

Committee 6
Science and Music: A Unifying Concept

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THE MISSING LINK OF THE QUADRIVIUM

The first time I ever heard about *Harmony of the Spheres* was at grade school; it did sound a bit like mythology. A few years later I found out about Laplace's cosmogony; it did make sense, in view of Saturn's rings, which we didn't know much about at the time. I began my studies in mathematics, with the intention of sometime or other dropping it and becoming a musician, unaware of the fact that music *is* mathematics. Meanwhile, astronomy was a lovely occasional pastime. I bought Kopal's *The Solar System* at a sale and read from time to time. The classical *Quadrivium* of the 4 Greek mathematical sciences (arithmetic, music, geometry, astronomy, *in this order*) did get my attention, and Helmholtz looked like a good place to start. It struck me that music, being both an art and a "máthema", was an easy victim for ousting from mathematics, a science nobody really ever loved except us mathematicians. And it seemed quite natural that the greatest loss ought to have affected contact with the other 3 sisters, including a complete severing of the old bonds to astronomy; it figures that they would in time sink into the hole of myth and of the occult.

One afternoon I was taking one of my peeks at Kopal, filled with the then daring ideas of resonances, when a certain diagram hit my eye. I knew I had seen it a week or so earlier, but I couldn't have, since I hadn't opened that book in months. It turned out that I was thinking of another diagram from Helmholtz. The former was Brouwer's diagram of asteroid orbits vs. their orbital periods expressed as fractions of Jupiter's, and the latter a figure of æsthetic harmony and mass of beats vs. musical interval. The match was great: the firmament puts nothing at euphonic distances and accumulates mass at dysphony and beats.

It was beautiful but catastrophic to my fantasy of a Pythagorean / Laplace geometry of space. And the villain was Kepler's third law, which formulates an analogy between cubes of periods and squares of average orbital radii. If there be a rational arithmetic for radial distances and another one for periods, these two arithmetics are interconnected in an irrational way. Could it be that nature can fit two watermelons under the same armpit, as a folk expression says? It turns out it can.

Please don't take my hunches here for completed research, because they ain't. They are very general, poor in physical equations and loaded with gross approximative thinking (e.g. circular orbits!). But, it's just that, you know, mathematics is all about looking around you and suddenly conceiving a pattern, for no reason: *mathematical intuition* is what they call it. And it has turned out over the centuries that our hunches must be damn good, otherwise we wouldn't be around any more. But my reasons for not giving up have an even more personal touch to them. My darling little model has kept predicting wonderful bizarre things that have been coming out true; this is much more than I can say about celestial mechanics, a science of people who are always flabbergasted at everything the Voyager and Hubble send them. If you want me to be bold and truthful, I am not at all impressed by the performance of celestial mechanics.

Chapter 1: Beats

Let two held notes be played simultaneously on two theoretical violins. We shall then hear a chord of two notes spaced at some musical interval. Each of the two notes sounds a fundamental frequency f and its integer multiples $2f$, $3f$ a.s.o. The double, triple etc. frequencies determine a set of pitches known as upper partials or overtones, whose amplitudinal profile largely determines timbre (i.e. tone-colour). For a "theoretical violin" we accept that overtones exhibit a uniform drop, their amplitudes being inversely proportional to their ordinal number in the harmonic series, and after the 10th harmonic they are so soft that they are inaudible.

Whether the interval played by these two theoretical violins is thought of as pleasant or unpleasant largely depends on the *primary beats* generated by each pair of simultaneously sounded pitches, and this of course would refer to fundamental as well as overtones; e.g. the 3rd harmonic of one note might beat with the 7th one of the other. And what exactly are primary beats? They are due to two pitches of "neighbouring" frequencies, generating a very characteristic sinusoidally envelopped cosinusoidal graph, and aurally recognizable as an annoying and vaguely alarming rattle. The image is not complete, as the experienced listener would be in a position to assert that "slow" beats have the character of a not-so-unpleasant vibrato, while "very fast" beats sound more like actual notes, constituting what is known as *difference tones*; these are "partly objective and partly subjective", as they occur when the half-period of the spontaneous and objective sinusoidal *envelope function* falls within the famous audibility limits, the so-called subjectivity attributed to the fact that asymmetries of the eardrum and beyond allow our ears to hear envelopes to some extent, beside simple harmonic tones.

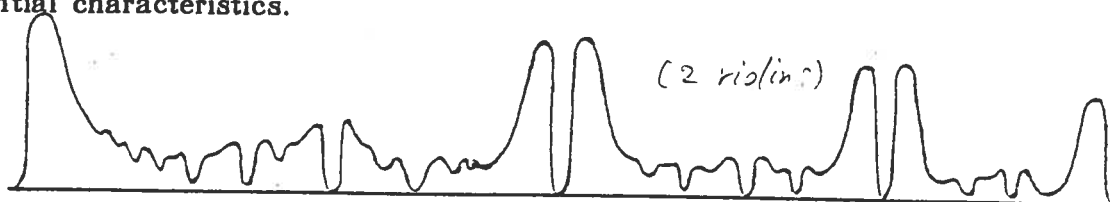
What do we mean by "slow" and "very fast"? Below 20 and above 70 cycles per second respectively. And, by general and uncontested consensus, the annoyance seems to culminate around 33 cps. Why this should happen is a complex mechanical and psychophysiological question involving how our ears are made, how our brain interprets stimuli, what size we are, how our species has survived etc. And why should we be concerned here with something dependent on so many shaky factors? For two paramount reasons, let alone the fact that the objective appearance of a beat graph is unmistakable.

The first of these reasons is as follows: regardless of where our perception puts boundaries and maximum annoyance, still we are always faced with a fine and very typical example of a law's *bracket of validity*. For the concept to be understood let us visualize the example of a long, thin steel cylinder that will vibrate as a string between two sizes of diameters only (all other factors held constant): a thinness at some deforming or breaking point and a thickness of total rigidity; let it also be mentioned that whether the death of a star will lead to a supernova, a nova, a white dwarf, a neutron star or a black hole singularly depends on similar brackets on some of its key parameters, notably its mass and volume.

The straightforward result of validity brackets tells us that a phenomenon is not subject to certain laws without constraints to its parameters, taken individually or in combinations. But the paramount repercussion is that *different* phenomena, within different validity brackets, may exhibit *similar or analogous behaviour*, the

adjectives mathematically interpreted as *isomorphic* axiomatic or theorematic formulations; such is the case e.g. with the basic theory of electric flux, which has been developed in full view of its *analogies* to basic hydraulics, electrons in lieu of molecules, implicating forces of differing natures and ranges etc.

The second reason has to do with the looks of the "roughness graph" thus obtained. Despite its deliberate descriptiveness and its not-so-quantitative nature (as it is a continuous, differentiable etc. function *custom-tailored to fit a subjective/objective physical/biological/aesthetic picture*), the curve has some very particular essential characteristics.



in the small scale the graph exhibits different types of features, according to elevation and curvature. As far as concavities go, depth (altitude difference from cliff-edge to bottom) shows strength of *euphonicity*, and elevation (*y*-intercept) of bottom shows degree of harshness. Thus on one hand we have steep, narrow valleys signifying smoother and rougher (deeper and shallower) strong *euphonies*, and, on the other hand, wide smooth "cols", aurally neutral areas, themselves heard as rougher or smoother contingent on depth of diagram locally. In convexities now there appears a perceptually simpler picture: high elevations stand for *dysphonicity* and designate *dysphonies* or *cacophonies* plagued by hosts of beats, all the more pungent as the bulges get higher, narrower and steeper.

In a medium scale some additional features become manifest. Typically a valley is contained between two roughly symmetrical bulges, whereas the converse is not true. The surrounding bulges may be two "single" high peaks or a more complex system, most interesting among which is a set of two higher hills on the left and right, the dual setup preceded and followed by a secondary pair of lower bulges etc.; thus, on the whole, a (typically less strong) euphony may be framed between two (roughly) symmetric "ridges".

The large scale is more apocalyptic. There appears a general scheme of euphonies falling exactly at points, whose distances from the origin constitute multiples of a certain *standard length r* by simple ratios, i.e. fractions *with small integer terms*: 2/1, 3/1, 4/1, 3/2, 5/2, 4/3, 5/3, 5/4 etc. Those ratios with unitary denominators (e.g. the first three) define a set of *perfect euphonies*: pairs of very high, steep and narrow hills enclosing valleys whose bottoms touch the horizontal axis. This feature will help us choose the appropriate typical *r* for each situation.

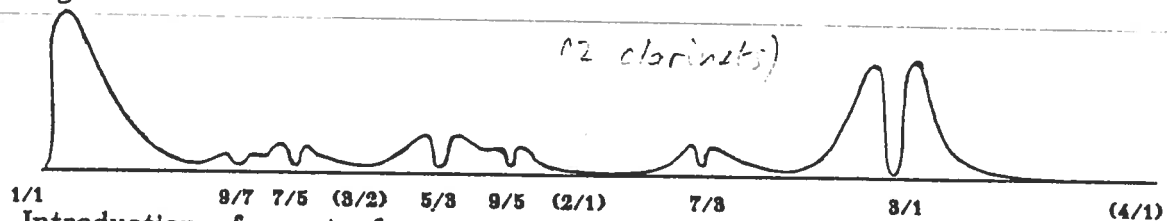
The overall spacing scheme of perfect euphonies to the right of the origin (for ascending musical intervals, the descending situation being symmetric to the origin) corresponds to that of the harmonics of strings or aerial columns. Not all simple ratios are occupied by euphonies, as some harmonics may be absent, i.e. their amplitude may be zero; in these cases we have harmonic profiles that are usually known as "poor" (with very few significant overtones, such as the flute or the french horn), "incomplete" (the best example being the "odd-harmonic" profile of the theoretical clarinet, lacking even-numbered harmonics) or both (e.g. the Pan-pipes). However not all of these positions will carry beat bulges; they may only be

crowned by mildly neutral wide and shallow cols. Here we concern ourselves with a very average case of a "full" timbre with ten significant harmonics, gradually dropping in amplitude (conversely to their ordinal number); this hypothetical sound source we have called "a theoretical violin", as it approaches the typical theoretical profile, "sawtooth" graph and sound of the actual tone-rich instrument.

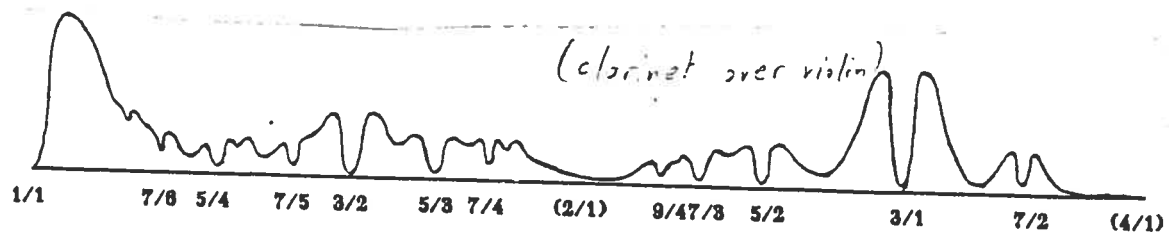
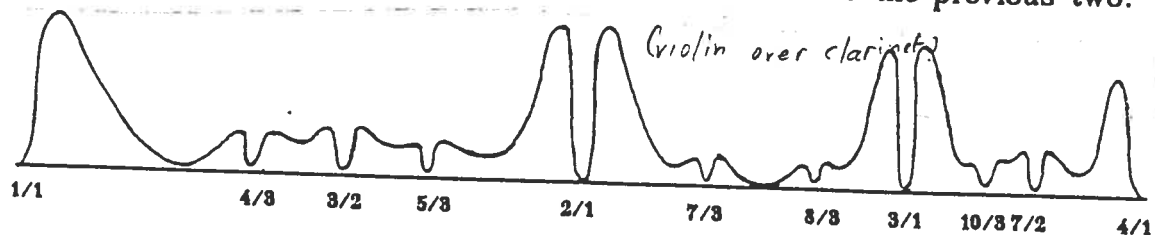
A pair of these violins exhibit perfect euphonies at each overtone. If λ be the distance of the first and most perfectly euphonious overtone (the *octave*) from the origin (the *unison*), the rest of them will be equitably spaced at $2\lambda, 3\lambda$ etc.; not *ad infinitum* though, only just a few, as our convention goes no farther than the 10th harmonic. In fact, any convention with any degree of realism will end somewhere.

What happens in between these perfect euphonies (which incidentally exhibit a gradual thinning trend as one moves farther and farther right) is the following: more euphonies arise at points measurable by simple rational multiples of π ; these are generally closer to perfect (milder, or "more beautiful") as the (integer) terms of the ratio are smaller ($2/1, 3/2, 4/3, 5/3, 5/4, 7/4, 6/5$ etc.).

Amplification/compression of extant harmonics in standard proportion will scale the diagram up/down in the vertical (y) direction. Altering the mix (proportion of amplitudes) of harmonics will change the relative heights and depths at and around euphonies. Elimination of a set of harmonics will shave off the relevant euphonies turning them into neutral basins; see figure for two clarinets with even harmonics missing.



Introduction of a set of new ones will add new euphonies. Enlargement of the harmonic compass with a large number of weak harmonics higher than the ones already there (e.g. introduction of harmonics 11 through 60 with low amplitudes) will drastically affect the smoothness of the curve, giving it bumpy zigzag-like looks, with hosts of new marginal euphonies at every ratio involving integers at most equalling 60; it will also extend the span of the curve to the right. Finally, coupling two dissimilar timbres will keep some euphonies and eliminate others according to the particular case; compare and contrast the two curves for violin above clarinet and clarinet above violin to each other and to the previous two.



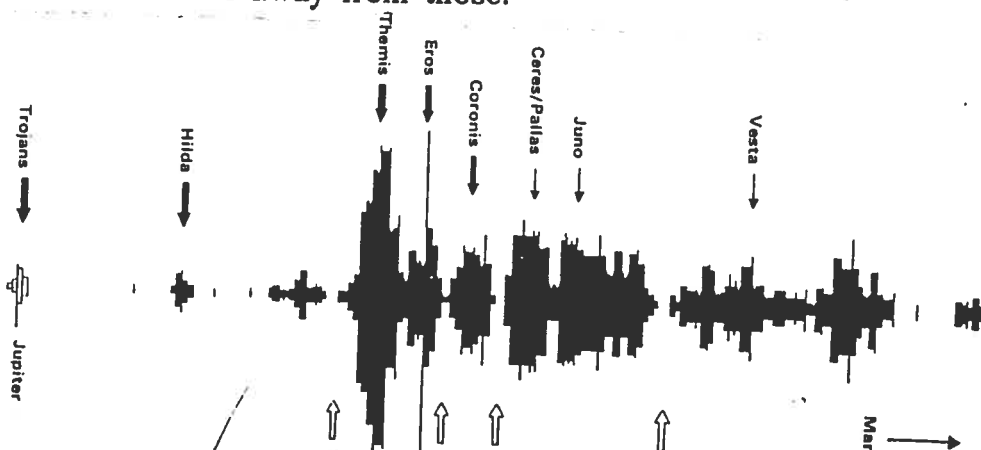
Chapter 2: Asteroids

Let us go back to the ripe stages of the preplanetary nebula. Matter can still be described macroscopically as finely chopped and mesoscopically as particles, gases, dust and boulders. While falling towards a centre of gravitation, at the same time it is already revolving (around some axis due to geometrical constraints) with an angular velocity diminishing away from the centre. Combination of inertia and gravitational and magnetic attractions and repulsions give rise to a compound structure of superimposed concentric geometric tendencies to assume the 3-dimensional shapes of a. sphere, b. torus, c. solid dilobe, and also the flat quasi-2-dimensional ones of a. circle (disk), b. flat dilobe.

The major part of the initial mass is condensed in satellites, planets and the Sun. On the other hand, current objects most closely resembling the state of matter in that stage are the present finely chopped areas of our solar system, namely a. magnetic fields, b. atmospheres, c. planetary toruses, d. rings and belts. Were one to start from flat configurations, one would be bound to concentrate on the more explicit examples of the asteroidal belt and the rings of major planets, Saturn in the first place, secondly Uranus and Neptune and thirdly Jupiter. It would be expected that these areas would speak most eloquently of certain structural particulars of the protoplanetary peripheral disk.

A simple viewing would be enough for the striking realization that the structural frame resides not in concentrations of mass/density but in the gaps, narrow strips of void serving as boundary between wide zones of material which, in a protoplanetary sense, would subsequently condense through a local whirling process. This would suffice to show that it might be wiser to try to deduce the integral structure of a planetary system's later fully formed stages by looking *not at the planets but at the gaps in between planetary orbits*.

Let us have a closer look at the asteroids, as time is ripe for particulars. The asteroidal belt, and especially its central, most crowded part, spells one thing very clearly: at various radii there are characteristic narrow voids known as *Kirkwood gaps*. So, were one to postulate that the original framework of the disk is reflected in this (archaic?) part of the planetary system, one would have to look at gaps rather than bodies. One would thus be tempted to conclude that the first thing to happen was a primeval breakup of the disk into rings by nodal matter-repellent boundaries, rather than a concentration of masses in matter-traps. In a simpler, more down-to-earth language, space chooses certain positions that it will leave empty, and shooes mass away from those.



And exactly how are these gaps spaced? Let us have a close look at Brouwer's diagram. In the scale employed there, gaps (e.g. at $1/2$, $2/5$, $1/3$ and $1/4$ in the vicinity of Mars) and near-gaps (e.g. at $3/7$ and the unmarked $2/9$ and $3/8$ in the middle portion) are spaced exactly in ratios of small integers, most faithfully reproducing the right half (upper octave) of the 2-violin dysphonies. This detailed and perhaps impressive resemblance may appear to be an ultimate basis to solving our problem. But this is only superficial. So far, all that the comparison of the two diagrams shows is a near-perfect correspondence of frequential correlations between two distinct phenomena, an aural and an orbital one; in other words we have come to the same result that has paved the road towards "resonance" theories. We are certainly not on the way to a geometric theory unless we introduce some geometric concepts, such as lengths, areas or volumes.

Actually, what are we told? nothing more than that at commensurabilities we get euphonies/gaps, and on either side of each one of them we get dysphonies/crowdings. We already knew that. For the analogy to attain geometric significance we must go over to radial distances, changing the scale of Brouwer's diagram to linear. So let us revert to linear scales.

One would duly ask two questions. If one were concerned with radial distances, why would one want to start with an orbital diagram like Brouwer's, rather than diagrams with a linear radial scale to begin with? Why go on reasoning along lines that have been shown devoid of validity? In fact there is a dual very real hazard in examining aspects of phenomena: overestimating the information they contain, and underestimating it. To correctly read what is inscribed, one ought to attain very delicate balances. Is that possible? and how? What is the *true* story told by the asteroids? Here one gets into a rather involved discussion.

Let us go over the logical alternative scenarios regarding the asteroidal zone, starting with the spatial parameter. First we notice that, with respect to an order of spatial magnitudes comparable to the entire solar system and beyond, halfway to our nearest stellar neighbours, matter in this zone would indeed appear finely chopped. In this sense it would reflect distributive and dynamic/kinetic laws to a considerable degree of faithfulness. Second we are obliged to observe that the zone is far from isolated, subjected as it is to the tremendous tidal pulls and pushes from its planetary neighbours: gigantic Jupiter and, possibly among other things, significantly sized Mars; furthermore, fine chopping of matter helps these forces be felt quite strongly by this archipelago of minuscule "particles".

Regarding the time elapsed between the early stages of the solar system and now, several possibilities can be stated and examined. Thus we can conjecture that initially the asteroidal zone was empty or it was finely chopped from the beginning; the distribution of the particles exactly or roughly resembled what it is now, or it did not resemble the present picture at all; as time went on, things did not change drastically, as no planet ever formed there; or, on the contrary, one or more planets did form, but are no longer there, because they either broke up or somehow wandered elsewhere, and this hypothetical event may have happened once or more than once, either by shattering and reassembling or by a series of successive formations-migrations. This "decision tree" just about covers the lot.

In the midst of all this we don't want to lose sight of our main objective here, that is to discover and formulate a possible geometric hypothesis. So if there be a

geometric predisposition in the form and structure of the solar system, it is either strong and inelastic to other forces or it is not, in which case these other forces will have prevailed by now, at the local or global levels. If the geometric aspect is ultimately strong, disruptions will tend to be settled over time and the system will exhibit a trend to restore the geometry wherever the other disruptions do not assume a permanent status; in this case, though, the later geometry would reflect the earlier one. And if, in our effort to formulate a viable hypothesis, we are exactly seeking such strong geometric predispositions, as this is the only context in which a hypothesis of this kind could be of any importance, then we need only be concerned with realities that are valid now as they were in the early stages.

Therefore, since the asteroidal zone is in a tidal tug-of-war, a meaningful correspondence of geometry and appearance will have to be sought elsewhere. And where would that be? I suspect that a good place to look is the zone of major planets, itself once finely chopped, yet endowed with huge quantities of material (if judged by the outcome) and thus relatively self-contained as regards mechanics. Since this zone has subsequently condensed in the four giant planets, and if significant displacements after condensation can be pinpointed, then we may conclude that their present orbital radii can reveal a lot about their preplanetary state.

The asteroidal zone is made extremely vulnerable to external influences by its very nature; the same holds for planetary rings. If there is a law governing the zone, it should be clearly depicted in its geometry at any time. Similarly, if two (or more) simultaneous laws are active there in an overlapping manner, what we see must be their joint/cumulative effect, and the wise thing to do is turn to the analytical method for disentangling them and imparting upon each its own.

There may be a key here to an eloquent qualitative picture of how such a swirling "dusty" zone turns out subject to some distributive law. And this is where Brouwer's diagram proves valuable. It shows that there exists an adjustment of scales (a domain where x 's are as radial distances raised to the $2/3$ power) that can bring out the similarities between two dissimilar phenomena in the already discussed context of two different brackets of validity of two different laws producing similar appearances; and, what is more, the similarity is so strong that it incurs the necessary dynamics for raising a fresh line of enquiry. Besides, if it weren't for this diagram, one might not have even suspected the likely viability of a geometric law, left thinking in terms of but mechanics.

It may indeed be notable that the greater portion of primitive material resides in the area closer to Jupiter (Cybeles, Hildas, Thule, Trojans), or what is left of it, since a good part of whatever was once there presumably has been driven off or swallowed by the giant planet over the aeons. It might be even more interesting in the light of our attempted geometric viewing that a lot of the more abundant material in the part away from Jupiter has probably been getting replenished, which is an argument in favour of factors tending to restore a disrupted geometric mould to some type of configuration of a relatively permanent status.

This restorative trend is of paramount importance. Not very much unlike the healing of wounds in living organisms, it can be viewed as performing three tasks: in primitive animals/areas it grows a new limb where one is lost, while in highly sophisticated ones it partially restores the disturbed region, often stabilizing an

impairment and typically patching up the damage with abundant scar tissue. Its importance, firmly bound to the concept of an innate predisposition, lies in the fact that when we look at a distant solar system, a perennially self-restoring or predictably disturbed geometry will tell us one story, whereas none will be told by a cumulatively self-disfiguring one.

If the zone was initially non-empty and finely chopped, what did it look like? Was it formed in a Lagrange manner, with ringlets surrounded by empty margins, as subzone *O* is now, or was it rather "resonantial"-looking in the fashion of subzone *M*, with empty nodal circumferences, each framed on both sides by fringes filled with material? Odds favour an eventuality of the latter type. The entire space occupied by the asteroidal zone has always been the locus of an interface and the battlefield of a tug-of-war, jammed as it is between two most essentially dissimilar zones. A mental relinearization of the asteroids' radial distribution will deliver a creditable translation of the present configuration to an intrinsic and durable geometric pattern, contingent only on the reliability of a nodal format. And when that is achieved, we can proceed to deciphering secondary facets.

So, did any planets ever condense there? Not counting planetesimals conceivably broken down to asteroid families, there is no decisive argument to the contrary. We know that asteroids have wandered great distances off the zone, in order to collide with or to be captured as satellites to Mars (Phobos and Deimos), Jupiter and other major planets. What excludes the possibility of larger planets having done likewise? Probably nothing, although the event has not attracted the attention of specialists to a sufficient degree so as to have afforded a focus of research as much as it probably merits.

Chapter 3: Anti-strings

A somewhat hypothetical object will be called upon to serve as the next general celestial analogue: a "fluid cymbal". The actual marching band cymbal, with its vibrational mode involving nodal circumferences and diameters, affords the eye with a rather direct reference. But why a fluid one?

Firstly the rigidity of actual metal cymbals, largely due to cohesive adjacent molecules not only along radii but across and in every oblique direction too, makes diameters vibrate more like bars than like "infinitely flexible" strings. Secondly, the weak (gravitational) cohesion of particles permits the envelope profile of a standing wave to be depicted much more faithfully, unlike a string whose elastic pull progressively damps overtones. Thirdly, unlike a string that has to be clamped at both ends, the rim of an unsupported (unmounted, not suspended) actual cymbal vibrates freely, and in this sense is not nodal.

Summing up, the whole object would be akin to some sort of cymbal, each of whose diameters vibrates like a loose "anti-string", oscillating in ways that permit loops at both ends (opposite rim-points) and maximum amplitudes at any point in between nodes; the centre could coincide with either a node or a loop. Oscillations of this object are a dynamic phenomenon of evolution in time, in what regards slow transitional processes from stage to stage, especially the initial formatory and transformatory phases; here is another property reflecting the onset behaviour of an actual cymbal.

In order to progress this discussion, a small diversion is necessary. Cymbals are theoretically classified as round metal plates, along with gongs, tamtams, bells and chimes of various sorts; the feeling of a recognizable pitch goes in this direction from "indefinite" to "definite". Of course a metal plate, being two-dimensional, never produces musical notes in the strict sense, its overtones lacking the required frequential relation of integer multiples to a fundamental. The feeling of definite pitch relies solely on a perceived prevalence of one fundamental frequency and on more or less acceptable numerical relations amongst a relevant set of clearly audible harmonics; this is why the bellmaker's craft is largely determined by a wise varying of thicknesses in the intent of a purer and more melodious sound. Constitution also plays a key role in the emergence of an identifiable musical pitch.

The point and mode of excitement is of paramount importance to the sound quality of a cymbal or any other elastic plate; these factors have to do with whether for example the instrument is hit at the rim or at the centre or in between, at a node or at a loop, with a hard or soft, wide or narrow stick or mallet. The initial excitement procures a running wave that, undergoing repeated reflexions, ends up in a standing one. Sound quality is a cumulative sensation chiefly dependent on the particular harmonic mix, which is greatly influenced by the stroke. The harmonic mix does not remain constant over time, the most dramatic changes happening at the onset. This description befits any sonorous body.

The wavelength incurred by this process can span the entire body or parts of it, whence the perceived pitch will be appraised as lower or higher. Likewise a string, if violently bowed, especially near the bridge (*sul ponticello*), sounds like a chord of overtones; if lightly touched near a node, it rings at a higher harmonic

(*flageolet*) of the fundamental; if "badly" constructed with haphazardly non-constant thickness or differential density it may sound poor, noisy and out of tune. Similar things happen with a pipe that is overblown or leaking or improperly bored.

As time passes, certain harmonics may decay faster than others, with a net result of a progressive alteration in timbre. An artfully made church chime usually loses the perceived fundamental and some harsh overtones, leaving a long mellow tail of a few well-tuned harmonics. After a while the perceived pitch is almost impossible to identify, or a harmonic may be named in lieu of an extinct former "fundamental". A string that is plucked while being touched at a node may keep ringing at the overtone pitch, even though the finger be removed upon plucking. A bowed string may continue being touched likewise or a flute may be kept overblown until the end of a note; the actual pitch perceived is that of the overtone.

These facts are relevant for reasons connected with the nature of celestial systems. Time periods required for their formation are on a cosmological scale, contingent upon extremely long-range forces carrying their own influential parameters in norm and geometric positioning. So onset and final balance phases, interspersed with temporary equilibria, settlements and transitions, are both long and determinative. For our task to be considerably disburdened, we shall attempt to deal with wave phenomena along 1-dimensional spaces, i.e. lines, holding the obligation to separately examine surfaces and solids temporarily alleviated; whatever results we obtain will pertain to discs and spheres as well. A complication is that one can no longer speak in terms of frequencies, reverting back to the older language of wavelengths, as the two are not connected via any simple relation in a 2- and 3-dimensional environment. Yet right now the loss is not so serious, especially in view of what one gains in only having to talk about diameters.

What is of essence now is to describe the behaviour of a vibrating fluid cymbal's diameter. To do that in the proper analytical way is to begin with the standing wave incurred by one single simple harmonic oscillation. For convenient study, let us take any diameter and place it on a trigonometric axis between $-\pi$ and $+\pi$, the centre occupying the origin, the y -axis showing displacement; this theoretical model is an *anti-string* as it were, with loops at both endpoints for each simple harmonic constituent. Supposing that the diameter vibrates at nothing but its lowest specific fundamental frequency, any one of its points at any instance should be expected to be found inside an envelope shaped by plus or minus specific maximum displacement for each abscissa. Immediately we should distinguish between two generic cases, involving whether the centre/middle is a node or loop.

In the first case the envelope of the simplest vibration possible obeys formula $\pm A \sin(x/2)$ (where $-\pi \leq x \leq \pi$), node at 0. In the second case, ignoring a trivial "simplest" no-node sub-case ($x = A$) for vibration to exist in the sense we mean it here, we must go directly to $\pm A \cos x$, nodes at $-\pi/2$ and $\pi/2$. They look like this:

In cases like a planet, or our solar system, or a usual galaxy, concentration of mass at the centre suggests the latter. The simplest case, a geometrically allowable one, implicates what we might call a "0-th" harmonic; its equation is $y = \pm A \cos 0x = \pm A$ and its envelope of an axial section is the trivial no-node subcase: two parallel straight lines defining a slice of space, containing information on orientation and maximum thickness. The 1st harmonic, a true fundamental, is given by equation $y = \pm A \cos x$.

See how the two curves are open at the rim (fig. I). If however we sum the two, $y = \pm (A + A\cos x)$, the rim closes into the simplest case available for a system of this type (fig. II). In adding more harmonics, i.e. integer multiples of the fundamental, we get a variety of solids of revolution closed at the rim, provided the sum of amplitudes of all odd harmonics equals that of all even ones. Varieties of shapes with a few harmonics (in fig. III) are exciting to a topologist, but also to anyone familiar with photos of star β Pictoris. Coming back to the "simplest case possible" above, look at fig. IV for a 3D plotting of the solid's upper surface, and check it against two real objects in the skies. Shape and proportions match to perfection.



Fig I

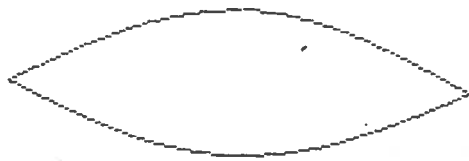


Fig. II

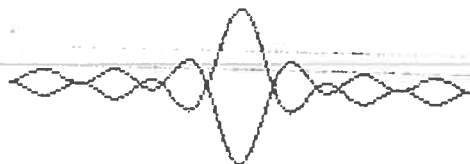
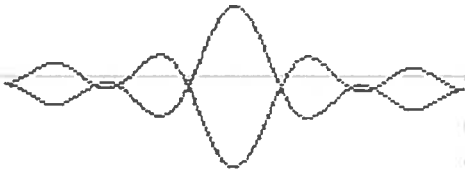


Fig III

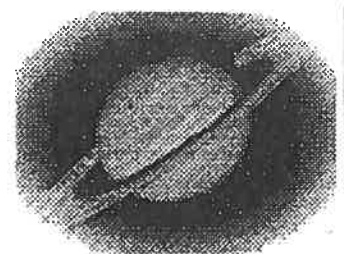
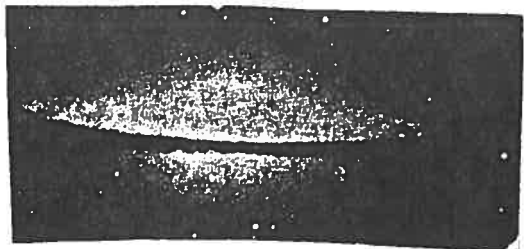
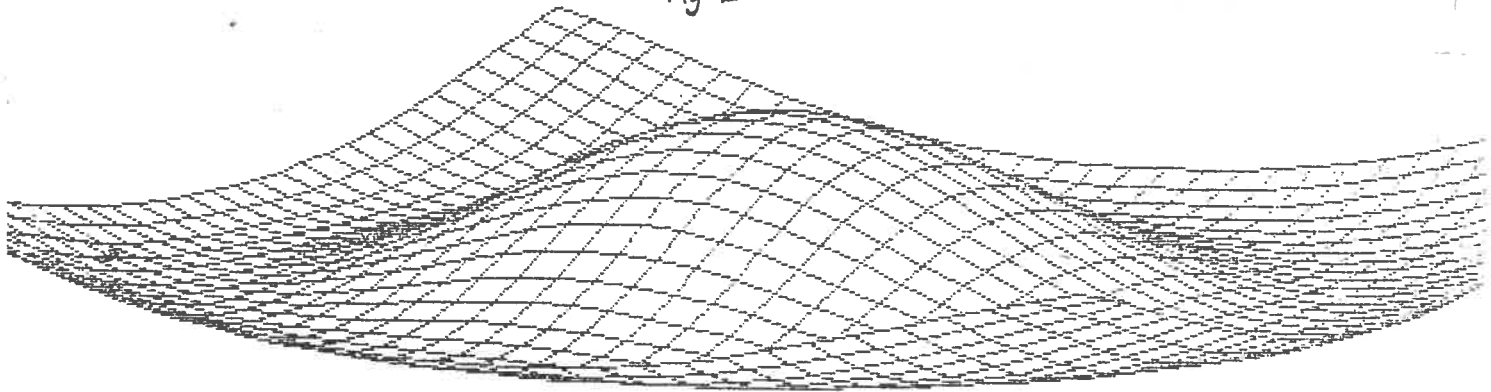
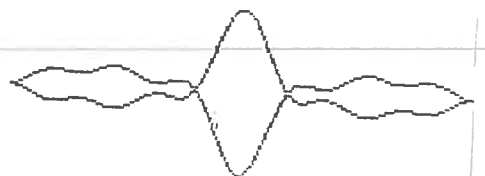


Fig. 4

Chapter 4 : Twins

As was pointed out in chapter 6, contingent on emergence of the system's true geometric skeleton, the leading feature of our dysphony model with respect to such matter-repellent nodes is the formation of two roughly symmetric and mutually slanted mass-accretions on either side of each one. This feature is expected to prevail in the interior. Does the exterior exhibit any degree of beats as well? It turns out that this is much more than plausible, since a standing wave presupposes reflexions at the rim and, if there be a borderline between interior and exterior, then some degree of imperfect reflecting outside this rim ought to develop; strict analogy carries us back to an actual cymbal with a considerably thicker ring at a certain radial distance, and thence to the bellmaker's trick for controlling over-tones. Symmetries here are not meant axially, as with the two spiral arms of a galaxy (that would be a bit too easy); running along a radius, they take on the truly remarkable qualitative meaning of pairs of "like" bodies in successive orbits. This is the cornerstone of a radial geometric pattern involving ubiquitous *dualities* in celestial systems.

If these dualities be as ubiquitous as our line of thought suggests, we must be able to spot them everywhere. To better define our terms, we ought to state right away that "qualitative" is used in the sense of not yet taking into account exact interrelations of actual orbital radii, while "like" has the meaning of comparable sizes, masses and chemistries. In looking for such *twin* planets, it would be mechanically ostentatious to take all possible successive pairs and to ultimately discard the hypothesis on the basis of "Mercury not matching Venus", "Neptune being quite unlike Pluto" etc. An open-minded observer cannot miss three lucid twin pairs hitting the eye and immediately recognizable as such beyond a shred of statistical randomness: Venus and Earth, Jupiter and Saturn, Uranus and Neptune.

It is not the author's flaming desire that makes him see things where they don't exist. Literature has frequently spoken of Venus as a *twin sister* to the Earth, of Uranus and Neptune as *twins*, of Jupiter and Saturn as being two *very*

alike planets; what has passed unremarked is the obvious fact that these twin pairs of planets already constitute a leading majority of six out of nine; furthermore that these pairs rotate in *successive* orbits, a constraint that lifts possible charges of randomness or grouping observations at will. Besides, in light of the new scene, these are the very ones interlocked in "resonances", a further argument favouring this particular grouping and not another one; indeed, "resonances" exist in this configuration, not between say Mars and Jupiter or Saturn and Uranus. Last but not least, these also happen to be the six largest planets. The statement that ought to have been made long ago is that *the solar system is extensively characterized by pairs of twin planets in successive orbits.*

Qualitatively speaking, we are in fact looking for say one, two, three etc. twin pairs of rather sizeable planets, radially arranged one after the other in a row. The image of the outer solar system with its two grand twin pairs of Jupiter/Saturn and Uranus/Neptune aptly fulfils this vision, with the extra specification that the two members of the outside pair are appreciably smaller than the inside ones, all without breeching a basic comparability in sizes, masses and chemical compositions encompassing both pairs.

The same "dual duality" pattern of two roughly comparable successive pairs of successive twin rotating bodies is also the leading element around Jupiter (Io/Europa, Ganymedes/Callisto) and Uranus (Ariel/Umbriel, Titania/Oberon), only here the more massive pair is outside. As for single dualities of a possibly analogous nature, though not on a disc (it is a bit early for this discussion), one cannot fail to mention what revolves around the *two* major members of our local galaxy group; our Galaxy and M31, the great Andromeda nebula, are each accompanied by a twin pair of small *satellite galaxies*: the two Magellanic clouds and the two elliptical nebulae NGC 221 and NGC 205 respectively.

If the inner disc be viewed as generally conformant to some beat function of partial reflexion provenance and of dysphony diagram looks, we ought to allow for the possibility of a different dual duality scheme of twins: *a pair within a pair.* This pattern is consistent with a Venus/Earth pair within a smaller Mercury/Mars one. If this be taken as a viable indication, we shall have accounted for yet two more planets, leaving but one behind: Pluto. Given present rising dispute of its planetary status, the outermost planet is unique in more than one ways. According to most recent data it is no single planet, nor can it be aptly described as a double one; it would be more appropriately called *a planet in two*: Pluto proper and "satellite" Charon, pieces of a ruptured predecessor in all likelihood.

Reverting back to literature, there is a nearly unanimous characterization of Pluto as identical to Neptune's satellite Triton; their physical and chemical properties match, while size and mass of "pre-Pluto" (pre-Pluto-Charon that is) amount to another Triton. Particulars can wait, yet there are lessons to be learned here: planet/satellite twin pairs are conceivable, and twin couplings of big planet-sized satellites are a perfectly legitimate complement to planetary ones.

So, from a certain size up, there emerges a general scheme with supplements:

Chapter 5: Controls

The complex set of correlations in our solar system resides in but a few simple constituents, easily set apart by analysis. Thus we have referred to three main kinds of controls: attractive/dynamic, coordinative/kinetic, spatial/geometric. These controls, in no way independent, are in fact largely contingent upon one another, especially in a historical perspective.

The Sun acts as chief attractive pole, subduing the entire system to its centripetal pull, making everything fall in a coiling fashion or attain inertial equilibria in specific orbits obeying the laws expressed by Kepler. The chief kinetic control proceeds from the principal zone of Jupiter and Saturn, where globally maximal angular momentum resides. As a consequence, the whole solar system behaves like a grand gyroscope, resisting changes in the orientation of the *invariant* plane. Geometric moulding of standing wave provenance comes in the two types of beat diagram and fluid cymbal already discussed in detail, both exhibiting empty (orbital disequilibrium) harmonic nodes and crowded (orbital equilibrium) dysharmonic areas. Twin bodies in proximal successive orbits are a direct result of the former, more dominant in the interior than in the exterior. The two types of geometric moulds differ in that the former induce great orbital stability, while the latter, prevailing in the exterior in the long run, afford easier sliding. Synergies of the two geometric elements are especially manifest in cases of common (pseudo-)nodes.

To repeat all facts regarding synergies and antagonisms at this point would be tiresome and redundant. A brief analytical-synthetic probing of crucial factors can and should be carried out though, starting with centripetal attraction that tends to give subsystems a spherical form. Thus the Sun, individual planets, satellites etc. past a certain size (given by Wildt's formula), and the entire solar system as a whole, exhibit a spherical tendency. Conversely, in a gyrating system, inertia is felt as an escaping or *centrifugal* factor. Inertia now is the same everywhere, while attraction diminishes with the square of the distance from a gravitational mass centre. We begin study of their interplay at the global centre of a celestial system, specifically at the central entity. If rotation be so fast as to totally overcome attraction, balancing out is attained at the centre and we get an 8-shaped prostrate toroid of revolution with no central sphere and, consequently, a sinusoidal envelope. If the two influences balance each other out inside the body we may see an oblate sphere with perhaps two little hollow regions at the poles.

Note that the present analysis concerns a single modestly sized isolated primæval nebula with an identifiable punctual centre, not a greater one destined to form sibling stars in a genuine 3-dimensional arrangement, like close-range small-period twin stars or similar stellar swarms with partial centres arranged in a containing solid, as is the case with the Pleiades for example, where 4 approximately equal bigger stars roughly occupy 4 peaks of a tetrahedron, let alone several nebulae at close range, forming miscellaneous stars that will eventually drag one another into large-period double or multiple systems.

Coming back to the nebula in question, the better part of excess material shall form a massive peripheral zone. If the quantity of material still be quite sizeable, i.e. comparable to central mass, though possibly differing in density (denser?) and

Other physical/chemical properties, the zone may or may not split into 2 and then form peripheral whirlpools igniting to produce double or triple stars, akin to a magnified and much more equitably split Sun-Jupiter or Sun-Jupiter-Saturn setup; in any case margins are narrow, as anything a few times the mass of Jupiter will ignite. Each particular configuration is a consequence of combinations of inherent parameters and/or external influences from the immediate neighbourhood.

During the nebulous stage, one massive peripheral flat torus or bulgy zone will concentrate most angular momentum, producing a gyroscopic analogue and turning into our familiar principal zone, whose considerable mass accrual will attract all the more mass to itself, going on to forming two domineering whirlpools, and subsequently two big planets; these planets will continue exerting attraction, only as solid bodies now.

We shall now go to the inner solar system, radially pictured in scale below up to and including Saturn, putting planets and zones in their present positions.

<i>MVE</i>	<i>M</i>	<i>As†</i>		<i>J</i>		<i>S</i>
o o o	o†.....	.	0	.	0
0	$\frac{\alpha}{16}$ $\frac{\alpha}{8}$ $\frac{3\alpha}{16}$ $\frac{\alpha}{4}$	$\frac{3\alpha}{8}$ $\frac{\alpha}{2}$	$\frac{5\alpha}{8}$ $\frac{3\alpha}{4}$	α	$\frac{5\alpha}{4}$	2α
	$\frac{\alpha}{12}$ $\frac{\alpha}{6}$	$\frac{\alpha}{3}$ $\frac{5\alpha}{12}$	$\frac{7\alpha}{12}$ $\frac{2\alpha}{3}$		$\frac{4\alpha}{3}$	$\frac{5\alpha}{3}$
	$\frac{\beta}{8}$ $\frac{\beta}{4}$ $\frac{3\beta}{8}$	$\frac{\beta}{2}$ $\frac{5\beta}{8}$ $\frac{3\beta}{4}$	$\frac{7\beta}{8}$ β	$\frac{3\beta}{2}$	2β	$\frac{5\beta}{2}$ 3β

Inner pair: Mercury and Venus
 upright axes: 2°, 177°.8
 long sidereal days: 58^d.6, -243^d.1
 inclined to the ecliptic: 7°, 3° 24'
 no satellites

Outer pair: Earth and Mars
 slanted axes: 23°.4, 25°.2
 shorter sidereal days: 23^h.934, -24^h.623
 hardly inclined to the ecliptic: 0°, 1° 51'
 satellites

Main pair: Venus and Earth
 bigger and heavier
 more alike

Auxiliary pair: Mercury and Mars
 smaller and lighter
 less alike

less eccentric: 0.0068, 0.0167
 ratio of diameters: 19:20
 ratio of volumes: 6:7
 ratio of masses: 13:16

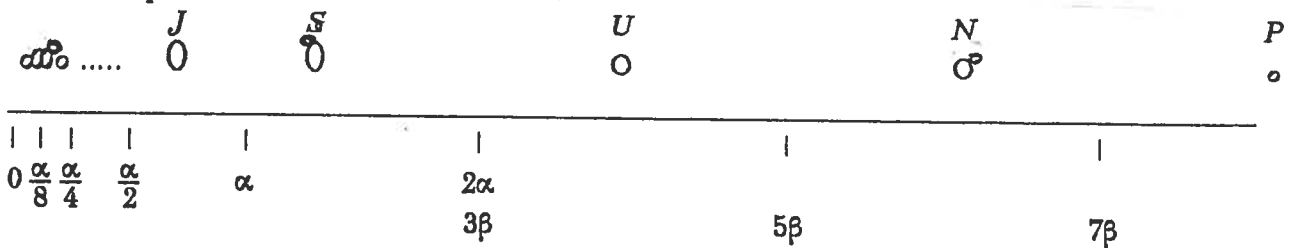
more eccentric: 0.2056, 0.0933
 ratio of diameters: 5:7
 ratio of volumes: 10:27
 ratio of masses: 14:29

Naturally, going away from the Sun there are decreasing solar and increasing Jovian controls. The Earth has one disproportionately large satellite, the Moon, with some interesting characteristics. Locked in a 1:1 spin/orbit synchronicity, it permanently shows the Earth the same hemisphere. Presumably the Moon has come from elsewhere. Its density (3.34 kg/lit) is close to martian, its seismic properties are reminiscent of a metal or baked clay bell and it has circular *maria* only on the side facing the Earth. Mutual motion of the two bodies is not stabilized yet, as the Moon is in a process of increasing its radial distance from the Earth, also tidally slowing down our planet towards a terminal sidereal day of about 60 pre-

3/2	perfect fifth	G
8/5	minor sixth	A \flat
5/3	major sixth	A
7/4	subminor seventh	B \flat (sub-flat)
9/5	minor seventh	B \flat
2/1	octave	C'
9/4	ninth	D'
7/3	subminor tenth	E \flat '
5/2	major tenth	E'
8/3	perfect eleventh	F'
14/5	subdiminished fifth	G \flat '
3/1	perfect twelfth	G'
16/5	minor thirteenth	A \flat '
7/2	subminor fourteenth	B \flat '
18/5	minor fourteenth	B \flat '
4/1	double octave	C''
14/3	subminor seventeenth	E \flat ''

Saturn, taken as approaching a 2/5 resonance, would be placed a major tenth below Jupiter at note A \flat ; the fundamental would lie another octave deeper than that at A \flat . According to what we have been seeing, the solar system seems caught in warfare between two sound and well-organized sides: on one side stand two geometric systems operating on the principle of a fluid cymbal's radial segments, viewed as wavelengths and as fields of beat accumulation; the other side is held by planetary frequencies. The quest for approximative reconciliation through powers 3/2 and 2/3, impossible in the exact sense, betrays an effort to put two irrationally related rational arithmetics in the same basket.

What seems to be a big concern for planets has been troubling human thought for centuries too. It is the battle painstakingly fought in arithmetic and geometry by ancient mathematicians, centered on the alarming rational-irrational impasse and persistently recurring in the famous problems of doubling a cube (the *Delian* problem), squaring a circle and trisecting an angle. The exact same battle was then being fought in music between the number-theoretically oriented Pythagorean school and the perceptually oriented school of Aristoxenus, only to be taken up anew in the late Middle Ages, the Renaissance and all the way to the present day, except this time it is called a war between "just intonation" and "equal temperament", the latter having temporarily (I hope) predominated through the assistance of mass production and consumerism.



If the case of planets is hard to tackle, moons are considerably more complicated, though by no means prohibitive; moons are vulnerable besides, and their history is a tormented one. I shall only briefly cover features grouping the biggest ones in pairs of twins (in successive orbits as always). First comes Jupiter's "Galilean" quartet.

Inner pair: Io and Europa
 smaller
 ratio of diameters: 8:7
 ratio of volumes: 3:2
 ratio of masses: 3:2

Outer pair: Ganymedes and Callisto
 bigger
 ratio of diameters: 14:13
 ratio of volumes: 19:15
 ratio of masses: 8:5

And here come Saturn's original 3 pairs, according to the present approach.

Mimas and Enceladus
 smallest
 least alike
 ratio of diameters: 7:9
 ratio of volumes: 10:21
 ratio of masses: 16:31

Tethys and Dione
 intermediate
 ratio of diameters: 15:16
 ratio of volumes: 9:11
 ratio of masses: 10:17

Rhea and Iapetus
 biggest
 most alike
 ratio of diameters: 17:16
 ratio of volumes: 6:5
 ratio of masses: 13:10

Coming to Uranus now:

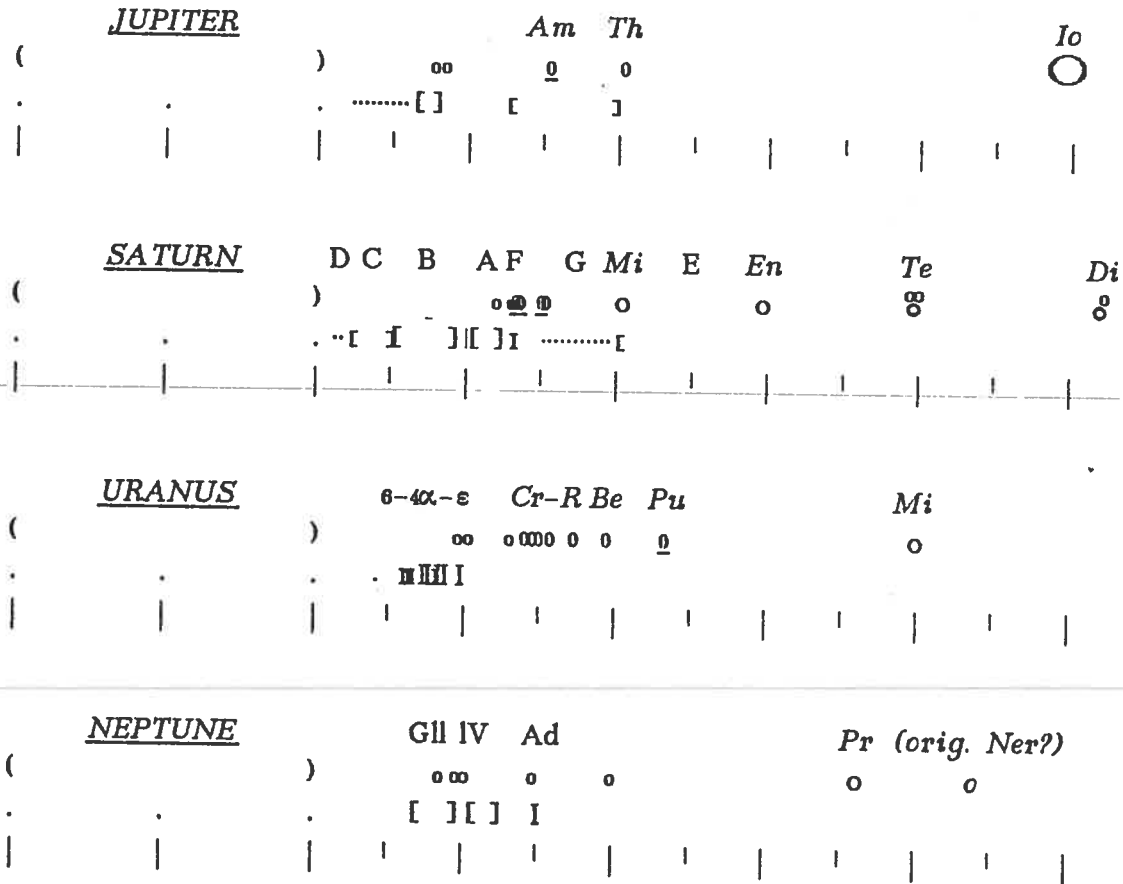
Inner pair: Ariel and Umbriel
 smaller
 ratio of diameters: 28:25
 ratio of volumes: 7:5
 ratio of masses: 7:3

Outer pair: Titania and Oberon
 bigger
 ratio of diameters: 117:116
 ratio of volumes: 40:39
 ratio of masses: 7:4

Triton is to Neptune what Titan is to Saturn. And the question lurks: what ever has happened to Neptune's own Uranus-like quartet? That question is definitely not an unanswerable one.

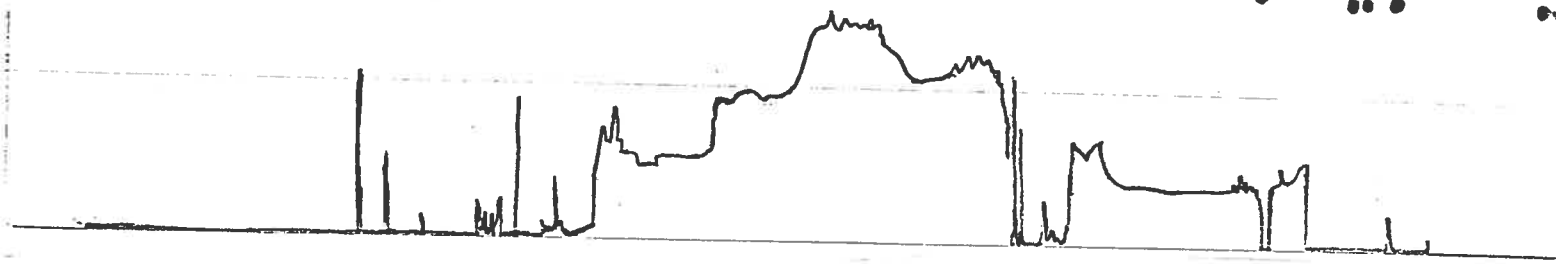
Chapter 6 : Comparing and contrasting

Shown below is a quartet of figures, one for each major planet and its private neighbourhood; each partial figure is differently scaled, so as to visually equate the planets' radii for comparison. Striking geometric similarities in the four planets irrespective of sizes, masses, densities *and extraneous influences* are here for us to see. Parentheses indicate equatorial planetary diameters; circles and ellipses signify moons; square brackets enclose the width of individual rings (I's for narrow ones). Underlined moonlets are large.

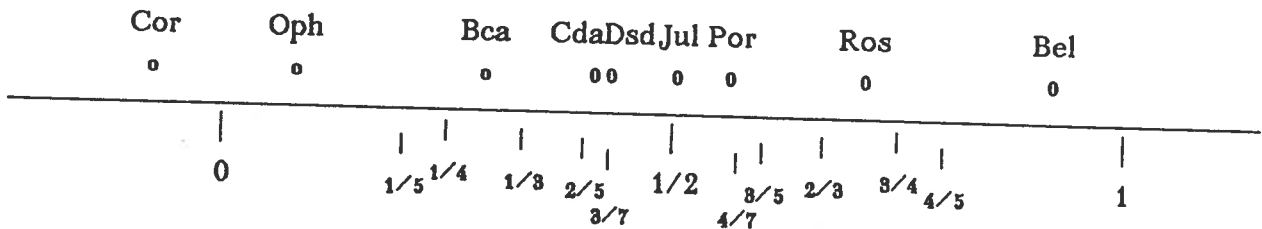


Ring systems are best suited as a detailed study object for a faithful manifestation of whatever laws govern the situation. Being loose and dusty, their fine particles subject to very weak interaction, they exhibit a harmonious coexistence of a sovereign geometric frame and subordinate yet clear dysphony configurations, too numerous to dismiss. Regardless of the diversity of their physical parameters, they all have sharp cuts at euphonic radial ratios, each according to the particular mix of harmonics which is thus traceable; these gaps are typically framed by two moonlets, or two density peaks, or one of each. They all have a main gap at the distance of 1 radius from the surface (Cassini gap or its analogue) and then an outer set begins at 1 radius and ends at 2 radii. The inner and outer rings, possibly carved into ringlets and euphonic grooves, are varied mixtures of moonlets and sub-rings. The Uranian ring system is best to study in this respect, as the inner half is practically all ringlets and the outer half all moonlets. From talks I've had with astrophysicists, they seem truly amazed at this fact, so far unnoticed by their science. A fuller analysis is impossible due to space

limitations. For a better look at intra-ring harmonic synteses, take a peek at an exaggerated profile figure of Sarurn's rings, heights showing "radial density of matter" (i.e. concentration of mass along the radius) and decide for yourselves which part of which dysphony diagram from ch. 1 they match. This will be your homework.



Uranus's exciting "discrete outer ring" (swarm of moonlets) presents us with this telling set of symmetries; I should point out that the "asymmetrical" arrangement of the 4 in the middle is even more eloquent for a very specific set of reasons.



EPILOGUE

This whole idea, presented on this paper, is a mathematician's feeling that a certain angle of tackling a problem is viable: a sovereign harmonic geometric skeleton filled in with intricate fringy zones of beat accumulation, two rational geometries connected in an irrational way. And, mind you, I have said nothing of fully 3-dimensional objects like galaxies, or nodal diameters which do a lovely job in cases like asteroids Trojans that are Jupiter's tiny companions, like Neptune's arc of a ring or like a galaxy's spiral arms; or even sinusoidal harmonic equations with a node in the centre, potentially ideal for black holes and singularities; or other exciting stuff. There is no such thing as a true geometrical topological model for all this, and I have a strong hunch that it may just work if and when more work is done. A great many details have already been checked and re-checked with encouraging results here. Of course what you heard here is only a rough outline.

Reception of this particular approach has so far been one of interest, puzzlement, scepticism or overt rejection. This is all understandable, as I provide no complicated differential equations, non-linear perplexities, chaotic considerations, fancy advanced mechanics or other current scientific tools. I could but I won't. This is my reaction to a bitter fact: all of us, caught up in our specialties as we are, sometimes lose perspective of what science is all about. What else, if not a *decision* that a certain direction or model, imagined in somebody's mathematical *intuition*, can match observation if elaborated on to a sufficient degree?

The heliocentric model of antiquity was well-respected until it was shown to misperform; Ptolemy provided an improved version, the *epicycle* model, which seemed fine for its time. That one *was* doing the job. But later times showed that it was more advantageous, meaning *simpler* and *less fancy and involved*, to think in terms of a heliocentric model. Could the geocentric one be improved so as to do better and better? Undoubtedly so. However, later scientific consensus *decided* that the question had been asked "wrong", meaning *not to the advantage of study*. And one sometimes wonders how many questions, answered today in their advanced particular ways, will be declared "wrong" and obsolete tomorrow. That is why we all keep getting *intuitively* inspired.

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