, to define the invariant according to the π -theorem, we need to present matrix Q_{ij} in the following form:

$$\mathbf{Q} = \frac{\mathbf{R}}{\mathbf{P}}, \quad (6)$$

where \mathbf{R} is nongenerate matrix of dimension $M \times M$, and \mathbf{P} is a matrix of dimension $(N-M) \times M$. Then, the determinant matrix ζ_{ji} for $L = N - M$ invariants π_l should be calculated as follows:

$$\zeta_{ji} = \left| \left| \begin{array}{cccc} -1 & & & \\ & \mathbf{R} & & \\ & & \mathbf{P} & \mathbf{I} \end{array} \right| \right| \quad l=1,2,\dots,N-M, \quad (7)$$

where \mathbf{I} is the unit matrix $(N-M) \times (N-M)$.

For our example, we will obtain matrix :

$$\zeta_{ji} = \left| \left| \begin{array}{cccc} -1 & 1 & 1 & 0 \\ & & & \\ & & & \\ -1 & 2 & 0 & 1 \end{array} \right| \right|, \quad (8)$$

and two invariants:

$$\begin{aligned} \pi_1 &= s^{-1} t^1 v^1 a^0 = vt/s \\ \pi_2 &= s^{-1} t^2 v^0 a^2 = at^2/s. \end{aligned} \quad (9)$$

Now let us transport this formalism to the stoichiometry of chemical reactions. Again, to make things clear, let us use a simple example of a chemical system with four constituents: $X_1 = A$, $X_2 = B$, $X_3 = AB$, $X_4 = AB_2$ and as components of the system, let us choose A and B. In this case, the molecular matrix a_{ij} may be written as:

$$\begin{array}{r}
 \text{A} \quad \text{B} \\
 \text{A} \quad 1 \quad 0 \\
 \text{B} \quad 0 \quad 1 \\
 \text{AB} \quad 1 \quad 1 \\
 \text{AB}_2 \quad 1 \quad 2
 \end{array}
 \tag{10}$$

and $L = N - M$ “chemical invariants” can easily be defined in the same way by means of equation (8). As a result, we will obtain the following expressions for the two “chemical invariants” in this example:

$$\begin{array}{r}
 \Pi_1 = X_1^{-1} X_2^{-1} X_3^1 X_4^0 = X_3 / X_1 X_2 = K_1 \\
 \Pi_2 = X_1^{-1} X_2^{-2} X_3^0 X_4^1 = X_4 / X_1 X_2^2 = K_2
 \end{array}
 \tag{11}$$

The analogy between the equations of the thermodynamic law of mass action law (1) and the π -theorem based on the equivalency of matrixes a_{ij} and Q_{ij} , $\nu_{i\alpha}$ and $\zeta_{\alpha i}$ is thus self-evident. Of course, there are significant differences in the meaning of the dimensionless invariants π_i and the constants K_i or the “chemical invariant”- Π (the chemical invariant does have dimensions in the sense of the theory of dimensionality). Nevertheless, we intend to use the mathematical equivalence between chemical thermodynamics and the theory of dimensionality. As was shown, equations (3) were derived in classical thermodynamics from the principle of minimum free energy (or maximum entropy) for a mixture of ideal gases. This assumption imposes a strong constraint for use of these equations for a condensed system. If we take the standpoint of the theory of dimensionality and suppose that “chemical invariants” reflect the independence of the thermodynamics constants K_i on the initial concentration of M components of the system, we can apply equation (1) to any system of chemical reactions, without constraints, that has been based on ideal gas assumptions. If the initial system of chemical reactions does not give an adequate description according to the theory of dimensionality, we need to change our initial hypotheses about the mechanism and add some more components and constituents, or just change the type of chemical reaction.

Another recommendation coming from the theory of dimensionality is the claim for invariants to be constant (not to depend on time, initial concentration, concentrations of constituents, etc.) for similar systems. If a new situation arises (for example, if from simulation system kinematics we intend to analyze the dynamics), we need to add a new component and variables in the matrix ζ_{ji} (kg as a main unit for measurement of forces in this particular case, and force, energy, and power as a new variables q). Following this rule to move to the description of the dynamics of the system, we need to add a new “time component” in the molecular matrix a_{ij} for the simulation of dynamics from thermodynamic equilibrium. The details of such an approach can be found in ref. [10]. If we do so for a closed system, we will obtain, after simple operations with the matrixes, the time-dependent functions $\pi_i(t_q)$ on the right side of the equation (1), with $q = 1, 2, \dots$, instead of the constants K_i . For the concrete type of time dependence $\pi_i(t_q)$, it is reasonable to choose $\pi_i(t_q) = K_i \exp[-W_i/t_q]$, and therefore equation (3) should be rewritten in the following form:

$$\prod_{i=1}^N X_i(t_q) = K_i \exp[-\omega_i/t_q] \quad (12)$$

which together with equation (4) constitutes a complete system of N non-linear algebraic equations for N unknown trajectories $X_i(t_q)$. The reason for our choice of an exponential function for $\pi_i(t_q)$ can be found in the asymptotic behavior of closed physicochemical systems [4]. Equations (2) and (12) have a unit solution for all $X_i(t_q) > 0$. This is the way in which we can construct a system of algebraic equations for mathematical simulation of the dynamics of N concentrations $X_i(t_q)$ without using differential equations for any mechanism of the chemical reactions given by matrix ν_{ij} . Numerical calculations performed to compare solutions from differential equations for the kinetic mass action law and the proposed equations for the same mechanisms of chemical reactions gave good qualitative agreement. System (12,2) is much simpler from the computational point of view—no problems with “stiffness” of the differential equation—and can serve as good approximation to the solutions to the differential equations.

Now to advance to the mathematical simulation of open systems involving catalysis, we intend to include in our molecular matrix a_{ij} a new component reflecting a time delay interaction, which, after simple transformation, will bring us to a system of difference equations. In this case, π_l becomes a function of all concentrations X_i , calculated at a previous moment in time t_{q-s} , where $s = 1, 2, \dots$. For an open system, in which t_q changes from t_0 to t_c - constant (characteristic time for any particular system, for which a steady state is reached), system (12,4) transforms from an algebraic expression to a set of non-linear difference equations. In the same way, we insert a space coordinate r through the dependence $X_i(r \otimes)$ in our matrix a_{ij} , and the final function for π_l may be written in the form:

$$\pi_l(X(t_q), r) = K_l \exp\left\{-\sum_{i=1}^N \alpha_{li} X_i(t_{q-s}) + \sum_{i=1}^N \beta_{li} X_i(r \otimes)\right\} / t_c \quad (13)$$

where α_{li} , β_{li} are empirical parameters, characterizing, respectively, the influence of the intensities of feedback influence in time - α_{li} , and influences of the space-distributed concentrations of the neighbors $X_i(r \otimes)$ β_{li} on the l -th chemical reaction. We have now constructed a full system of basic equations for simulation of the dynamics of all types of complex physicochemical reactions, starting with the initial hypothesis of the mechanism given by matrix v_{li} .

Using the fact that equation (1) is equivalent to the minimization of free energy, we intend to generalize this principle and formulate a new extreme principle for the dynamics of chemical reactions: reactions in multicomponent physicochemical systems proceed in such a way that at each instant of time t_q ($q=1, 2, \dots$) and at every point of the space $r(x, y, z)$ under consideration, the function:

$$F(t_q, r) = \sum_{i=1}^N X_i(t_q, r) \{\ln X_i(t_q, r) - f_i - 1\} \quad i=1, 2, \dots, N \quad (14)$$

reaches a minimum in the concentration space X_i , subject to the to the law of conservation of mass (4).

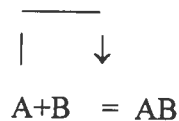
Here,

$$f_i = \begin{cases} 0 & i=1,2,\dots,M \\ \ln \pi_i & i=M+1, M+2,\dots,N. \end{cases}$$

and $\pi_i(X(t_q), \mathbf{r})$ is defined by (13).

5. Mathematical modeling of the dynamics of living and thinking systems

Now, after presenting the principles of discrete chaotic dynamics and their mathematical model in the form of systems of nonlinear difference equations (2, 12, 13), let us discuss why this approach may be useful for describing the dynamics of complex and living systems. It is well known that, in contrast to non-living systems, living systems can reproduce themselves and can collect, exchange, create and process information [1-3]. If we consider chemical transformations, for example, we may suppose that as a result of interactions between the constituents of any specific system (atoms, molecules, clusters, ions, etc.) a new constituent or a new element will appear (birth process). Chemical reactions, by their very nature, encompass, in a formal way, the fact of the creation of a new element from the initial elements. Another characteristic of living systems is information processing. We think that the proposed new mathematical model can be effectively used for describing the dynamics of such systems because the parameters π_i are functions of all the concentrations X_i . This means that all the constituents of chemical reaction $A+B \rightarrow AB$, i.e., A, B, and AB:



are interconnected by the influence of π_i , according to (13), which we have named an “information exchange” (shown by arrows). Our term “information exchange” is quite different from the term information or negentropy that has been used in classical thermodynamics or the theory of information. We consider our mathematical formalism, equations (2, 12,13), as having two levels of mathematical modeling of the behavior of complex systems: the first is the mass (concentration) exchange as $A+B=AB$ and the second

is the information exchange controlled by the influence of concentrations X_A , X_B , X_{AB} on parameters π_i .

As is already well known, the set of difference equations represents all types of complex behavior: complex oscillations within chaotic regimes, bifurcations, and self-organized structures. Therefore, it is important to define the physical meaning and the means of construction of the initial equations. We believe that living systems are closest to the ideas of mass and information exchange presented by equations (2,12,13). Small changes in the parameters π_i resulting from the influence of concentrations in the previous moments of time can exert a significant effect on the general dynamic behavior of a system: this is reflected in the fact of the living systems have a memory about the past as well as in the significant interaction between physicochemical processes and information exchange. Therefore, we can expect that our approach to be more productive in the simulation of living systems, because from first principles we have included in our basic equations the specificities of living systems, in contrast to the traditional way of applying physical equations, derived for non-living systems, to the dynamics of living systems.

Coming to the dynamics of thinking systems or mathematical modeling of the human brain, we need only evaluate the mechanism of biochemical reactions and their interactions in nerve cells. This mechanism can be formally presented by matrix ζ_{ij} or matrix v_{ij} in a formal way:

$$\begin{array}{c}
 \overline{} \\
 N \quad | \quad \downarrow \\
 \sum_{i=1} v_{ij} A_i = 0
 \end{array}$$

where A_i - a list of constituents participating in the "thinking process" and the arrow reflects the hypothesis of neuron interconnections. Starting with one particular mechanism, we can apply our basic equations to the different processes of complex systems dynamics including brain creativity mathematical modeling in a form of ornaments and mandalas. In Figures 2, 3 we present the symmetrical patterns—ornaments and mandalas—generated by the proposed approach. In Figure 4, 5 we demonstrate the growth dynamics of complex patterns formation and on Figure 6 —dynamics of solitons.

6. Meaning of time and space in complex systems

Looking at our basic equations (2),(12) and (13), we can find different times involved in this model: t_q is reminiscent of our conventional astronomic time, but it appears in our approach as one-way time, changing from 0 to ∞ . In contrast to the classical meaning of Newton's reversible time, our time has only one direction and cannot be changed without forsaking the meaning of $-t$ in the equations. A second time scale appears in our difference equations, not directly, but via dynamic or evolutionary processes that can be written briefly as: $X_{n+1} = F(X_n)$.

The solutions to this equation, as is well known, could be complex discrete periodic, non- periodic, or chaotic oscillations. The spectrum of these oscillations consists of many frequencies, which are related to the spectrum of the different times involved in the process. Because the particular oscillations are a product of the mechanism of interaction between the constituents, the resulting spectrum is a function of our hypothesis about interaction in the system and is therefore not related to a fixed time scale, as it would have been in differential equations. From such a consideration, we reach the conclusion that the time scale in complex systems—when they are modeled by difference equations—is a function of the internal properties of the system and nothing more. After calculations have been performed, we can relate our results to conventional astronomic time, solely by calibrating our spectrum of obtained times. New discrete "internal times" in complex systems reflect the specificity of the interaction of system components on the level of matter or mass exchange and information exchange.

We may reach similar conclusions about "discrete space". The kind of patterns or forms the systems will produce depend only on the composition of system and the interactions between components and their neighbors (either in proximity or situated far from each other). The boundary conditions or initial geometry are not important for the dynamics of complex and living systems. The dynamic behavior in space and the final distribution of the elements of the system depend solely on the character of the interaction between them. We can thus say that complex systems will produce forms and structures according to their internal dynamic properties and the external conditions if they can effect an interaction between themselves.

7. Conclusions

As it was pointed out above, thermodynamic entropy in closed systems reaches a maximum as the system reaches equilibrium, whereas non-steady state systems tend to behaving the opposite way. This is one of the most interesting effects that is responsible for all the creative processes in the universe, reflecting the internal laws of nature. Formulating and understanding these laws can bring us to the better understanding of the dynamics of complex systems, especially living systems, whose appearance, existence and development are in contraction to the principle of increasing entropy. Despite obvious similarities between living and non-living systems, which are based on the same constituent elements, such as atoms, electrons, etc., they can produce completely different dynamic behavior. We need to concentrate on their differences rather than on their similarities in order to define correct biological laws and their mathematical formulations. According to our theory, an important element has been missed in the description of the dynamics of complex living systems, i.e., the “information” exchange between the constituents of the system, which proceeds in parallel to the exchange of matter and mass.

We feel that the new discrete chaotic dynamics of physicochemical reactions can shed some light on this problem. To include “information” exchange formally in our basic equations, we postulate a dependence of the model parameters π_i on the constituents participating in physicochemical interactions and transformations.

The dynamic equations related to the new extreme principle may be considered as a mathematical model that can contribute to modern understanding of the meaning of time and space in complex systems. In opposition to the continuous time and space of differential equations, we obtain the "discrete" time and space of difference equations. Astronomic, continuous time disappears from our equations, and a set of "internal times", as a characteristic of the temporal behavior of a complex multicomponent system, emerges from the proposed approach. We are now not constrained by our watches as the sole time scale, but a set of "internal times" are obtained as a result of our calculations. (We may scale our obtained "times" to astronomic time when needed). The same can be said about the space dynamics of complex systems—the geometric forms that result from the calculations of our equations can, and should, be subsequently used in the conventional way, after scaling to real experimental space. Space patterns reflect the mass and "information" exchange according to the particular mechanism of the reactions taking place and will present forms according to our extreme principle (10).

We trust that the new calculus presented in this paper—the "calculus of iterations", will be developed further and used in a same way as calculus of infinitesimal in differential equations. Being much simpler from a calculation point of view (in comparison with differential equations), the proposed mathematical model can be used when real-time calculations are needed.

Our model is deterministic—does not use any probabilistic approaches—for simulating the dynamics of chemical reactions in space and time. The set of difference equations, by their very nature, provides chaotic regimes, which play the same role as the introduction of the random number generator in differential equations, and are therefore suitable for simulations of complex stochastic behavior of a system.

We understand that we are standing at the very beginning of the new discrete dynamics of complex and living systems, but even the preliminary results we have obtained by the proposed basic algebraic equations—and subsequently difference equations with clear physicochemical meaning—have given us the confidence to believe that we have entered a field with great potential.

References

1. I. Prigogine, "From Being to Becoming: Time and Complexity in the Physical Sciences", W.H. Freeman, San Francisco, 1980.
2. H. Haken and H.P. Koepchen, "Rhythms in Physiological Systems" Proceedings of the International Symposium at Schloss ELMAU, Bavaria, October 22-25, 1990. Berlin, Springer, 1991.
3. F. Cramer, Chaos and Order. The Complex Structure of Living Systems. VCH, Germany, 1993.
4. E. Lorenz, "Deterministic nonperiodic flow", J. Atmos. Sci. 20, 1963.
5. E. Schrödinger, What is life?, London, Cambridge University Press, and New York, The Macmillan Company, 1945.
6. L. Brillouin, "Life, thermodynamics, and cybernetics" *Am.Sci.* 37, 554, 1949.
7. R. May, "Simple mathematical models with very complicated dynamics", *Nature* 261, 459, 1976.
8. M. Feigenbaum, "Quantitative universality for a class of nonlinear transformations", *J. Stat. Phys.* 19, 25, 1978.

9. L. Brand, " π -theorem of the theory of dimensionality", Arch. Rational Mech. Anal., V. 1, 1957, pp.35-43.
 10. V. Gontar, "New theoretical approach for physicochemical reactions dynamics with chaotic behavior", in "Chaos in Chemistry and Biochemistry" R.J. Field and L. Gyorgyi, eds. World Scientific, London, 1993, pp. 225-247.
-

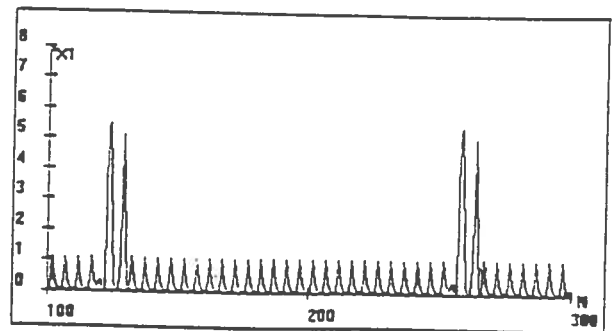
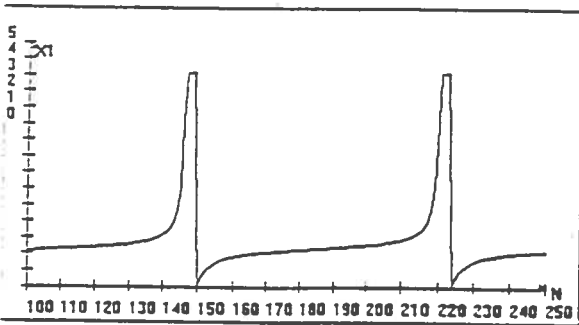
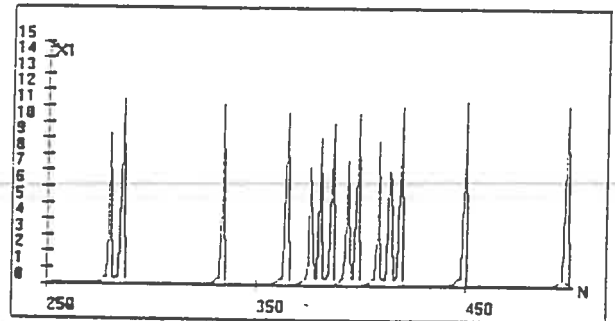
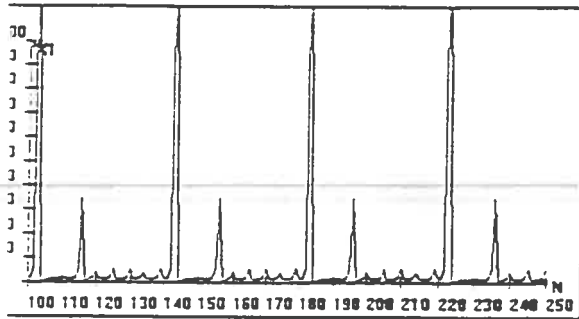
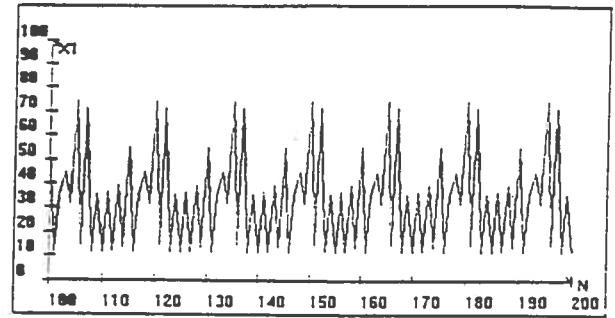
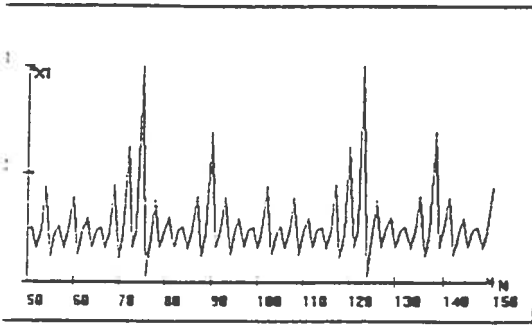
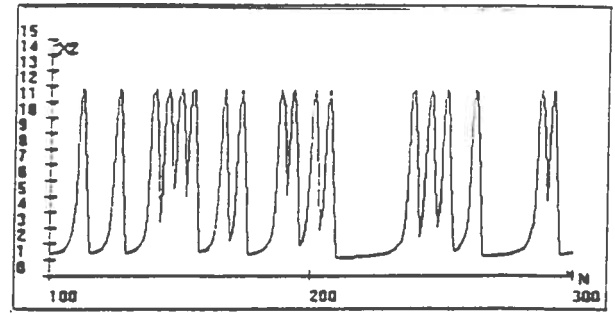
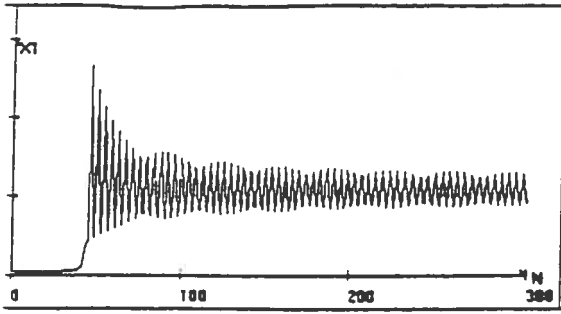


Figure 1. Different types of oscillations obtained by the discrete chaotic dynamics

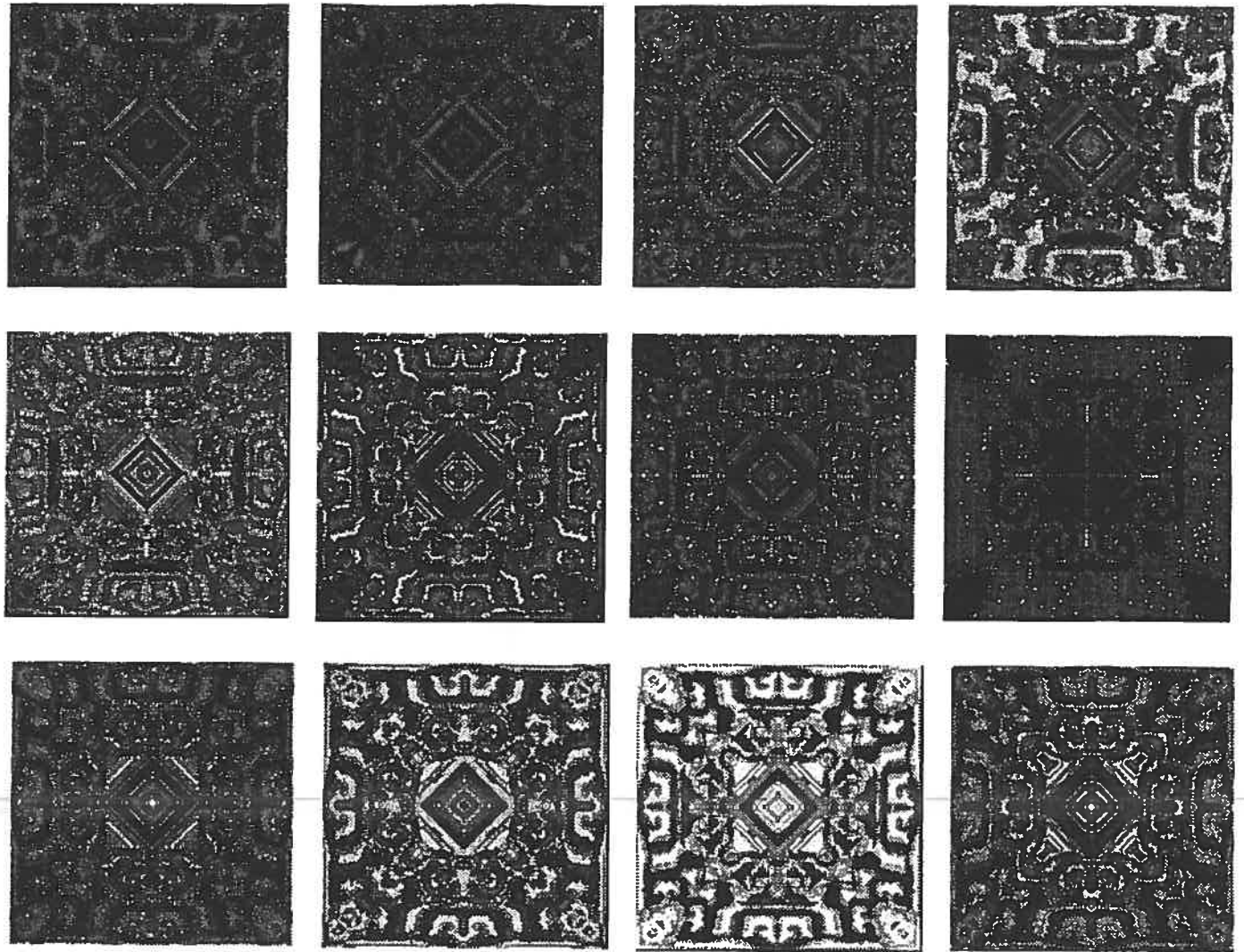


Figure 2. Ornaments generated by the discrete chaotic dynamics algorithms

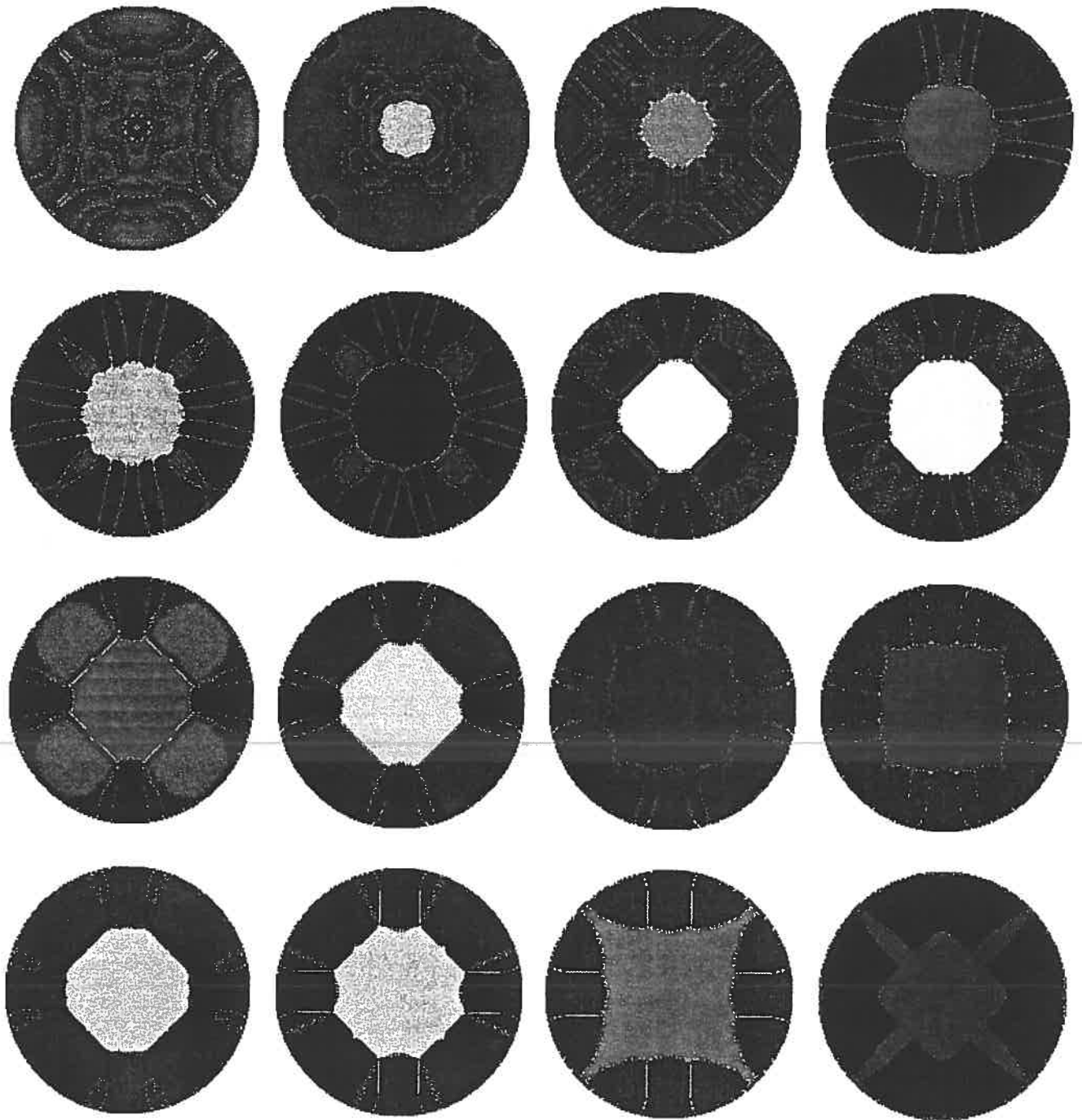


Figure 3. Mandalas calculated by the discrete chaotic dynamics algorithms

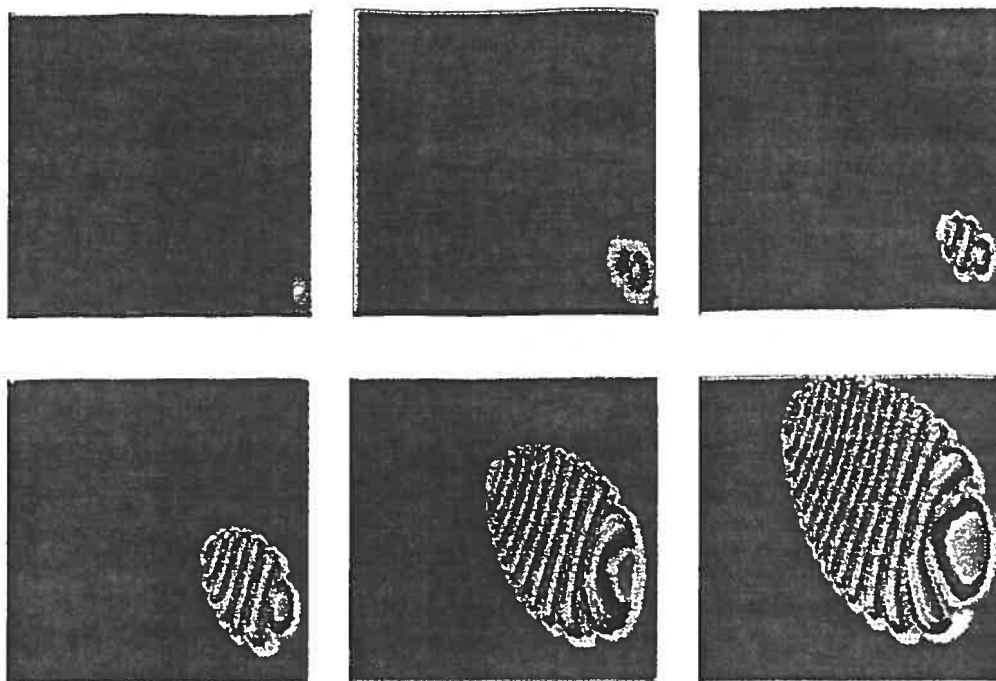


Figure 4. Dynamics of pattern formation, calculated by discrete chaotic dynamics
For the mechanism of reaction: $A \rightarrow B \rightarrow C$

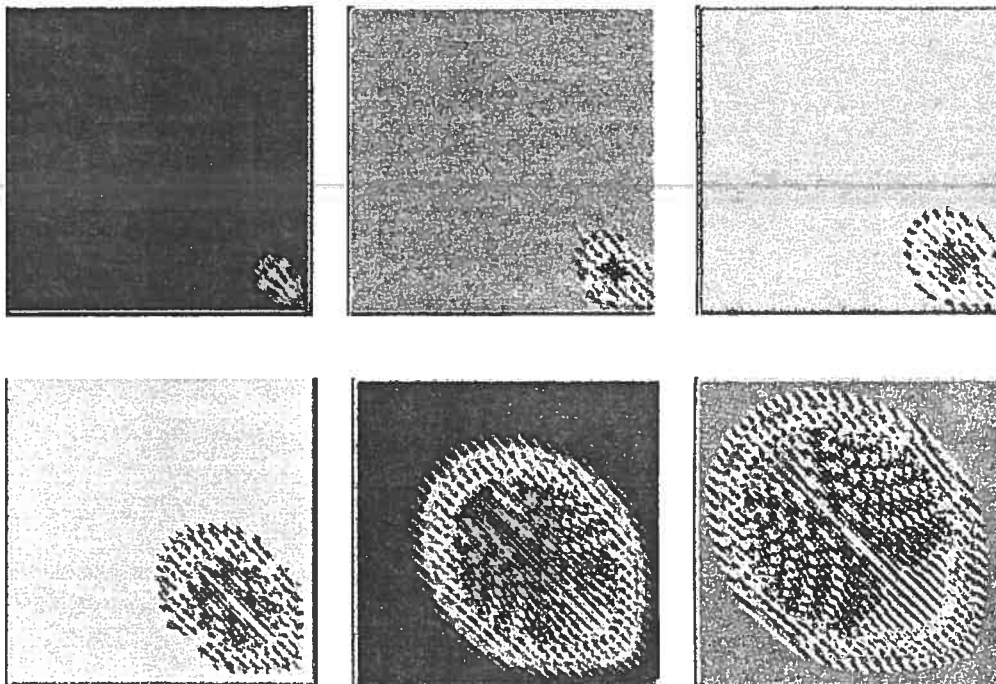


Figure 5. Dynamics of pattern formation, calculated by discrete chaotic dynamics
For the mechanism $A \rightarrow B \rightarrow C$, but with different from presented on fig.4
parameters

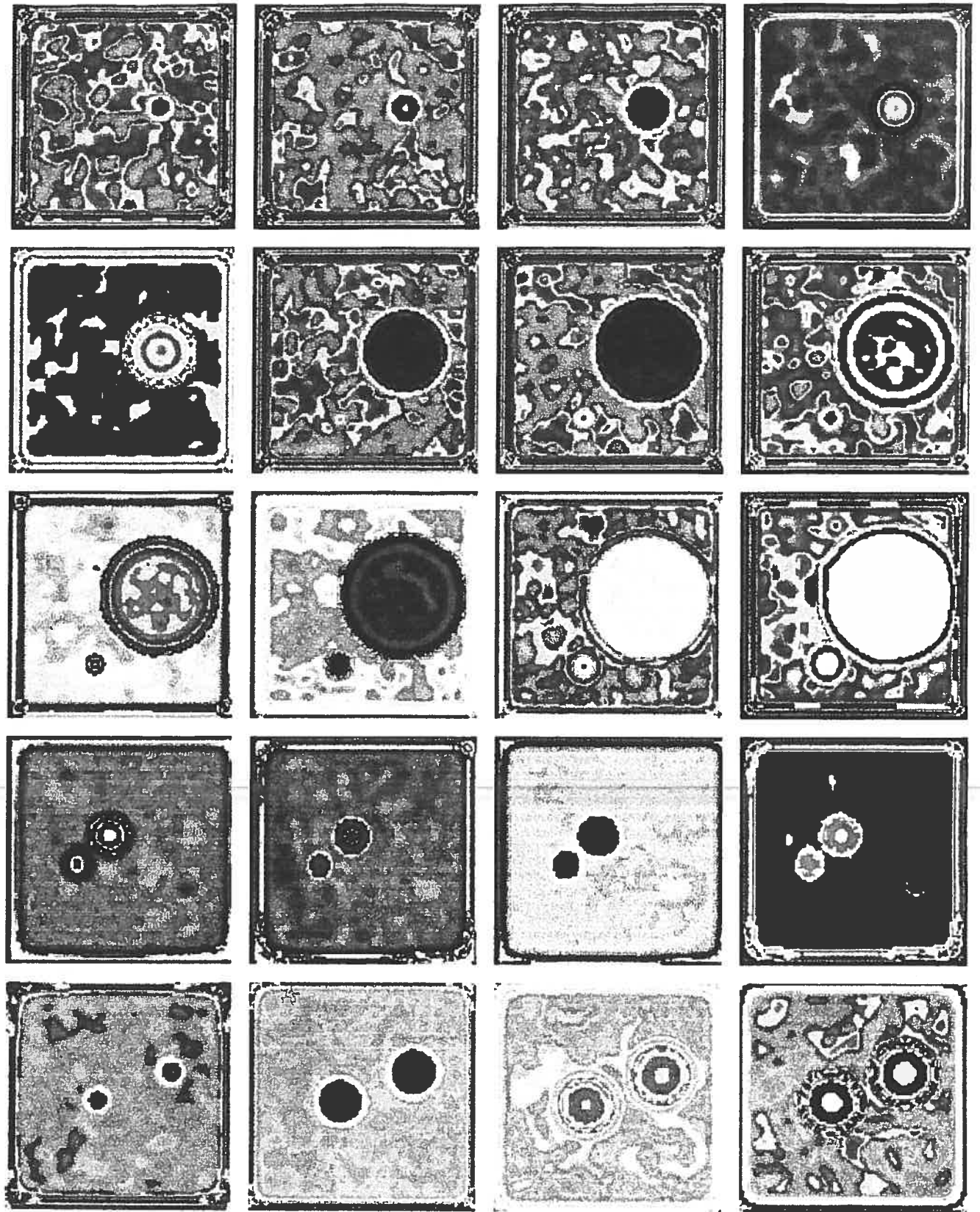


Figure 6. Solitons, calculated by discrete chaotic dynamics