Committee 6
Science and Music: A Unifying Concept

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The Music World Before, During, and After Pythagoras

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Very often, in conferences like this, it becomes necessary for me to talk about the rôle and workings of mathematics. Ideally this should not be happening. However one finds that most people mistake a physical version of mathematics as the discipline itself. It is not my intention to put things straight here but, judging from the two subjects I am to talk about, it may be inevitable at certain points to stop and review what mathematics does and what it doesn't do.

The subject here is music, the second of four classical branches constituting the mathematical *Quadrivium*. In order to uncover the majestic structure of ancient Greek music, first we must ponder on the nature and essence of the art, as well as its parallels, differences and interconnexions with other fields and sub-fields of human knowledge and activity. Music then is an art conceiving, building, executing, enjoying and appraising a certain category of sound event. What kind? Structure requires laws and rules; sound, in general, abides to one analytical law offering perspectives for building structure: periodicity as described in Fourier's famous theorem. If it weren't for that, all rules pertaining to music would have to be arbitrarily chosen or borrowed from something else, essentially extraneous to music. The choice is clear: either periodicity or lawlessness. To the latter add the following: if there exists an analytical law, then lawlessness boils down to one of three things, a. total lack of relevant *and available* information, b. failure to grasp its applicability, or c. refusal to accept it. In an era like ours, they all amount to the same really.

What should one expect from a musically "lawless" culture? A fully chaotic state of sound sources, arbitrary and short-lived rules, failure of justifiable and lasting aesthetic criteria, absence of a founded sense of value, a detached subjective historical, anthropological, social and psychological tone to everything. On the contrary, a "lawful" culture going by the appicability of periodicity as basic musical law would be expected to construct musical sources playing notes with harmonic profiles, so these sources would necessarily turn up one-dimensional and elastic. And that is for continuous stimuli interpreted as pitches over 20 or so Hertz; as for periods around there and below, the same culture woud be expected to employ objects making short dry noises that could be periodically repeated, giving a discrete sense of rhythm. Simple pitches would pile up in timbres (within one note) and chords (among different notes) and all those would interchange in melody, progression and counterpoint, put together in forms with elements of an even slower periodicity. It's all right there from the very start.

And why is that? Because period involves cycle of repetition and the time it takes, as well as its inverse, which is frequency. Periodicity becomes number and, all of a sudden, the full package of number theory is available as music theory. The ear deepens the connexion: it participates in this plot by perceiving ratio as interval through an admirably coordinated complexity of mechanical and perceptive mechanisms, so enter logarithms.

It is important to grasp two basic facts here. A. The choice of even one of these tools and attributes indicates a conscious conception of periodicity and the choice for making music thereby. Certain styles have done away with this principle here and there, but they are betrayed by the fact that they abide by it in other areas, or constantly keep approximating it or manifestly and deliberately avoiding it. But really now, how balanced amd coherent can one be when one accepts and rejects the same

thing at the same time and, even more so, when one pretends that an extant law doesn't exist, or that an arbitrary rule is "preferable"? B. Given that basic periodicity manifests itself as law, ground material and frame of finished product, all three in one, the Æsthetics of music, if such thing existed, would show a most decidedly *ontological* face. But, does it exist? And how do philosophers react to this idea?

Philosophy originally meant a general quest for knowledge, including a study of the physical world. Around Pythagoras's time, the field got segmented, but its pieces were still viewed as separate disciplines under study by the same people. It is therefore incomplete and pretentious, on our side, to approach pre-classical and classical Greek art, mathematics and philosophy (in the current sense) without one another. It is fortuitous to start right here then.

Music is typically studied as an art within the realm of Æsthetics. But this philosophical branch has been developed mainly on considerations stemming from "the art of speech", i.e. literature, with extras from the pictorial arts. A vicious circle surfaces here. The markedly ontological flavour of music has not been allowed to contribute to the development of Æsthetics, which, nevertheless, has been called upon to settle issues pertaining to music. Were we to stick to the most basic requirements of abstract reasoning, we'd have to reach the verdict that Æsthetics, if formed *in absentia*, is clearly not pertinent to music. This simple fact has not been duly understood however, so Æsthetics has consistently been applied to music. Devoid, as it is, of ontological elements, and one-sidedly "artistic", Æsthetics has kicked back by denuding music of its most essential and formal feature, the pure mathematicity of music theory, and restricting it to its more superficial level, that of creative imagination, emotional response, enjoyment, social function etc. Thus music has been reduced to an "artistic endeavour" in this limited sense, and philosophy has missed a great chance of building a fuller branch of Æsthetics, incorporating the dazzling Gestalt of music and understanding its workings and its dialectic bonds to other human activity. The loss is great, the hole is huge and something has to be done about it: perhaps philosophy need rebuild Æsthetics from the roots up.

No matter how the ear works and how the brain interprets, perception is the issue here and, as everyone knows, a phenomenon *is* before it is perceived; the standard and established way of going into such matters is to begin by examining what *is*. I don't want to go into a theoretical discussion here about what *is* vs. what *appears*. I'd rather answer another pressing question: what have we to gain by tackling music in this fashion? *Everything*, I think, and I'll try to illustrate by a most relevant example.

You are all familiar with efforts to decipher and dissect ancient Greek music. How have we been going about it? In the usual way of studying excerpts and fragments of scant sources, where they exist, drawing conclusions and formulating hypotheses towards a model of understanding and reconstructing. This is the staple way of experimental – empirical sciences, nowadays called just simply "The Sciences" and revered as applicable everywhere. Are we forgetting something? Most definitely so. Music theory, being a "mathema" (= learnable discipline), at least in those times, cannot be duly approached without regard to its own mathematical arsenal, which dwells on the exact opposite: it assembles stimuli from the (perceived) real world, grasps the idea of a model through *intuition*, forgets about the real world, develops the model in a formally sound way and only then does it return to reality, in order to interpret and apply.

In understanding ancient Greek music, then, we shouldn't seek sources, because we don't need them for anything truly substantial. We could look for the principle

instead. First we must ask ourselves why Pythagoreans should have attempted to formulate music theory, and why in the particular way they have. And then, knowing what music actually is, we can begin wondering about their keenness in hearing and deciphering the principle correctly, and rigorously utilizing it in building its structure. Once we've made sure, we can safely proceed to finding solutions to such age-old and "impossible" problems as why Pythagorean intonation sounds so harsh, why Archytas's "non-Pythagorean" tetrachords are not rejected by his fellow Pythagoreans, why Aristoxenus assumed the position he did, what the music sounded like and so on. What is the advantage? Our depot is no longer at level zero, but at a fully adequate knowledge of the state of the art.

Being so lucky as to have such advanced equipment at our disposal, let us now go about examining the eve of pure abstract mathematics, an original Greek conception and invention, music included. Why do I say that? A little historical background could be of assistance. Basic analytical mathematics had been common knowledge in Mesopotamia and Ægypt for a long time. Did the high priests in these advanced countries know theorems and formulae? They must have, otherwise their computational tables and practical geometric feats would have been inexplicable. There is evidence that a worldwide cultural revolution was attempted in the sixth century b.C., instigated by those who had the power to start something as massive as that: perhaps the priests of these two sides, initially in common agreement that probably soon turned into competition, rivalry and war. Hypothetically speaking, and judging by the outcome, it would appear that two ethno-geographical areas stood up and decided to dance to their own tune: Jews, who were caught in the middle, and Greeks, way off the side. I'll decline checking possible causes. I shan't attempt to touch upon the Hebrew side either, though I am partly Jew myself. But I'll try to paint as full a Greek picture for you as I can.

Where is the Greek intellectual revolution and why is pure mathematics Greek? Here it comes. The first philosopher to introduce an abstract methodology is an Ionian, Thales of Miletus, upon his return from his studies in Ægypt. He shall project a fresh way of viewing space, position, shape and size which is no longer an idealized description or version of reality as in Ægypt, but an ideal and abstract model for reality. These are the beginnings of pure theoretical geometry. Thales shall formulate four historical theorems, still valid today, *furnished with their proofs*.

Let me digress here, to focus on this very feature. You see, what *makes* pure abstract mathematics is the necessity to prove sentences that are provable, accept them if and only if we understand the proof and agree with it, reject and denounce them if we don't. All Greek mathematicians since Thales have gone out of their way in stressing *proof* as the quintessential feature of mathematics, and none of them has dreamt of formulating a theorem without proof. This attitude reflects a specific psychological, political, historical and geographical feature of Greeks, and grows into basic ideology: it is the *right* and *duty* of every citizen, regardless of origin, class, birth, finances etc., to argue, discuss, learn, accept and reject, in a vertically integrated social frame. It must be understood by us that the unprecedented Greek scientific and philosophical revolution is not a technical but a *social* and *political* one. The knowledge acquired from the East and the traditional culture of the Greeks combine to form what is known as *reason* and *dialectics* and culminate in Periclean Athens, materialized as socio-political system under a new name: *democracy*. The mixture generates a creative big bang and the world will never be the same. This principle is absolutely conscious and vibrantly radiates in the laws of Draco and Solon, in the

political philosophy of "historian" Thucydides and in the sum total of Æschylus's sublime dramatic poetry. What if it was incomplete and skewed and short-lived in its actual implementation? The principle was there, it was felt and declared as right and it was passed on to us in writing as eternal untarnished heritage.

Let us come back to Thales. His great first step needed completion, as, from what we can see now, it was not free of logical voids. Whereas his theorems were proved, other things were taken for granted, say enumeration (e.g. *one* straight line intersected by another *one* forms *four* angles, *two* of which etc.) and geometrical precedents of the proofs (e.g. the existence of straight line and plane, their infinite extensibility, their parallely etc.).

Enter Pythagoras of Samos. The younger graduate from Ægypt must have found geometric shapes and sizes already too realistic, though he was a conscientious geometrist himself, having provided us with formulation *and proof* of the famous theorem bearing his name. Number on the contrary, a concept employed by Thales but left unexploited, appeared to precede in importance and supercede in intellectual abstraction. There was another difference: the geometric model is a *continuum*. The *discrete* nature of whole number, capable as it is of turning continuous through analogy, is therefore even more abstract than geometry, and *counting* surpasses and comprises *measuring*.

In an explosion of fertile creativity, Pythagoras reckons that the cosmos, visible and invisible, material and immaterial, is born of numbers and analogies of numbers, specifically the most perfect kind, integers, constituting the basis of an innate ability, that of enumeration or counting. But whole numbers themselves are contained in and born by *unity*, a concept indicating the element and the whole: □□ □□, □□ □□ □□□ □□ □□□ (being, one and all). These ideas provide the impetus and context for the development of a *sui generis* theology, according to which all proceeds from godly unity and numbers, divine entities surrounding it. As for visible and tangible extant material bodies, Pythagorean attitude towards them is given by Aristotle in the exact words «□□□□□□□ □□□□□ □□ □□□□ □□□ □□□□□□□», i.e. what exists is in *mimesis* (imitation, transform) of (ideal) numbers.

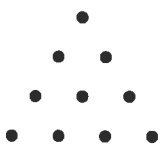
Thalean and Pythagorean mathematics remaining valid to this day, one has a sound basis for studying even the mystical side of their teachings if one has access to its numerical foundation, by simply reverting to the finished product of arithmetic and geometry, even if one lacks "sources". Pure mathematics, as a structure and as a vehicle of attitude, has a lot to tell us. Let us listen, starting from pure arithmetic. Since unity occupies the topmost place, it makes sense that the closer a number to unity, the higher its position in hierarchy. *Small numbers* are therefore closer to perfection than larger ones. Two *consecutive* numbers now, k and k+1, fulfil certain relations. First of all, their *pair / couple* consists of an *even* (female) and an *odd* (male) one, whose *ratio* (quotient, analogy, fraction), k+1 over k, keeps approaching unity as they get larger and larger. Ratios of this type are called *superparticular* (2/1: *superprimal* or *duplex*, 3/2: *supersecundal* or *hemiolic*, 4/3: *supertertial*, 5/4 = *superquartal* a.s.o.) and claim an utterly special place in Pythagorean arithmosophy. Unity stands on the summit alone, so the first and hierarchically supreme couple of consecutive numbers is the smallest of all: 2 and 3. Number 3 is connected to its predecessors through the weaker and more independent bond of summation (3 = 1+2), the stronger and more dependent one being that of multiplication. An infinity of multiplicatory (factorial) combinations of numbers 2 and 3 can yield as good an approximation to any other number or ratio as we wish. For instance, superparticular

ratio 5/4 (= 1,25) is "satisfactorily" close to ratio 81/64 (≅ 1,2656), equalling 3^4 / 2^6, and "much closer even" to ratio 8192/6561 (≅ 1,2486), equalling 2^13 / 3^8; this is followed by an infinite sequence of other fractions, with ever more and more 2's and 3's, which will draw even closer to any desired degree of approximation, whether the approximand be rational or irrational. Since everything maintains a state of *mimesis* anyway, and since numbers 2 and 3 suffice in order to approximate (or be approximated by) every other analogy, their pair assumes the status of an archetypal fertile universal parental couple. They are followed by 4, a (female) member of their immediate family (= 2+2 and 2×2). Thus 5 (= 2+3) is the (male) number introducing a category which is inferior and decidedly more "impure" and "worldly".

Geometry is another field where Pythagorean arithmosophy seeks the perfection of integers upon principle. This is why they are viewed in terms of discrete elements, introducing an interpretation of *continuum* through *discretum*: here the topmost position will be occupied by the *point*, an immaterial entity without size but with an identifiable location, and unity will be assigned to this; then a straight line, a plane and space will be seen as *loci* determined by 2, 3 and 4 points respectively, a fact which also happens to suit arithmosophy just fine.

Why is all this covered in such detail? It will be clear as soon as we proceed to the next step. The most perfect material / immaterial expression of number lies in music, which is raised to the glory of the purest and most abstract mumerical manifestation in this way. Music theory is exalted to a private mathematical science: not third in rank and value, but second only to arithmetic, before and above geometry which is thus squeezed into third place. The fourth place is taken up by the remotest and divine geometric manifestation, which is astronomy. Given the sovereignty of arithmetic over all, relations of music to geometry and astronomy will be investigated. Music shall beget solid bonds to geometry and its abstract ground elements, i.e. point, straight line and length, through the string; she shall also be recognized as ruling factor of astronomy: celestial bodies move in accord to the laws of music. It is extremely important to realize that the arrows of this particular idea point in this direction and not the opposite: music governs the universe rather than being governed by it, which is why the idea is called *Harmony of the Spheres*, and not, say, 'Cosmicity of Music' or something to that effect. The entire workings of the universe are depicted in the Pythagoreans' supreme symbol, the "sacred tetractys", a graphic design containing, among other things (elements of nature, planets, perfect solids etc.), the basic ideas from three mathematical sciences in one.

- From arithmetic: unity (1) plus all the purest numbers (2, 3 and 4); also, indirectly, ten which is the basis of our numbering system (1+2+3+4 = 10).
- From music: the complete set of all perfect consonances, i.e. the *tautophony* (unison) (1), the *diapason* (octave) (2), the *diadodeca* (twelfth) (3), the *dis diapason* (double octave) (4), the *dioxeia* or *diapente* (fifth) (3/2) and the *syllaba* or *diatessaron* (fourth) (4/3).
- From geometry: all basic elements: the point, the straight line, the plane and space (1, 2, 3, 4 respectively).



This is the foundation upon which Pythagoreans assemble music theory, each in his own way. And while number is introduced to theosophy as a supreme regulating factor, musical research enters a two-way course. Music is subject to the entire philosophical and mysticist structure of arithmetic; at the same time, the music-theoretical suprastructure returns to direct research and formulation of arithmetic, reaching, through her, all the way to philosophy and theosophy. Musical law assumes cosmic dimensions and turns to a form of religion: what is harmonic is / ought to turn into a model and subject of *mimesis* for each and every side and aspect of the extant world, not excluding the *psyche*. And we can all realize that, unless there were a very real correspondence to accepted musical values, all this would be nothing but a remote construct by the few for the few.

Music history has a firm idea of basic Pythagorean intonation. Yet there is a general feeling that we are missing something. Let me first of all briefly outline the system by means of its alleged theoretical and practical origins at their proper dimension. I shall concentrate on *diatonic* tetrachords and their suprastructures for now. First af all we do agree, I hope, that four perfect concords within the octave (unison, fourth, fifth and octave) are universal constants, as witnessed by their recorded prevalence in all cultures, including the ones labelled "primitive". Their absolute tuning, as prescribed by the theory of oscillations, finds a rightful ally in the human ear: should they be mistuned, the ear is warned by annoying beats, given a single necessary condition: that the tuning be attempted in simultaneous sounding, otherwise no warning, no beats, no annoyance, no correct and incorrect tuning!

Given this, a number of operations handling perfect consonances (at least) cannot help being universal themselves.

- Piling *cycles* of like intervals (cycles of octaves, fifths and fourths).
- *Intraposing* a narrower interval within a wider one (a fourth or a fifth within the octave, a fourth within the fifth). This furnishes a more specific interpretation and its converse.
    - *Fission* or *splitting* of an interval (the octave is split into fifth and fourth).
    - *Fusion* of two or more intervals (fifth and fourth are fused to give an octave).
- *Reduction* of an interval within a narrower one, typically the octave (a *ninth*, fusion of two fifths, is reduced to a *second*).

This last operation provides the octave with the extra status of something like a "higher self" of the unison. The implied *(octave-) cyclic principle* enjoys no direct theoretical backing, but appears to have held in practice universally and throughout the ages, from aboriginal tribes to Schönberg's serialism and beyond.

However, products of these operations are by no means universal if they are not perfect themselves, or if they cannot very directly be traced back, in an immediate aural sense, to their perfect point of provenance and reference. Thus the ninth of the last operation above (not to mention its reduced counterpart, the second) is not universally fixed, unless one can literally hear the note in relation to another one actually heard or implied a fourth or fifth away. Non-perfect and process-generated intervals thus assume a variable ("movable") and approximate status within certain "accepted" or "implied" limits. Starting from the unison (say C), notes a fourth and a fifth up (F and G) are a precise fixed *tone* away from each other. But a like interval inside the tetrachord, C to F, is no longer that; the notion is transformed to one of a "movable", "approximate" tone at best.

We do realize that this makes perfect sense. But we must also realize a fundamental fact that keeps escaping us for some reason unknown to me: the above mentioned operations and all the successive generations of their products are already there, in the nature of periodic sound and in the mathematical theory supplying the model for them. It isn't as though we had so sit down and get inspired and invent something; it's more like discovering and pragmatically seeing it; it's like the oxygen we knew nothing about but were breathing before Lavoisier found out it was there, essential as it may have been for our biological survival.

At any rate, Pythagoreans gave a formulation of a diatonic tetrachord as consisting of *a limma and two tones* in all three possible arrangements, the limma being what is left of the fourth take away two tones. But this is hardly all. There is also a long and involved discussion of arithmetical and philosophical principle, as well as suprastructures forming:

- a pentachord out of:
  - a *proslambanomenos* tone and a tetrachord, or
  - a tetrachord and a *disjunctive* tone,
- a nucleic *systema* out of:
  - two conjunct tetrachords (*synaphe*), or
  - a tetrachord, a disjunctive tone and another tetrachord (*diazeugma*),
- a *lesser systema (hendecachordon / dodecachordon)* out of an optional proslambanomenos, two conjunct tetrachords, a disjunctive tone and another tetrachord, or something similar.

All this is known. But questions arise. Why is the octave underrated and the fifth provisional under the absolute rule of the fourth? Why two tones and a limma? What is fixed and what is movable and how? What happened to simultaneous sounding? And the most crucial questions of all: why did Pythagoreans decide to impose mathematical order upon music practice, and why did they do it in this particular way (which is far from general in method and choices)? It must be clear that, if we fail to answer these questions, sticking to the numerical recipe and tuning pitches accordingly, we understand little of a school of thought of such mathematical excellence after all. The answer to the imposition of order, I daresay, is rather obvious: one imposes order on something that is in disorder. As for the elaborately specific nature of a suprastructure thus formed by articulated tetrachords, we may wish to look for an object that produces just that. And, mind you, the string of a monochord is not it. To really grasp the secrets and whole meaning of Pythagorean thought, views, history and intonation, we need check what the musical situation was that Pythagoras inherited. In other words, the full detail of prehistoric and archaic music in the Greek peninsula. Do we do that by recursion to sources? This *is* a laugh. Do we decide that we can't after all? By no means. I promised a mathematical reconstruction with the due historical background. To do that, I need digress once more.

Geography, geology, climate, archæology and linguistics, each in its own way, strongly suggest that Europe got inhabited by homo sapiens from the southern tips of Mediterranean peninsulæ upwards. Thus it was an initial Ægean population, also known as Pre-Hellenic or (Proto-)Pelasgian that first thrived in the Southern Balkans, the islands and eventually in Western Anatolia (the Ægean coast of present-day Turkey). In their penetration inland to the north, they soon split into two wings that diverged from each other in just about every feature. Settlers of the western rugged

mountainous side (western Greece, Albania and the entire Adriatic coast) rooted themselves to small lots, made a point of land ownership, remained more faithful to tradition and original language, grew static and conservative and developed patriarchy; they are known as Pelasgians proper, Epirotes, Illyrians and possibly Venitians and Western / Goidelic Celts, with linguistic enidence suggesting affinities to the Balts. The Eastern side of the Balkans (Northern Ægean, Black Sea coast), a land blessed with vast plains, helped form a sharply contrasting character: mobility, nomadic life, migrations, colonialism, versatility and matriarchy: Thracians, Dacians, and, quite possibly, with all due reservation, the ancestors of Slavs, of Germanic peoples and of Eastern / Brythonic Celts. Because of a series of factors, topped by the devastating eruption of the volcano of Thera and the resulting cataclysmic tsunami, the ancestors of Phrygians, Lydians and the Greek tribes (Ionians, Achæans and Dorians) roamed back south and blended with the Pelasgians left there, through long waves of war and peaceful settlement, imposing the dominant traits of their own culture (e.g. language) and absorbing quite a few more archaic elements in a dynamic synthesis. This part of history is clearly reflected in mythology as well.

What happened to music? The question to ask is: have we any reason to believe that those cultures had a good musical ear? The answer is in the affirmative. Now then, as we know what they were supposed to be hearing and have a guarantee that they were indeed hearing it correctly, what more do we need but a few scant archæological clues revealing their choices?

Patriarchal Pelasgians went by the octave and the fifth, developing a sound basis for no-semitone pentatones (3 tones and 2 trihemitones to the octave) in exactly the same way as the Chinese allegedly did some 4,000 years b.C. But Mr Moraitis will be telling that exciting story. On the other hand, matriarchal Thracians went by the fourth divided into 3 miscellaneous intervals, possibly exposed to infiltration and / or influences from the peoples in the East: Caucasians and Semites. But where did all those cultures come across their intervals (and how do we know they heard them well)?

The answer given by archæology is unanimous: they found them *on flutes*. You see, primitive animistic creeds worshipped the sound produced by the wind on hollow bones of the dead, whether people or animals, as the voice of their spirit or their lost breath of life, hence Sanskrit *ánimi* (= I breathe), Greek □□□□□□ (= breeze, wind), Latin *animus – anima* (= spirit – soul). Overblowing in flutes yields easy and generally accurate overtones; their possible inaccuracies are rendered intolerable by buzzing beats, but are easily corrected by a number of tricks such as a tempered end or side-blowing. Closed flutes produce only odd overtones, i.e. *male* ones, hence Pan's syrinx, an instrument very clever with fifths, represents the virile sexual force of nature. And what about open flutes?

The principle of boring holes along the basic open flute has been exactly the same and unchanged from palæolithic times to the present day, whether in its authentic form or with minor adjustments which are only too easy to account for: 3 holes for 3 fingers for one hand, bored at equal distances and with equal diameters. Experience shows what the diameters should be, so that a precise octave be comfortably and neatly produced upon overblowing the second harmonic. Now let us see what will happen if we bore the three lower (right-hand) holes so as to get the third hole from the bottom at a perfect fourth. Why a perfect fourth? Because the left hand can get another perfect fourth, so the total natural range of two hands can complete a diatonic scale and safely land on the overblown octave. You have noticed of course how the

idea of 7 degrees to a scale is interconnected with the idea of 6 fingerholes to the flute. But let us forget about the left hand for now.

If all goes well with the flute, we know exactly what notes it produces, with the right hand alone. Supposing it is in D, we get a tetrachord of fundamentals [D, Ew, F+, G], its 3 steps being Tw (*condeficient tone*, 12/11), Tm (*deficient tone*, 11/10), t (*grave tone*, 10/9). What about overtones?

Version I

D□ Ew□ F+□ G□

A+□ Bw□ C+□□ D□□

D□□ Ew□□ F+□□

G□□

The tetrachord is the obvious origin of all *spondeion* forms, including one first formulated at a very late period (2nd c. a.D.): it is Ptolemy's *equal diatonic* (*homalon diatonon*). Notice a handful of emerging facts that tie beautifully among themselves and to other facts we know about music of the Eastern Mediterranean. 1. One hand suffices for all notes, therefore the left hand can hold something else: another (end-blown) flute, a drum (cf. fife-and-drum) or crotal, a religious relic in a festive parade. 2. The suprastructure literally and faithfully replicates our basic disjunct – conjunct pattern. 3. The *synaphe* from A+□ to G□□ has a setup of the so-called *mixolydian* type, the term perhaps implying what comes out when one tries to play a lydian set of fingerings on a Phrygian Aulos. 4. The single tetrachord of fundamentals calls for a drone at upper end G, while the *systema* is at best with a drone at D□□ with auxiliary possibilities for drones at G□□ and G□. So, drones in the treble. 5. The left hand can play the drone in the treble, or whatever is called for by a careful 2-part voicing. What adjustments can we make?

The major adjustment we can make is add an extension, playable by the so far idle little finger (pinky), extending the range lower by a whole tone. This extension can bear the name *proslambanomenon*, a participle meaning "taken up". Fundamentals now form a pentachord [C, D, Ew, F+, G], steps become T (epogdoon tone, 9/8), Tw (12/11), Tm (11/10), t (10/9), a cross- or half- fingering on the second (Ew) hole also produces an extra note in the vicinity of Eb↓ (E subflat) and overtones are:

Version II

C□ D□ Eb↓□/Ew□ F+□ G□

G□ A+□ Bb↓□/Bw□ C+□□ D□□

C□□ D□□ Ew□□ F+□□

G□□

There is a slight problem: C+□□ and C□□ clash. A flautist or aulete need not worry so much, as differences of a comma are easily corrected with the lips or breath, so they are taken as practically negligible. But if one wants to be theoretically straight, or has to deal with precise fingerings on a string, one may wish to avoid the discrepancy. One could then revert to lowering the F+ in the original tetrachord by a comma, by making the correspondent hole a bit narrower. So version III here calls for a perfect fourth from C to F ([C, D, Ew, F, G]), steps T (9/8), Tw (12/11), Tm- (*grave deficient tone*, 88/81), T (9/8). The cross- or half- fingered Eb↓ gives an additional diatonic tetrachord of fundamentals, with a very different character: notes [D, Eb↓, F, G], steps □ (*Archytean "diesis"*, 28/27), T↑ (*supertone*, 8/7), tone; overtones are:

Version III

C□ D□ Eb↓□/Ew□ F□ G□
G□ A+□ Bb↓□/Bw□ C□□ D□□
C□□ D□□ Ew□□F□□
G□□

But now there is another conjunct (– disjunct?) interpretation of the same notes or part thereof plausible,

Version IIIb

C□ D□ Eb↓□/Ew□ F□
F□ G□ A+□ Bb↓□/Bw□
C□□ D□□ Ew□□F□□
G□□

with an "out-of-tune" fourth from F□ to Bb↓□/Bw□ (F□ to Bw□ strongly in the direction of the *syntonolydian* genus) and so on and so forth. Behold the disorder, made worse by the fact that amplification of sound at some point necessitated the attachment of a double-reed mouthpiece, turning the flute into the classical Aulos, but changing overblowing into a feat for the virtuoso. And the sum total of the flute's technical facilities, including half- and cross-fingerings, would have become a nightmare, had there been applied a conical bore for smoother quality. This is why, incidentally, I avoid calling the Aulos an "oboe". But let me point out the presumably long exposition of the ancient musician to traditional systems and their problems, as well as the evidently extreme antiquity of elements typically assigned to later ages.

Version II above, roughly the original Phrygian Aulos, fits the archaic "enharmonic" tetrachords of Olympus and Damon's spondeion scales, version II coincides with the Islamic and Byzantine intervallics of Zalzal and Chrysanthus, while III / IIIb can indeed adjust the Ew upwards by changing the position of the hole up to an occasionally flatulent E, and with a frequent D- (D lowered by a comma also, considering the "negligibility" of differences of a comma). But then the half- or cross-fingered second hole will produce true Eb or its grave counterpart (Eb-). The former can agree with drones either above (G□) or below, whereas the latter variant only sounds well with the C□. But it ponts towards an alternative interpretation as a multiple conjunction, since notes C, F, Bb-, Eb- are a partial cycle of perfect fourths from one another. Anyway, here we come to Lydian types of Aulos, fluctuating between two extremes, the E-'s and B-'s possibly even lowered further down to Em (*middle* or *neutral* E), whose ratios to C and D- are 11/9 and 11/10 respectively.

Version IV

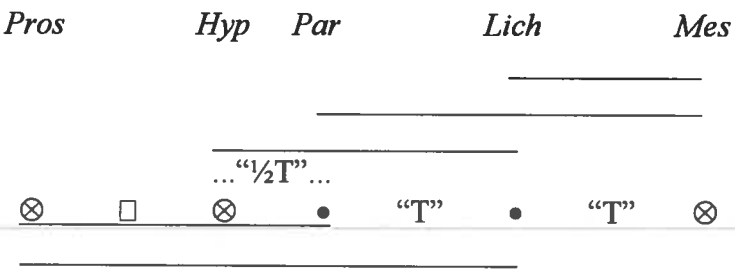
C□ D□ Eb□/E□F□ G□
G□ A+□ Bb□/B□ C□□ D□□
C□□ D□□ Eb□□/E□□
F□□ G□□

Versions IVa

C□ D-□ Eb-□/E-□ F□ G□
G□ A□ Bb-□/B-□ C□□ D□□
C□□ D-□□ Eb-□□/E-□□
F□□ G□□

I made a point about the drone being in the treble. Now I must add that attachment of the extension and the proslambanomenos, together with an adjustment of intervallics, allows the drone to go in the bass. The significance of this is far from negligible. Just think of the drone as an accompaniment for the melody. Drone in the treble (/ bass) means the important voice below (/ above), therefore men (/ women) tribally and / or ritually chanting it respectively. There are visible traces of the patriarchy / matriarchy interplay here. As for the *single* diatonic Aulos, intervallic recipes are applied by the left hand directly on the upper part of the instrument's body, therefore we have conered a full octave with fundamentals. An often double "broken drone" can be intermittently inserted at rhythmically strategic places, in the standard playing style of island bagpipes.

As for classical Greek enharmonic and chromatic tetrachords, they are easily produced on the flute through simple cross- and half-fingerings, but the Aulos can only make do with difficult and treacherous half-fingerings and fluctuations of the breath, the enharmonic suited somewhat better on the Phrygian types of Aulos and the chromatic on the Lydian. But mind one more thing: the notes of the enharmonic and chromatic *pycnon* are necessarily executable on the Aulos *with one and the same finger*, utilizing a possibility of double holes, a fact clearly reflected in ancient music notation and lost on strings and later string-derived terminology. The classic pattern of harmonic relations is depicted by the horizontal lines in the figure, at least for the diatonic tetrachord.



[A demonstration of figures and instruments, and illustration of overtones, tetrachords etc. through actual sound]

Let us come back to Pythagoras. This has been an outline of the inherited tradition that must have come down to him. He and his school decided to standardize demands, ideas, problems and solutions, transferring the experience of the Aulos onto the string (Dionysus to Apollo). Given all the above, and the demand for certain notes to be perfect fourths away from other notes, the end of the road was to build diatonic scales out of tones and limmata. Given the archetypal factorization of ratios, the certainty of double voicing as evident from the pictorial profusion of double Auloi, a consequent demand for consonance and Pythagoras's own law of small numbers, it is highly improbable that Pythagorean intonation could ever have been meant as actual and practically applicable. Besides being archetypal, it also gave a measure of how one is to classify genera, judging from which Pythagorean tetrachord they were "reasonably close to". So I believe that a literal interpretation and pedantic persistence in an accurate application of this intonation as an actual tuning system suits a typically *anti-mathematical* indoctrinated approach, which also seems to me extremely removed from Pythagoras's spirit. It sounds too harsh besides.

Musicians have always played to the approval of their own ear, whether by instinct or by training. Anything that came out reasonably close to ["half-tone", "tone", "tone"], though perhaps not exactly [limma, tone, tone], was rightfully acknowledged as a valid version, or *chroa*, of dorian diatonic. The most convincing witness to this view, besides Philolaus's own loose attitude towards the venerated enharmonic, is one of the most prominent and respected Pythagoreans of all ages, and one of the greatest and most versatile personalities of all ages: Archytas of Tarentum.

Philosopher, general and governor of Tarentum, mechanic, inventor, mathematician and Aulos player, Archytas excelled in all these sectors. His contribution to music is huge. He is the founder of musical acoustics, being the one to formulate the physical law according to which *sound is due to vibrations of air reaching the ear, and, the faster the vibrations, the higher the pitch.* This happened in the early 4th century. His own diatonic genus has three superparticular steps with factors 2, 3 *and 7*: [28/27, 8/7, 9/8]. His two "tones" are unequal: the *superseptimal* tone or *supertone* of 8/7 in the middle exceeds the usual (*superoctal*, *epogdoon*) *tone* of the (old and sacred) pentatones by about 13% (is round 1.13 □). This is exactly what we found in version II above. And there is nothing stopping us from supposing that such a percentage of deviation was perfectly legitimate and acceptable. The "diesis" of 28/27 (2^2x7 / 3^3), common to all three of Archytas's genera, is perceptibly less than a limma (nearly equals .7 limmata) and barely covers .31 epogdoon tones. The situation is similar for the rest of his genera. Archytas being an accomplished aulete and actually playing this consonant form, could couple his theoretical endeavours with the approval and enjoyment of his listeners.

[*a demonstration of Archytas's tetrachords on the flutes*]

The theory and history of classical Greek music passes through the antithesis and synthesis of ("western") Pelasgian pentatones and ("eastern") Thraco-Phrygian diatones. The struggle between the two sides and its outcome are reflected in many aspects of history and culture, from the *Iliad* to the myth of Apollo and Marsyas to the juxtaposition of Apollonian and Dionysiac elements to the enigmatic legendary figure of "the greatest musician of all times", Thracian Orpheus to the constituency of Greek blood, character and culture to the very feeling of "being Greek". Without this understanding, there is no satisfactory way to interpret and explain the intervention of Pythagoras and his school and the detail of what followed. Soon the bulk of Pythagoreans turned to a class or irder of meticulous music-theorists without music. Among other things, this involved actualization of precise (harsh) Pythagorean tuning and dismissal of others as sacrilegious or heretical, a fact that was to push all music in the area to then corner of a literal or pseudonymous monophony. Reaction to this was not slow in surfacing.

On Aristotle's deathbed, his brilliant disciple, Aristoxenus of Tarentum (also), bitter with rage for not having been appointed as Aristotle's successor, will declare a new doctrine: *music belongs to musicians and the supreme and final judge of music is the ear*. Manifestly disgusted at Pythagoreans of his own time, he propagates a new system of "model" intervallics, according to which one is not to go by far-fetched ratios but by subdivisions of the tone. Now these subdivisions are not to be taken as literal and exact either, let alone as a "temperament", but as a simpler approximative / classificatory system suited to the needs of musical art. Was Aristoxenus right or wrong?

The "good" side is that musicians are free, at long last, from being caught up in a compulsory distressful and time-consuming obligation of learning an impossible theory, to which they may not be able to relate, and can now spend their time and talent on unhindered practicing. The "bad" side is that they now specialize in handling a tool with great spiritual, emotional and moral powers, without actually knowing what it is they are handling, with no access to its (possibly unexplored) potentialities. They are just simply left alone to "feel" along the lines set out for them by another "specialist" teacher of theoretics, who "is supposed" to "know". What will musicians go by, now, except a mechanical acceptance of "givens" and their own subjectivity? What choice do they really have? What happens to the deception of the senses, appearance and actual essence?

Aristoxenus has done the same as many other pioneers. As long as he lived and taught, he was perfectly aware and knowledgeable of what he was eradicating and dismantling, and he had things under control. But, after his death, teachers of the next generation would treat his teachings as doctrine, without really understanding it. Are there mistakes to be made in such a "specialist", "professional" and "labour-union"-oriented context? Let me mention but one.

Playing a stringed instrument, of the lute or violin family, calls for memorizing the physical activity of fingerings. The mental information contained in this process is this: the part of the string *silenced*, rather than the one sounded. This information forces musicians to perceive their activity in a way not just simply crooked, but totally and conclusively reversed. As we go to woodwinds, this misconception and misinterpretation is coupled. Not only does the musician measure lengths from the wrong end of the horn, but even so memorizes wrong lengths too: the note does not come out from the point fingered, but from the next hole down.

The spectacular success of Aristoxenus and his school has indeed been great, and the impact lasting. All later "theorists" shall repeat his own point of view, or at least keep referring back to it. Such would not have occurred, had he not met the demands of his time, and would not still be lasting, were demands not the same today: demands for specialization towards meticulous yet removed researchers on one hand, and active and functioning yet uninformed (ergo ignorant and unfree) practising musicians on the other; a demand for a divorce of music from her own mathematical and philosophical theory, a severing of all bonds and mutual contributions from and to everything else and a restriction to the artistic side, to (non-directional) subjectivity and to (manipulable) public response (and consumerism). If music be the ideal gateway to grasping and internalizing the workings of the universe, this gateway is tightly shut, in spite of partial and sparse attempts to peek through it. The extremist attitude assumed by later Pythagoreans was in no way innocent of this outcome. But the picture is rather more general: Greek antiquity is marching towards decline at a fast pace. Does this ring a bell?

Demetrios E. Lekkas

THE MUSIC WORLD BEFORE, DURING AND AFTER PYTHAGORAS

Figures and tables

The Pythagorean Tetractys

Right hand flute overtone scales

Version I

D´ Ew´ F+´ G´ A+´ Bw´ C+´´ D´´

D´´ Ew´´ F+´´ G´´

Version II

C´ D´ Eb↓´/Ew´ F+´ G´

G´ A+´ Bb↓´/Bw´ C+´´ D´´

C´´ D´´ Ew´´ F+´´ G´´

Version III

C´ D´ Eb↓´/Ew´ F´ G´

G´ A+´ Bb↓´/Bw´ C´´ D´´

C´´ D´´ Eb↓´´/Ew´´ F´´ G´´

Version IIIb

C´ D´ Eb↓´/Ew´ F´

F´ G´ A+´ Bb↓´/Bw´

C´´ D´´ Eb↓´´/Ew´´ F´´ G´´

Version IV

C´ D´ Eb´/E´ F´ G´

G´ A+´ Bb´/B´ C´´ D´´

C´´ D´´ Eb´´/E´´ F´´ G´´

Versions IVb

C´ D-´ Eb-´/E-´ F´ G´

G´ A´ Bb-´/B-´ C´´ D´´

C´´ D-´´ Eb-´´/E-´´ F´´ G´´

Harmonic relations in a Greek diatonic tetrachord

Pros *Hyp* *Par* *Lich* *Mes*