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THE EVOLUTION OF THE CONCEPT OF ENERGY AND ITS ROLE IN SYSTEMS OF INCREASING COMPLEXITY

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1. The Concept of Energy

Energy is probably the most important concept which interconnects the different branches of science. The concept was developed in physics starting with simple mechanical considerations in the late 17th century. The great leap forward however came much later, in the mid 19th century when it was realized that heat was energy tied up in the motion of the myriads of atomic particles constituting matter. At that time came the momumental discovery, or perhaps rather the bold postulate, that energy is always conserved. One kind of energy can be transformed into other kinds, but nothing is ever lost.

Energy considerations have since then been indispensable for the description of the behaviour of gases, liquids and solid bodies. Such systems, which from a mechanical point of view are exceedingly complex, can be treated simply and elegantly by the general methods of thermodynamics. Thermodynamics can be formulated entirely in terms of macroscopic quantitites as pressure, volume and temperature. But the microscopic picture of heat as motion of atomic particles has been of immense value for our understanding of all kinds of processes in physics, chemistry and even biology. This understanding rests on an atomistic view of matter combined with statistical methods for dealing with systems composed of many particles. The successes of the statistical approach to thermodynamics were of decisive importance for the acceptance of atoms and molecules as real objects and not just philosophical entities.

The two great contributions of our own century to man's understanding of nature are the theory of relativity and quantum mechanics. Both theories have added to the concept of energy. From the theory of relativity came the quite unexpected equivalence of energy and mass as expressed in Einstein's equation $E=mc^2$. From the quantum theory came the equally surprising quantisation of energy. The quantum theory gave a new universal constant to physics, the quantum of action h, which has the dimension of energy multiplied by time. In quantum theory the fundamental Schrödinger equation connects the entire physical state of a system to its energy.

Quantum theory has been the key to understanding matter in all its different macroscopic forms. Gas, liquids, solids, plasmas. On the microlevel we have learned how the atoms are built and how they can be combined to chemical compounds. Some of them simple, as a molecule of ammonia. Some of them exceedingly complex as the macromolecules of certain proteins, which can contain several hundred thousand atoms. Also at a deeper level quantum theory has given us new insight. We understand a great deal about the structure of the atomic nucleus and today we are well underway towards a new understanding also of the constituent particles of matter and their interactions.

The concept of energy has been absolutely central in all these developments. There is no branch of physics where the term energy is not used over and over again. It is therefore an important part of the education of every physicist to learn to recognize and handle energy quantities. In spite of this it is difficult to answer the simple question: What is energy?

Why is it, that the concept of energy, which unifies all branches of physics and those of neighbouring sciences, cannot be given a clear and satisfactory definition? True enough, in all the well developed subdisciplines of physics we know how to express energy in terms of other quantitites. We have formulas which serve that purpose: The <u>kinetic</u> energy of a particle of mass m and velocity v is $\frac{1}{2}mv^2$. If the particle is placed at a height h above ground its <u>potential</u> energy is mgh where g is the acceleration in the earth's gravitational field. If a point charge q is placed in an electric potential V, say between the plates of an electric condensor, its <u>electric</u> energy is qV. If a mole of an ideal monatomic gas is heated to a temperature of T degrees Kelvin, its <u>internal</u> energy due to the motion of its N ($=6.02\times10^{23}$) atoms is $\frac{3}{2}kNT$ where k is Boltzmann's constant. A photon, the quantum of light, of frequency ν carries <u>radiation</u> energy $h\nu$ where h is Planck's quantum of action introduced in

1901. If an electromagnetic field in vacuum has the electric field strength E and the magnetic field strength H its electromagnetic energy density is $\frac{1}{2}\varepsilon_o E^2 + \frac{1}{2}\mu_o H^2$, where ε_o is the socalled vacuum permittivity and μ_o correspondingly the vacuum permeability. As a final example: If a particle has mass m it represents a mass energy mc^2 according to Einstein's equation from 1905.

All these formulae and several more are among the most basic in physics and well known to all physicists. It does however require a fair mastery of the subject to see, how each of the many expressions for energy comes into physics and how they are all interrelated. And even such an insight does not bring us much closer to an answer to the elementary question: What is energy?

Nevertheless, when apparently dissimilar quantities as $\frac{1}{2}mv^2$, $\frac{3}{2}kNT$, qV and mc^2 all are taken to express a common property, it is of course not without foundation. The unity rests on two general premises: Firstly, that the many different forms of energy can be transformed into each other, at least partly. And secondly, that energy is always conserved.

There is something which is conserved. That, according to H. Poincaré[1)], is about as close as we can get to a definition of energy. Admittedly a poor definition within a science which prides itself of exactness and rigor. But then, as remarked by H.A. Kramers[2)]: In the world of human thought generally and in physical science particularly, the most fruitful concepts are those to which it is impossible to attach a well defined meaning.

2. The Evolution in Retrospect

With a few notable exceptions the history of science is not pedagogical. The following discussion of the evolution of the concept of energy is therefore not a scholarly exposition based on the original sources. It should rather be considered an attempt in retrospect to illustrate how we have learned to use the concept in various fields of physics and other sciences, and how energy in this way has become a great unifying principle in man's understanding of nature. The trend in the exposition naturally will be from simple examples into more complex situations. This, of course, is also a trend which can be recognized in the historical development, although the route followed was circuitous and many dead ends had to be explored.

2.1. Mechanics

The concept of energy is decidedly of mechanical origin, but it came into mechanics at a rather late date. Although mechanics could be developed without the concept, one can wonder, why the importance of energy was not discovered much earlier. Newton (1642-1727) might well have discovered the conservation of mechanical energy, but according to diligent readers of his writings he did not[3]).

Every first year student of physics today knows how in a few lines of calculation to derive the conservation law for mechanical energy. Say for a freely falling body. The calculation is based exclusively on three of Newton's great contributions to physics: The second law of motion, the law of gravitation and infinitesimal calculus. It might therefore be worthwhile to present the few steps in this elementary derivation:

Conservation of Mechanical Energy.

A Newtonian Calculation never done by Newton:

$$F = m \frac{dv}{dt} \quad \text{(second law)}$$

$$F = mg \quad \text{(gravitation)}$$

$$mg = m \frac{dv}{dt} \rightarrow mgdx = m \frac{dv}{dt} dx = m \frac{dx}{dt} dv = mvdv$$

$$\rightarrow mg \int_o^h dx = m \int_o^v vdv$$

$$mgh = \frac{1}{2} mv^2 \quad \text{(calculus)}$$

The final equation expresses, that a body of mass m initially at rest at a height h above ground by a free fall to ground will acquire a kinetic energy $\frac{1}{2} mv^2$ which equals the potential energy mgh it originally possessed. The situation just before the fall, when the body is released, is of course quite different from the one just before the body strikes the ground. But from the point of view of total energy nothing has changed. Mechanical energy is a conserved quantity in this case, and in all other cases where the forces are conservative (the work is independent on the particular path followed).

Historians undoubtedly can give many reasons why Newton did not discover energy. For example, that Newton nowhere has written his second law. At least not in the form force equals mass x acceleration[4]). (This is the kind of disillusion often experienced when one studies what the great originators really said. It proves the point made earlier, that the history of science is rarely pedagogical).

But some of Newton's contemporaries came closer to the mark. Huygens (1629-1695) and Leibnitz (1646-1716) did from studies of swinging pendulums and colliding billard balls realize the importance of the quantity mv^2 called vis viva (living force) and made use of its conservation.

There is however a long step from the early ideas about the conservation of vis viva to the general law of conservation of energy, which goes much beyond just mechanics. But even within mechanics it took a long time to clarify the meaning of concepts like force, work or what we call potential energy, all essential for the general energy concept. A small but decisive detail, as the inclusion of the factor 1/2 in the definition of kinetic energy, came rather late (Coriolis 1829).

In spite of the lack of a clear concept of mechanical energy, energy played an important role in an entirely new formulation of mechanics. This was primarily the work of a unique group of French mathematicians centered around École Polytechnique.
While the Newtonian mechanics is concerned with vectorial quantities as force and momentum, the new formulation, today known as the Hamilton-Lagrange formulation, is based on a single scalar function, L, which is an energy. If this function is known, ex-

pressed in suitable coordinates, the behaviour of even quite complicated physical systems can be derived by standard mathematical methods. The Hamilton-Lagrange formulation of mechanics plays an important role in statistical mechanics and in quantum mechanics.

2.2. Energy is conserved

Around 1840 the time was ripe for one of the most important discoveries in the history of science: The conservation of energy.

It is not easy to say who discovered energy conservation. Indeed, afterwards the question of priority gave rise to a bitter controversy where great scientists abused each other and primitive nationalistic instincts were excited.

In a classic paper[5]) T.S. Kuhn names four European scientists - Mayer (1814-78), Joule (1818-89), Colding (1815-88) and Helmholz (1821-94) - who, mostly independently of each other, announced the hypothesis of energy conservation. Kuhn adds further names to the list and concludes, that twelve men within a short period of time grasped essential parts of the concept of energy and its conservation.

The time was ripe. There was the accumulated experience, that the various "forces" of nature could be transformed into one another. Coal could be burned in a furnace to raise steam which could turn an engine. Batteries could, at the expense of metals dissolving in acids, produce electric currents, which again could

could heat wires and even produce light. Ørsted (1777-1851), in his search for unity in nature, had shown that currents could cause motion of a magnetic needle. In biology it was known, that there was a relation between the inspiration of oxygen and the heat losses of the body.

All these observations pointed to an underlying unifying principle of nature, and it is not surprising, that several men more or less simultaneously should arrive at the law of conservation of energy, although by quite different routes.

Today the principle is universally accepted in all branches of science. It has been and still is, a lodestar, when science expands into virgin areas. Sometimes the general validity of the principle has been questioned, even by men like Niels Bohr (1885-1962). But in the end the conviction, that energy is always conserved, has persevered.

The acceptance of heat as energy associated with molecular motion was essential for the general conservation law for energy. Energy and temperature became interconnected and a consistent new theory, thermodynamics could be created.

2.3. Thermodynamics

The conservation of energy is often called the first law of thermodynamics. It can be formulated in several ways. One of them is, that the change in internal energy of a system equals the heat added minus the work done by the system.

The first law of thermodynamics:

If ΔU is the change in internal energy of a system when a quantity of heat ΔQ is added and the system performs mechanical work ΔW one has

$$\Delta U = \Delta Q - \Delta W$$

It is clear that the three quantities must be measured in units of energy. Although the law as here formulated obviously is inspired by the working of a heat engine (heat in - work out), the law is quite universal. The internal energy can be associated with molecular motion but e.g. also with radiation.

The special usefulness of the internal energy U is connected to the fact, that it depends only on the state of a system. For a gas the state can be described by e.g. its pressure and volume. If we go from one state A to another state B we can follow many different pressure-volume paths, each giving its own ΔQ and ΔW. But the difference always yields the same change in internal energy, $\Delta U = U_A - U_B$. The internal energy is said to be a function of state.

The first law of thermodynamics can be put into words in different ways, which however all are equivalent. Such wordings help us to attach meaning to the word "energy": Energy cannot be created or destroyed, only transformed. The energy in a closed system is constant. A perpetuum mobile of the first kind is impossible (such a device would, operating in a cyclical manner, produce work without any input of heat or other work). Or, with a daring extrapolation: The energy of the universe is constant.

Thermodynamics is essentially a science concerned with systems at different temperatures. Temperature, although introduced in physics long before the advent of thermodynamics, does not have any meaning in mechanics. In thermodynamics it is a key concept.

The beauty of thermodynamics is its generality. It describes "systems". Systems can be almost anything which can be isolated from the surroundings and which contains many particles. Even immaterial particles as photons enclosed in a suitable box. A gas in a container, a battery, a star or a mouse are examples of systems about which we can learn something interesting by the methods of thermodynamics.

Equation of state.

Thermodynamics describes a physical system in terms of a few macroscopic variables as pressure, P, volume, V, and temperature, T. Other variables can be used, but the three mentioned are well suited for the description of basic thermodynamic systems involving gases or vapors. The variables are not independent. They are connected through relations called equations of state. For the case of an ideal gas, where the molecules interact only by brief elastic collisions, the equation of state is:

$$PV = nRT.$$

The R in the equation is called the gas constant. It has the physical dimension of energy per degree. The n is the number of moles.

Temperature does not enter directly in the first law. The first law is an accounting rule for energy transformations, but it does not tell us whether or to what extent a particular transformation actually occurs. To obtain such insight we need the second law of thermodynamics.

Historically the essence of the second law was discovered, before the first law, by Sadi Carnot (1796-1831) in 1824. It is interesting to note, that while the British were busily building the steam engines of the industrial revolution, a French engineer was reflecting on the ultimate efficiency of such a machine. Carnot pointed out, that it was the temperature difference between the boiler and the condenser which was decisive for the amount of work that could be extracted from the engine. Thus cold is as essential as heat for the functioning of a steam engine. But Carnot went further than that. He also devised the ideal machine, which, although it can never be built, serves as a reference for judging the performance of practical engines.

Carnot's ideal engine in many respects resembles a real steam engine. It operates in a cycle, it absorbs heat at a high temperature, it converts some of that heat into useful work, but never completely. There will be an inevitable heat loss to the condenser. The ideal machine is different from practical machines because it is reversible. At any point in the cycle it can be made to run backwards. In a reversed steam engine one would crank the shaft and the engine would absorb heat from the condenser and deliver heat to the boiler. The backwards running engine in fact is a refrigerator. Carnot pointed out, that no engine could be more efficient than the reversible one. Otherwise a combination of a forwards running "super" engine and a backwards running reversible engine would be a perpetuum mobile.

Carnot's ideas were later amalgamated with the concept of energy as a conserved quantity into a consistent theory of thermodynamics. The main figures in this development were William Thomson (Lord Kelvin, 1824-1907) and Rudolph Clausius (1822-88). Their work led to the formulation of the second law of thermodynamics and to the creation of an entirely new concept in physics: Entropy.

The word entropy was coined by Clausius in analogy to energy. In a loose sense, energy describes activity, entropy transformation.

Energy, although no easy concept, is something we to a certain extent can sense and about which we have accumulated some everyday experience. Entropy is much subtler. Even in physics it is difficult to grasp its full meaning, and in philosophy it is likely to lead astray.

Entropy in Physics.

Clausius expressed entropy in terms of two concepts already familiar: Heat and temperature. If a small amount of heat, dQ, is added to a system having an absolute temperature, T, the entropy, S, of the system will change by

$$dS = \frac{dQ}{T} .$$

If we go from a state A to a state B of the system the change in entropy can be calculated as

$$\Delta S = \int_A^B \frac{dQ}{T} .$$

This integral can be evaluated only for a reversible path, where the system at any point is in equilibrium.

The function S is useful, because it, just as the internal energy U, is a function of state. The integral from A to B is independent on the particular reversible path chosen. For a cyclical path, where one returns to the point of origin, the integral is zero. However, if irreversible processes are involved, such as the sudden cooling of a hot body in a bucket of water, the total entropy of a closed system is always increased.

The second law of thermodynamics, just like the first, has been given a number of different formulations: Entropy will not decrease. Heat cannot flow from a colder body to a warmer. A perpetuum mobile of second kind cannot be constructed. (Such a device would, operating in cycles, take heat from a single source and transform it into work). The second law of thermodynamics is, in the words of Max Planck (1858-1947) a statement of impotence[6]).

2.4. Heat as Motion

The idea, that heat is some kind of motion - translational, vibrational or rotational - of atomic particles is quite old. Daniel Bernoulli (1700-1782) thus gave the first version of what was to become todays kinetic theory of gases. But this and other attempts did not contribute much to the development. For many years most scientists were inclined to believe in heat as a kind of fluid called caloric.

First the enunciation around 1840 of the general law of energy conservation paved the way for the concept of heat as molecular motion. The general acceptance of this idea then led to a rapid development. Clausius and Maxwell (1831-79) were among the first to realize, that the picture gave important new insight. Atoms and molecules became real objects with properties which could be calculated. Probability for the first time got an important place in a physical theory and gave, through the works of Maxwell and Boltzmann (1844-1906), a new approach to thermodynamics: Statistical Mechanics. Statistical mechanics again was the route which led to quantum mechanics. All these developments, which are among the most important in the history of physics, can directly be traced back to the principle of energy conservation.

Equation of State Revisited.

The equation of state for an ideal gas (p.11) follows directly from the picture of a gas composed of molecules moving randomly in all directions bouncing off each other and the walls of the vessel.

Let the velocity of a molecule with mass m have velocity $\vec{v}$ with components v_x, v_y and v_z. For each component, say v_x, the opposite component $-v_x$ is equally probable. The mean velocity of the molecules is therefore zero. However the mean square velocities, say $\langle v_x^2 \rangle$, are positive and all equal:

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

where the last equality results from $v^2 = v_x^2+v_y^2+v_z^2$.

The pressure on the walls is the result of a continuous bombardment of molecules. Let a molecule strike the wall with a perpendicular momentum mv_x. It bounces back elastically with velocity $-mv_x$. But momentum is conserved. Thus a momentum $2mv_x$ has been transferred to the wall. We experience this as a force on the wall equal to the momentum change per unit time. The molecules striking each unit area of the wall per unit time are contained in a column of gas of height v_x. The number of molecules per unit volume is N/V where N is the number of molecules in the vessel and V its volume. Only half of those have a positive v_x. The momentum transfer per unit area and unit time is what we call the pressure P. Consequently, on the average:

$$P = 2mv_x \; v_x \frac{N}{V}\frac{1}{2} = m\langle v_x^2\rangle \frac{N}{V}.$$

Finally, by the use of $\langle v_x^2\rangle = \frac{1}{3}\langle v^2\rangle$:

$$P = \frac{1}{3}\frac{N}{V} m\langle v^2\rangle$$

or

$$P\,V = \frac{1}{3} Nm\langle v^2\rangle$$

We recognize here the equation of state for constant temperature.

If we introduce the average kinetic energy of a molecule we have

$$P\,V = \frac{2}{3} N(\frac{1}{2}m\langle v^2\rangle)$$

But according to the experimentally verified equation of state PV=nRT. If we have just 1 mole in the vessel n=1 and the N is the Avogadno number.

Thus

$$\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}\frac{R}{N} T = \frac{3}{2} kT$$

which directly, through the Boltzmann constant links temperature to mean kinetic energy of a molecule. For an oxygen molecule at room temperature we calculate an average velocity of 675 m/s.

The example above illustrates, how a molecular quantity can be derived from the combination of the macroscopic and microscopic picture of a gas. No wonder, that the kinetic theory of gases was so important in giving credence to atoms.

Much more was accomplished along these lines, and at the end of the 19th century thermodynamics could be seen as the macroscopic manifestation of a microscopic structure of matter. A whole new physics, statistical mechanics, was born.

Statistical mechanics shows many new and surprising facets of the concept of energy. Indeed so many, that almost all of modern physics would have to be included in even the most cursory description. Only the entirely new picture of entropy due to Boltzmann shall be mentioned here.

Boltzmann focused attention on the fact, that systems tend to develop from states of order to states of disorder. As when orderly energy in a sliding body by friction is converted into heat, that is disorderly molecular motion. The opposite has never been observed. Processes of this kind are irreversible, although Newton's laws of motion are strictly reversible.

The fundamental reason for this, according to Boltzmann, is that the number of disordered states is so much greater than the number of ordered states. Boltzmann saw here a connection to the fact, that entropy is an always increasing quantity. Entropy, said Boltzmann, is a measure of disorder, and the increase of entropy reflects the trend towards increasing disorder. This connection found its mathematical expression in Boltzmann's famous formula reltating the entropy S of a system to the number Ω of states accessible to it:

$$S = k \ln \Omega$$

2.5. Energy in the Field

Energy is an abstract concept, difficult to define and difficult to visualize. Entropy even more so. Because of their abstractness the concepts are not tied to any concrete situation. They are general in character and unifiy our understanding of the physical world.

Another such concept is the physical field. The idea of the field dates back to Faraday (1791-1867) and came into prominence through Maxwell's formulation of the laws of electromagnetism. Today almost all branches of physics make use of field concepts, and the front line theories are field theories.

A field represents a physical quantity which can be defined throughout a region of space: The temperature field in a room, the gravitational field around the sun, the electric and magnetic field in a radio wave.

The field describes a continuously distributed quantity. This quantity can be energy and we speak about the energy density of the field. The energy density need not be stationary. Energy can move from one region to another, and we speak of energy flows.

Light, X-rays and radio signals are electromagnetic waves, and their propagation through space is described by Maxwell's equations. The passage through empty space of electromagnetic waves is counterintuitive. We can imagine waves, but only in a medium of some kind. Waves in vacuum cannot be visualized, but their presence can be detected. Radio signals can direct space probes through the rings of Saturn, because tiny amounts of energy are transmitted from earth.

Energy in the Electromagnetic Field.

In an electromagnetic wave there is an electric and a magnetic field. These fields can be described in terms of vectors $\vec{E}$ and $\vec{H}$ which are functions of time and location. The vectors are perpendicular to each other and to the direction of propagation.

The energy density in the wave is given by

$$U = \frac{\varepsilon_o}{2} E^2 + \frac{\mu_o}{2} H^2$$

and the flow of energy by the Poynting vector

$$\vec{S} = \vec{E} \times \vec{H} .$$

Note that the energy density is composed of a purely electric and a purely magnetic part, whereas the flow of energy depends on as well the electric as the magnetic field. The energy flow is thus truly electromagnetic.

It should be remarked, that the identification of the proper expressions for energy in the electromagnetic field is based on energy conservation. There is no unique way of telling how the flow of energy should be represented, but without the inclusion of the Poynting vector mentioned above, energy would not be conserved.

2.6. Energy and Mass

New insight is often obtained in a most unexpected way. If somebody, say at the beginning of last century, had speculated how one could extend the world's supply of fuel, he might have considered better wicks and tallows for candles or have invented improved paddle wheels for steamships. He would certainly not have started to think about how to set one's clocks in moving systems of reference.

Einstein (1879-1955) did just that. He realized that clocks should be synchronized by means of signals of light. The concept of absolute time was abandoned, but the synchronization ensured, that light moves with velocity c in all systems of reference irrespective of their state of uniform motion. This led Einstein to the special theory of relativity in 1905.

One surprising consequence of the theory of relativity, which was discovered by Einstein, is that mass and energy are equivalent. Mass and energy are different quantities, but mass has energy and energy has mass, according to the equation $E=mc^2$.

If a body is heated and thus increases its internal energy, it also increases its mass. That is, it gets more inertia. Two equal lumps of wet clay colliding and sticking together, will form a bigger lump with a mass increased by the combined kine tic energy divided by c^2. A proton and a neutron can combine through the process of radiative capture and form a deuteron. But the deuteron mass is about 0.1 percent less than the sum of the proton and neutron masses. In the course of the capture, energy is released in the form of a gamma ray quantum and a small recoil energy of the deuteron. The energy release just corresponds to the mass difference between the proton plus neutron and the deuteron multiplied by c^2. There are numerous examples of the mass energy equivalence.

Mass-Energy Equivalence.

The mass energy equivalence can be derived from the special theory of relativity in a number of ways. The most instructive, perhaps, rests on the requirement, that energy and momentum for a system of point particles must be conserved in all frames of reference[7]).

A dramatic experimental demonstration of the equivalence is the process of positron annihilation.

The positron is the electron's antiparticle. It has the same rest mass as the electron, but opposite charge. An electron and a positron brought near each other at (almost) rest will form positronium. An "element" similar to hydrogen where the positron and electron orbit around the common center of mass. Within a period of about 10^{-10} s this deadly embrace completely annihilates the partners, and two gamma ray quanta are emitted in opposite directions (conservation of momentum). Written as a reaction:

$$e^+ + e^- \rightarrow 2\ h\nu$$

The quantum energy $2h\nu$ exactly equals $2\ m_o c^2$ where m_0 is the rest mass of the electron (511 keV).

2.7. The Quantum of Energy

Quantum physics describes matter and radiation at the micro-level, where matter is no longer a homogenous substance but shows structure. At each level of resolution we can discern a characteristic variety of particles: Molecules are composed of atoms, which again consist of a nucleus and a swarm of electrons. The nucleus is composed of protons and neutrons which at the highest level of resolution again reveal a structure of quarks. The particles we meet at each level have distinct properties as charge, mass, spin and magnetic moment. Such properties take discrete values, they are quantized.

At the microlevel energy too is quantized. Quantization of energy indeed was the phenomenon which, at the beginning of this century, first attracted the attention of discerning physicists. Quantized energy was something entirely new which foretold, that at the atomic level the laws of nature are very different from those of classical physics.

There were three, today classic observations, which each revealed an aspect of energy quantization. They were the black body radiation, the photoelectric effect and the spectrum of hydrogen.

Quantized Energy - the Beginning of Quantum Physics.

Black body radiation (Max Planck, 1901): A hot body emits electromagnetic radiation in the form of infrared, visible and ultraviolet light. The spectrum of an absolutely black body (a small hole in a heated cavity) is a continuous, bell shaped curve showing intensity as function of frequency. Earlier calculations by Lord Rayleigh, based on statistical thermodynamics, could reproduce the low frequency part of the curve. For high frequencies Rayleigh's formula diverged (the "ultraviolet catastrophy").

Now, Planck succeeded in fitting the experimental curve by a mathematical expression, and later he derived the expression theoretically. This, however, he could do only, if he assumed, that energy from the hot walls of the cavity was transferred to radiation in bunches of a certain size, namely

$$E = h\nu.$$

The constant h, which was introduced an numerically estimated by Planck, turned out to be the fundamental constant of quantum physics.

Photoelectric effect. (Einstein 1905): Ultraviolet light which impinges on a metal surface causes the emission of electrons. This photoelectric effect had been known for some time, but could not be accounted for by the electromagnetic theory. Among other things the energy of the electrons was determined by the frequency of the light and not, as expected, by the intensity.

Einstein proposed a new concept, a particle of light, later called the photon. Einstein's hypothesis was, that photons of frequency ν have energy $h\nu$. In a collision-like process this energy is imparted to the electron. However, a certain work W is required to remove the electron from the metal. Consequently the maximum energy of the electrons released is

$$E = h\nu - W$$

This is Einstein's photoelectric equation which first introduced the photon concept into physics.

Atomic spectra. (Bohr 1913): The spectrum of hydrogen emitted by a discharge tube shows a number of distinct lines. The frequencies of these lines can be reproduced by a simple empirical formula discovered by Balmer in 1885.

The occurrence of sharp spectral lines was a mystery to classical electrodynamics. A moving electron in an atom had to radiate and thereby loose energy. The electron would run faster and faster, emitting a continuous spectrum and would soon end up in the nucleus. Matter would not at all be stable.

Bohr realized that the message of the spectral lines was, that the energy of the atom is quantized. He postulated, that an electron can move only in a number of quantum states, where it does not radiate. Each quantum state has a definite energy. A photon is emitted when the electron jumps from one quantum state with energy E_1 to another with the (lower) energy E_2. The energy of the photon and thus the frequency of the light, being determined by

$$h\nu = E_1 - E_2$$

This theory not only explained the spectral lines. It reproduced the observed frequencies to a high precision and gave stability to matter.

The early contributions of Planck, Einstein, Bohr and others created what today is called the old quantum mechanics. It had many successes in explaining the behaviour of atoms, but there were also serious shortcomings. These were removed by an entirely new mechanics, which was created around 1925 by Heisenberg (1901-1976), Schrödinger (1887-1961), Born (1882-1970), Dirac (1902-84) and others. Quantization of energy and other physical quantities is a natural consequence of the new quantum mechanics, which today is the firm foundation of all descriptions of matter and radiation.

Energy in Quantum Mechanics.

Quantum mechanics has been given a number of different formulations. In the Schrödinger formulation, often called wave mechanics, the entire state of a physical system, say an electron in an atom, is described by a wave function ψ. This is a function of the space coordinates and time.

The wave function has to satisfy a differential equation, the Schrödinger equation, which (in one dimension) is of the form:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

where $V(x)$ is the potential in which a particle of mass m moves.

If one assumes, that the solution of this equation is of the form $\psi(x,t) = f(t)u(x)$ the equation separates into two. In the process of separation a constant is introduced which (by foresight) is denoted E. The solution for f is a simple oscillatory function, $\exp(-iEt/\hbar)$.

The equation for $u(x)$ becomes:

$$-\frac{\hbar^2}{2m}\frac{d^2u(x)}{dx^2} + V(x)u(x) = Eu(x)$$

Quantization is connected to the fact, that only certain discrete values of E permit physically acceptable solutions. These values of E, which can be enumerated E_1, $E_2 \ldots . E_n \ldots$, are the quantum energies of the particle. They are the only possible outcomes of a measurement of the particle energy.

The last equation given above is called the time independent Schrödinger equation. It is the quantum mechanical analogue of classical energy conservation. The first term represents the kinetic energy, the second the potential energy and the term on the right side the total energy. Indeed, if the rules of quantum mechanics for the determination of averages are applied, the equation leads to

$$\langle T \rangle + \langle V \rangle = E.$$

Thus the average of kinetic energy plus the average of potential energy equals total energy. In the heart of quantum mechanics, the Schrödinger equation, we rediscover the great unifying principle of physics: Energy is conserved.

3. Energy in Systems of Increasing Complexity

The historical development illustrates how the concept of energy has been broadened to encompass all domains of the physical world. Energy is used to describe systems and interactions among systems. Systems can be simple as a hydrogen atom or a rolling ball. They can also be complex as a living organism or a whole human society. Even in the most complicated cases some kind of energy account can usually be made, which tells us something important. This section discusses a few examples of the use of the concept of energy in systems of increasing complexity.

3.1 Atoms, Molecules and Nuclei

Today science knows a great deal about the structure of matter and about the constituents of matter. We believe we know most of the basic laws which account for the behaviour of atomic systems. But for any but the very simplest of such systems, a detailed calculation of specific properties is beyond the reach of even the most advanced computers. Even if possible, such a calculation might not be very interesting, because the results cannot be phrased in terms of appropriate concepts. For a gas in a container, it is infinitely more helpful to know, that pressure times volume is a constant, than it is to know the precise location and velocity of 10^{23} molecules one second hence.

Energy is a suitable concept. Even if very little is known of a system, its energy can often be calculated and from the energy other important quantities can be obtained.

It is equally important, that energy, or at least change in energy, is something that easily can be measured. Energy is an observable. This is crucial for atomic systems, where most macroscopic methods of observation fail. Energy by its own nature provides a signal. It is no coincidence, that the three classical observations (see p.24), which provided the foundation for quantum physics, were all measurements of energy.

Most of our everyday experiences about the world can be traced back to one single physical force: The electric interaction. It is the electric interaction between the positive atomic nucleus and the negative electrons which, together with the laws of quantum mechanics, determine the structure and properties of atoms. At our planet, where the temperature is just a few hundred degrees above absolute zero, matter is built from atoms. So what we sense and observe, can be traced back to the electromagnetic interaction. The other forces of nature: the strong and the weak nuclear forces and gravitation are of minor importance. But of course we should not forget, that the nucleus provides most of the mass of an atom and thus, through gravitation, makes us keep our feet on the ground.

<u>Why are atoms to big?</u>

The size of an atom can be estimated by a simple calculation which rests entirely on energy, on the law of electric interaction and the principles of quantum mechanics.

Let us think of a simple one-dimensional hydrogen "atom" where an electron of charge -e is bound to a nucleus of charge +e. At a distance r the potential energy of the electron, is

$$V = -e^2/r.$$

We now bring in the quantum effects condensed into the Heisenberg uncertainty relation. The binding of the electron in the potential is a spatial confinement to an accuracy of order r. This, according to the uncertainty relation, means that the electron is wiggling forth and back with a momentum numerically of the order $\hbar/r$. The corresponding kinetic energy in $E_k = p^2/2m = \hbar^2/2mr^2$. The total energy of the atom is therefore:

$$E = E_k + E_{kin} = \frac{\hbar^2}{2mr^2} - \frac{e^2}{r} .$$

This function obviously has a minimum at a distance r=a. Left to itself, the atom will minimize its total energy, which by differentiation can be seen to occur for

$$a = \frac{\hbar^2}{me^2} .$$

This, somewhat fortuitously, corresponds exactly to the "Bohr-radius" of the hydrogen atom (5.3×10^{-11}m). The corresponding total energy becomes

$$E = -\frac{1}{2}\frac{me^4}{\hbar} (= 13.6 \text{ eV}).$$

The size and binding energy of an atom is thus a natural consequence of quantum physics and the electric interaction.

An atom is electrically neutral and two atoms at distances large in comparison to the atomic diameters will not interact.

But if the atoms are brought so close that their electronic clouds tend to overlap, the atoms attract each other and a molecule is formed. The atoms however do not completely coalesce, because the repulsion between the charges on the atomic nuclei dominates over attraction at close distances.

<u>Energy in Molecules.</u>

The formation of a molecule can be summarized in what is usually called its potential energy curve (although also kinetic energy terms are included.) Let us consider the approach of two atoms A and B which eventually will form a diatomic molecule AB. At a distance r between the nuclei the energy of the system of electrons and nuclei is E(r). At large distances E(r)=0, but if a stable molecule is to be formed, E(r) must decrease with distance until a minimum is reached. At still shorter distances the energy increases because of the nuclear repulsion. The distance r=R where the energy minimum is found is the equilibrium distance of the atomic nuclei. The depth D of the minimum is the energy it takes to dissociate the molecule or, conversely, the energy relased (eventually as heat) during its formation.

A simple diatomic molecule is already a vastly more complicated system than an atom. In a molecule we, like in an atom, have a system of electrons. These electrons do not belong to any of the individual atoms in the molecule. They are shared and exchanged among the atoms which makes a quantum mechanical calculation of the molecular energy much more complicated. In addition new forms of motion become possible in a molecule. The equilibrium distance of the atoms, say in a diatomic molecule, corresponds to minimum energy. Oscillations of the nuclei around equilibrium are possible and the molecule gets vibrational anergy. Likewise, a diatomic molecule can rotate around an axis perpendicular to the line forming the nuclei. This gives a contribution of rotational energy to the molecule. Thus a complete account of the energy of even the simplest molecules is a complicated matter. This is also evident from a study of the spectra emitted by molecules which are infinitely more complex than those of the individual atoms.

If we take the next step towards complexity we encounter chemistry. A science concerned with the formation and dissociation of molecules and other aggregates of atoms. In principle chemistry can be understood on the basis of quantum mechanics, but except for the very simplest systems, a detailed treatment is impossible, and one has to rely on other methods. On the macroscopic level energy and entropy accounts are indispensable for the study of chemical reactions, and thermodynamics in chemistry has developed into a subject of its own.

The atomic nucleus is an object where the linear dimensions are about 10^4 times smaller than those of an atom. Conversely the scale of energy we encounter in the nucleus is about a million times greater. Where energies in atoms and molecules are measured in eV the relevant unit in the nucleus is MeV.

This change in scale of energy, together with the fact, that the nuclear particles are much heavier than electrons, means that the properties of the nucleus in some respects are quite different from those of the atom. Furthermore, while the forces governing the behaviour of the atom are the well known electromagnetic forces, the strong nuclear forces binding the nuclear particles together are less well understood. Still it has been possible to reach a significant understanding of the nucleus as a quantum mechanical system, and today the accuracy to which nuclear properties can be calculated rivals that obtained for atomic systems.

Numerous examples can be given of the use of energy concepts in nuclear physics. The alpha-decay, typical of heavy nuclei, was intensely studied at the beginning of this century and at once made it clear, that nature here operated on an entirely new scale of energy. The beta-decay, with its continuous energy spectrum of the emitted electrons, for long was an energetic puzzle. Why did a process between two well-defined states of a nucleus not always release the same amount of energy? Was here a case where energy was not conserved? Pauli (1900-1958) would not sacrifice energy conservation. To save it he invented a new, almost undetectable particle, the neutrino, which was supposed to run away with the missing energy. The neutrino, later found experimentally, perhaps is one of the most surprising discoveries based on energy conservation.

Energy in nuclei has in many other respects affected the scientific, the technical and the political development. The masses of nuclei undergoing transformation provided the first experimental proof of the equivalence of mass and energy. The discovery of nuclear fission resulted in the nuclear weapons and a powerful new source of energy. The energy production in the stars and the synthesis of elements in the universe has been explained in terms of nuclear processes. In later years the detailed investigation of the nucleus as a quantum system with many, but not too many, particles has contributed significantly to the development of as well nuclear physics and quantum theory.

Quantized Energy in Nuclei.

The study of many-body systems is a theme recurrent in almost all branches of contemporary physics, ranging from macroscopic phenomena in condensed matter through molecular and atomic structures to nuclei and elementary particles.

The nucleus here has a special position, because it at the one hand is sufficiently complex to show a variety of collective phenomena, at the other is sufficiently simple to exhibit sharp quantum states.

The nuclear shell model provides an example of quantized energy in the nucleus. In this model one considers the nucleons (protons and neutrons) as moving independently in a common potential created collectively by all the nucleons. In the simplest version this potential is taken as that of a 3-dimensional harmonic oscillator, $V(r) = \frac{1}{2} kr^2$. The quantum mechanical solution of this problem is a series of states with equally spaced energies $n\hbar\omega$, where n is an integer. Each state, according to the Pauli principle, can accomodate (n+1)(n+2) nucleons of one kind. In the shell model one further takes into account the interaction of the angular momentum $\vec{L}$ and the spin $\vec{S}$ of a nucleon. This results in a splitting of the individual oscillator levels. The resulting level scheme and its occupation numbers is in qualitative and, with adjustments, even quantitative agreement with observed energy levels in nuclei.

3.2. Energy in Agriculture

Systems in physics can be rather complex, but for such systems energy remains a well defined quantity which in principle can be accurately calculated or measured.

If we look at systems which are not exclusively governed by the laws of physics, energy and flow of energy are still valuable concepts, but they tend to loose some of their mathematical rigor. Also, other aspects of energy than those of most importance in physics, are brought into the foreground.

Energy considerations are important for the study of living organisms. The term metabolism is used to describe the complicated cycles of conversion of energy and matter responsible for life. The study of such processes is completely within the realm of natural science. However, if we consider systems which in various ways are affected by mankind the situation becomes more complex. One example of such a system is agriculture.

In the thermodynamical sense agriculture is an open system which exchanges energy and matter with the environment. It differs from a natural ecological system because of the interference of man.

The basic process in agriculture is photosynthesis, the trapping and storing of solar energy by green plants. In principle photosynthesis is a simple process where the energy in light converts carbon dioxide and water into sugar and oxygen:

$$CO_2 + H_2O + nh\nu \rightarrow (CH_2O) + O_2$$

Here CH_2O represents a carbohydride, for example glucose, $C_6H_{12}O_6$, and the n indicates the number of photons needed to drive the process. The conversion of carbon dioxide into sugar requires about 5 eV per molecule. However, a photon in the visible region has a quantum energy of 2-3 eV, so obviously the process cannot proceed in one step. In reality at least eight quanta are needed per molecule of CO_2 which implies a quantum efficiency of about 30%

The efficiency by which solar radiation is converted by green plants is however considerably lower. Only a fraction of the spectrum can drive the photosynthetic process, energy is lost in respiration and the total area is not utilized. The total efficiency for green plants in nature is about 0.25%. Agriculture is man's method to improve this efficiency and to maximize the yield of edible products.

In agriculture man interferes in the natural processes mainly through the use of energy. Energy for preparation of the soil, for seeding, harvesting and storage. In modern agriculture these energy inputs are considerable and in excess of the nutritional energy of the crops. This is even more so in animal farming where the efficiency of conversion of vegetable feed into animal products is only about 10%.

The Energy Ratio.

One has attempted to judge the efficiency of various agricultural systems by means of the socalled energy ratio R

$$R = E_{out}/E_{in}$$

where E_{out} is the nutritional value of the food leaving the system and E_{in} is the input of energy for traction, chemicals and so on (but not solar energy). The table shows some examples of such energy ratios.

ENERGY RATIOS FOR AGRICULTURE

	E_{out} GJ/HA-Y	E_{in} GJ/HA-Y	R
Denmark	14	1	0.5
England	11	24	0.5
Holland	43	36	1.2
USA	9	13	0.7
Australia	6	2.4	2.8
India (example)	10	0.7	15
China (example)	281	6.8	41

E_{out}: vegetable + animal output

E_{in} : fossil fuel + human and animal labour

One should not uncritically judge the performance of agricultural systems on their efficiency of energy use. Agricultural production happens to be measurable in energy units and therefore it is tempting to relate output to input as in the energy ratio. That this ratio can exceed unity is due to the sun, which generously gives an energy subsidy to agricultural production. But a dimensionless ratio does not tell anything about absolute yield. Experience shows, that high yields invariably are connected to use of energy for work and fertilizer.

3.3. Energy and Society

Energy as a concept evolved within physics. But energy is also an important factor in technology, economics and politics. Energy is one of the few examples of a difficult and abstract scientific concept which has penetrated into everyday life and which is understood intuitively and often quite correctly by the common man.

No system can be more complex and bewildering than a human society. Certain aspects of the functioning of a society are described in economic terms. But the economic theories do not have the same predictive and analytic power as the theories in physics. Nevertheless it is instructive to attempt to unify the concepts of economics with the concepts of physics.

The first difficulty one meets in such an attempt is the problem of measurement. A scientist measures quantities of energy in joules and flows of energy in watts. In the interest of understandability he might accept other units as a liter of gasoline as units of energy. Energy would still be a measurable quantity.

In economics it is not so much the physical quantities which matter. It is value. Value, in its monetary sense, is a concept which is foreign to science. The value of something is expressed in its price. Price is determined at the market place through an iterative procedure where, at the end, supply (going up with price) and demand (going down with price) will meet.

Scientists of course are reluctant to accept such unreliable and even fluctuating unit of measurement as a dollar, a yen or a mark. Some scientists have therefore suggested, that energy should be taken as a measure of value. Energy is reasonably scarce, it is necessary for almost every product or activity in society. So why not introduce an energy theory of value?

Economists have shuddered at such suggestions and rightly so. One can no more suppress the law of supply and demand than one can suppress Newton's second law. So scientists by and large have come to terms with the concept of value.

The total value of all goods and services produced in a society is expressed in its gross national product (GNP) measured e.g. in $ per year. Each year statistical institutions add everything up and, by not too transparent methods, arrive at the GNP in constant dollars or other currencies.

Energy Intensity.

Energy consumption and GNP are rather strongly correlated. The ratio of energy use to GNP is often called the energy intensity of the economy. The table[9]) shows such intensities for the major regions of the world in 1975.

It is worth noting, that the per capita income in these regions differs by several orders of magnitude. In spite of this, the energy intensity is constant within a factor of two.

ENERGY GNP RELATION (1975)

	ENERGY EJ/a	GNP 10^9\$/a	ENERGY/GNP MJ/\$
North America	80.5	1678	48
Latin America	12.5	327	38
Japan	14.2	496	29
Western Europe	50.3	1695	30
Denmark	0.8	34	22
USSR, Eastern Europe	59.5	925	64
China	16.3	315	52
Asia (excl.China)	10.9	210	52
Africa	5.0	163	31
Middle East	4.1	153	27
Australia	3.1	94	33
World	256.6	6056	42

Energy from BP Statistical Review (1976).

GNP from World Bank Atlas (1977).

1 EJ = 10^{18}J.

The close connection between income and energy use is no mystery. Almost any activity in a society which is included in the GNP requires energy. But if one looks into the details of this connection certain trends can be seen. The rich countries tend to have lower energy intensities than the poor countries. This is not surprising in view of the different composition of the production. The rich countries have economies dominated by services, the poor countries have economies dominated by energy intensive agriculture and basic products as steel and cement.

Energy is of crucial importance to us all. This has been amply demonstrated during the last 15 years where the price of energy has undergone violent fluctuations. The survival of our societies, rich or poor, depends on a reliable and affordable supply of energy. This is the lesson we have learned.

There is a long evolution from the concept of _vis viva_ to todays space probes or nuclear power plants. Science and society have both contributed to this evolution. It has been said, that science owes more to the steam engine than the steam engine to science. But even if this statement were true, science has amply repaid its debt through its evolution of the concept of energy, which today penetrates science as well as society.