

ENERGY AND MATTER IN THE EARLY UNIVERSE

by

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1. Introduction

The main features of our universe are space, time and matter. The space we live in is three-dimensional, while time proceeds in one dimension. Einstein's discovery of the relativity of space and time led the way to our modern understanding of the four-dimensional space-time manifold, in which space and time are considered as a union. Yet a distinction still exists: our world develops in time, not in space, a process, which transforms the future into the present, while the present fades away by becoming past as time proceeds.

The third feature of the universe, matter and energy, exhibits itself by a multitude of phenomena. Matter does not only consist of the electrons and the other leptons, or the various quarks. Also the different forces in the universe, i.e. the attractive gravitational forces between lumps of matter, the electromagnetic forces between charged particles, the chromodynamic forces between the quarks and gluons deep inside the atomic nuclei or the weak forces among the leptons and quarks, are a manifestation of the matter in the universe.

The space-time manifold we observe is in general highly symmetric. For example, the space is isotropic; no direction in space seems to be preferred compared to any other direction. To a very good approximation space and time are homogeneous; no particular point in the universe seems to be distinct from any other point, and a particular moment on the time axis is as good as any other moment.

Matter does not exist on its own, but it is embedded in the space-time manifold. This implies that the symmetry of space and time imprints itself on the

dynamical behaviour of matter. The dynamics of the particles of matter must respect the symmetries of space and time. As a result certain dynamical quantities, most notably the energy and the momenta of a physical system, do not change as time evolves.

These conservation laws exhibit clearly the intimate relationship between the geometry of the space-time manifold on the one hand and the dynamics of matter on the other hand. This relationship, first discovered in the study of classical mechanics and later generalized to the quantum mechanics of atoms and the quantum field theory of elementary particles, is well-known. However several questions have to be discussed with respect to such a relationship:

- a. If one treats the whole universe as a physical system, what happens to the law of energy conservation? What can one say about the interplay between space, time and matter in the very early universe, a minute fraction of a second after the Big Bang?
- b. Is there a deeper connection between the space-time and matter, going beyond the fact that the space-time manifold provides the stage on which matter performs its dynamical evolution? Is matter simply a manifestation of geometry, displaying in a secret way the presence of a more extended structure of the space-time manifold?

In this article I shall be mainly concerned with giving possible answers to the first set of questions. Only at the end I shall discuss shortly some speculations concerning answers to the second set which, of course, are highly speculative and in no way supported by experimental observations.

2. Gravity and the Conservation of Energy

In general the energy of a physical system is not conserved, if it does not exist in isolation, but interacts with neighbouring systems. The conservation of energy is guaranteed only for an isolated system, e.g. a set of atoms, which does not interact with its neighbourhood. For such a system the flow of time is homogeneous, i.e. it does not matter, whether the system starts its dynamical evolution now, or after a certain time interval is elapsed. The result of the dynamical evolution in both cases is the same.

Not only the time is homogeneous, but also the space. The homogeneity of space implies another conservation law - the conservation of the momenta of an isolated physical system. Thus the motion of an isolated system through space is characterized by four conserved quantities: the energy and the three components of the momentum.

Thus far we have neglected gravity, which complicates the matter. According to Einstein's theory of General Relativity the gravitational attraction between different objects is nothing but a consequence of the curvature of the space-time manifold in the presence of matter. The geometry of the spacetime manifold depends on the dynamics of matter ; matter and geometry are intertwined. This connection is described by Einstein's equations:

$$G_{\mu\nu} = - \frac{8\pi G_N}{c^4} T_{\mu\nu}$$

(G_N : Newton's constant; c : velocity of light).

Here $G_{\mu\nu}$ denotes the Einstein tensor, which is entirely given by the geometry of the space-time manifold. If the latter is simply a flat space-time, $G_{\mu\nu}$ vanishes. On the r.h.s. of Einstein's equations one finds the energy - momentum tensor of matter $T_{\mu\nu}$ which can be constructed in terms of the various fields of elementary particles. One component of $T_{\mu\nu}$ describes the energy density. In the presence of matter the Einstein tensor cannot be zero, and the space-time manifold cannot be flat, i.e. it cannot be homogeneous. Since gravity destroys the homogeneity of space and in particular of time, the total energy of a system embedded in a gravitational field is not conserved (Fig. (1)). Constantly the system hands over part of its energy to the gravitational field, or receives energy by it. However the energy of the gravitational field cannot be defined in general, since gravity is simply the result of geometry.

Only if the gravitational field disappears outside a certain finite region, hence the curvature of the space vanishes and the space is flat far away, one is able to define a total energy of the system including the gravitational field, and this energy is conserved (Fig. (2)).

These considerations show that the concepts of energy and of energy conservation become rather intricate once the space-time manifold acquires a structure of its own due to the presence of gravitational fields. These problems persist if we turn to cosmology and look at the whole universe.

3. Cosmology and the Conception of Energy

Modern cosmology started when it was realized that the observed redshift of the distant galaxies is a global cosmic phenomenon. The recession velocities of the

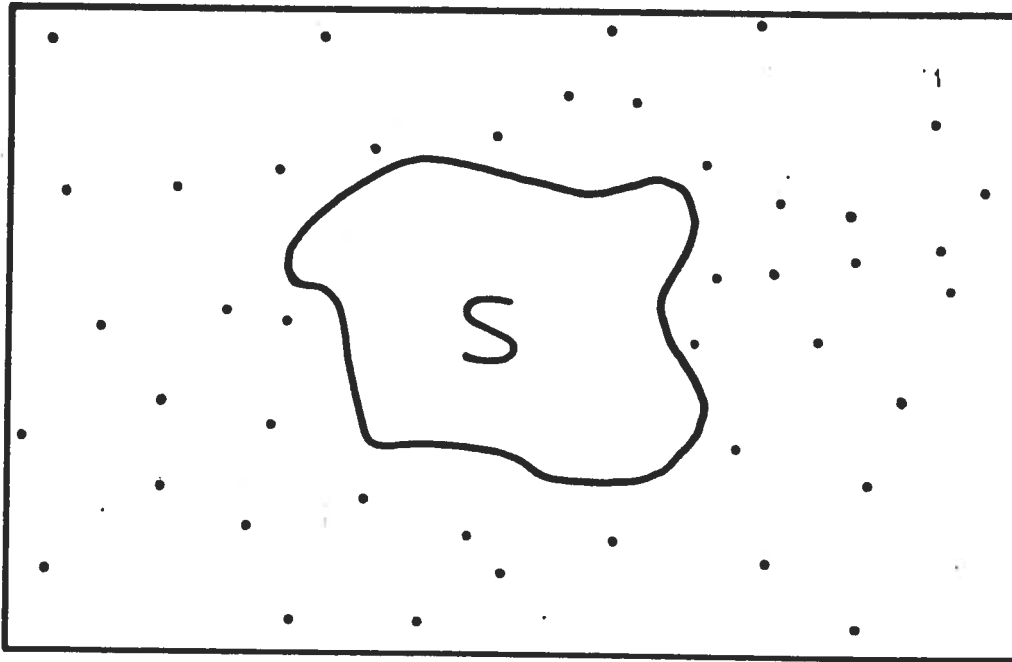


Fig. (1). The energy of a physical system embedded in a gravitational field of infinite extension is in general not conserved.

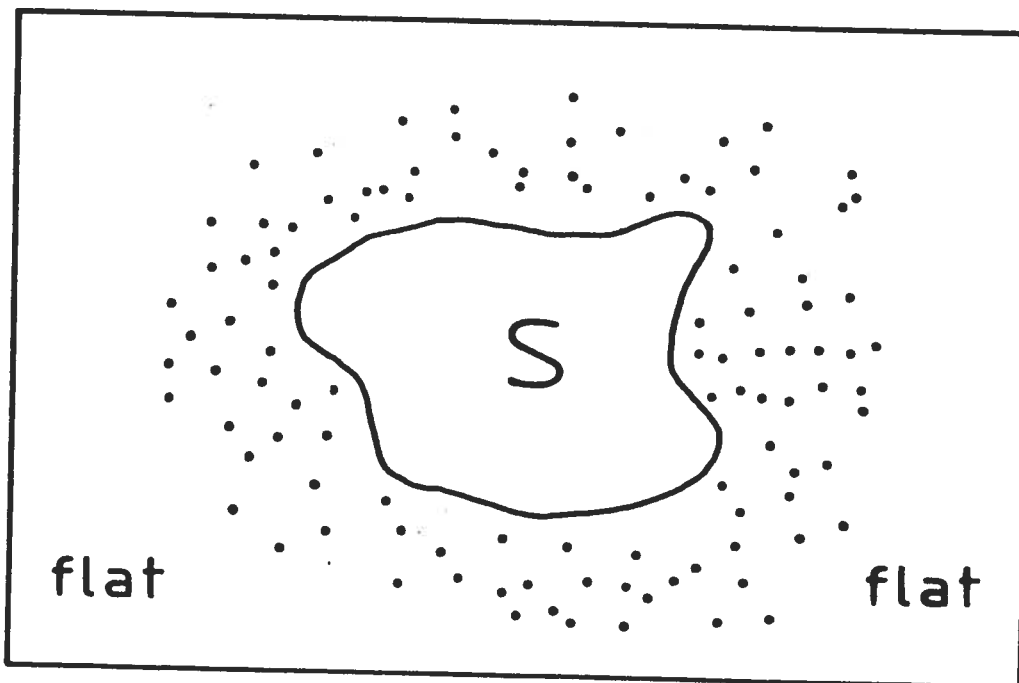


Fig. (2). If the gravity field ceases to be present outside a certain finite region of space, and the space is flat outside, the time becomes homogeneous again, and the total energy of the system, including the gravitational field, can be defined and is conserved.

distant galaxies are observed to be proportional to the distances which implies that all observed galaxies were concentrated in a dense region about 15 billion years ago. In cosmology this phenomenon is interpreted as a general expansion of the space. The Big Bang cosmology states that the universe, i.e. space and matter, were created about 15 billion years ago. Shortly after the beginning the cosmic matter was very hot and dense. Since then, the universe is expanding and cooling.

On a large scale the universe seems to be isotropic and homogeneous. When smeared out over sufficiently large regions (whose diameters are typically of the order of 100 million lightyears), the energy or mass density in the universe can be considered to be constant. Its observed value is of the order of $10^{-30} \text{ g cm}^{-3}$.

The rate of expansion of the universe can be described by just one number, the Hubble parameter $H(t)$, which is a function of the cosmic time t elapsed since the Big Bang, and which is simply the relative rate of change of any cosmic distance R in time:

$$H(t) = \left(\frac{dR}{dt} / R \right) = \frac{d}{dt} (\ln R)$$

Today the Hubble parameter is of the order of $50 \text{ (km)s}^{-1} \text{ (Mpc)}^{-1}$ [the astrophysical distance unit megaparsec (Mpc) is defined by: $1 \text{ Mpc} = 3.09 \cdot 10^{24} \text{ cm}$]. Thus the redshift of a galaxy, which is 100 Mpc away from us, corresponds to a velocity of $5\,000 \text{ (km) s}^{-1}$.

We emphasize that the interpretation of the observed redshift phenomenon as due to a recession of the distant galaxies from us is not quite correct. The

redshift is rather due to an expansion of space. The galaxies remain at rest but the space between the galaxies expands. As time evolves, all distances are universally stretched. It is interesting to note that such an expansion can only be measured if another way of measuring distances exists besides the cosmic distance ladder provided by the galaxies. This second way is provided by the quantum theory. One of the important consequences of the quantum nature of atoms is that all atoms of the same type, e.g. all hydrogen atoms, have the same size. The diameter of e.g. the hydrogen atom, which is of the order of 10^{-8} cm, remains constant in time. Thus the expansion of the cosmic space is the statement that all distances expand when compared to the diameters of atoms. (As Eddington once remarked, another interpretation would be that the cosmic distances remain constant, while the atomic scales shrink accordingly.)

The theory of General Relativity when applied to the universe as a whole predicts the time evolution of the Hubble parameter, provided the cosmic mass or energy density is known. In any case the expansion is expected to slow down in the future, i.e. $H(t)$ decreases. The observations indicate that such a deceleration is indeed present. However the data do not determine whether the deceleration is large enough such that the expansion comes to a stop eventually, and the universe starts to shrink afterwards. If the mass density is equal to the critical mass density, which is given in terms of the Hubble parameter and comes out to be of the order of 10^{-29} g cm $^{-3}$ (i.e. a density about five to ten times larger than the observed mass density), the expansion slows down and the Hubble parameter approaches zero as t approaches infinity.

We shall argue later that this case is of special interest. In view of the experimental uncertainties in the determination of the cosmic mass density and

in view of the possibility of new contributions to the mass density provided by weakly interacting neutral particles (e.g. neutrinos) as constituents of the so-called "dark matter" it is not excluded that the cosmic mass density is in fact equal to the critical mass density. In this case the three-dimensional space would be a flat ("Euclidean") space. Every cosmic distance R changes such that

$$\frac{R(t)}{R(t_0)} = \left(\frac{t}{t_0} \right)^{2/3} .$$

(see Fig. (3)).

The conservation of energy follows for an isolated system which is embedded in a flat empty space whose distances do not vary in time. This is not true for the universe. The universe is not an isolated system since it cannot be viewed as an island surrounded by empty space. Furthermore its dynamics is not homogeneous in time. Every instance in the cosmic evolution is unique, characterized by the cosmic time t , which in turn determines the Hubble parameter $H(t)$ or the distance scale $R(t)$ normalized in a suitable manner. In cosmology there is a profound difference between today and tomorrow. Tomorrow the universe is one day older than today, and tomorrow's universe is a bit larger than today's universe; for example, the distance between us and the Coma cluster of galaxies has increased by about one billion kilometers.

It is suitable to divide the space of the universe into cells, e.g. cubes with a side length of 1 km ("cosmic cubes"). These cubes are expanding like the universe, and the opposite walls of each cube are receding from each other with a speed of $H \cdot (1 \text{ km}) \sim 10^{-5}$ cm per year. Suppose we treat such a cosmic cube

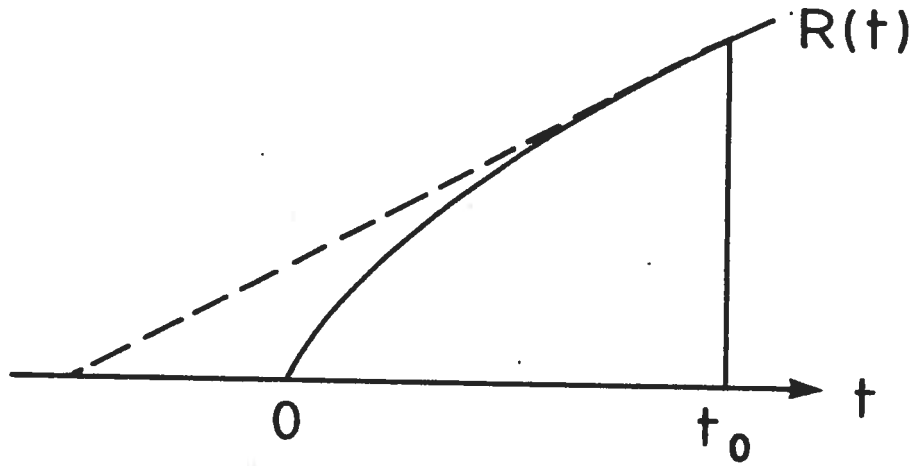


Fig. (3). The expansion of the "flat" universe. At the time t_0 a certain cosmic distance is given by $R(t_0)$. At any other time t the corresponding distance $R(t)$ is related to $R(t_0)$ by $R(t) / R(t_0) = (t / t_0)^{2/3}$. The Hubble parameter $H(t)$ is given by $H(t) = 2/3 t^{-1}$. It is infinite at $t = 0$ ("Big Bang singularity") and approaches zero as $t \rightarrow \infty$. Note that the time evolution of $R(t)$ is calculated within the theory of General Relativity. The mass density is supposed to be described in terms of noninteracting "dust" particles, an approximation which fails to be correct for the very early universe ($t < 1s$).

as a resonant cavity whose sides are able to reflect electromagnetic waves, i.e. the photons composing the electromagnetic waves are bouncing back and forth between the walls of the cube. Each time a photon is reflected by the receding walls of the cube it loses energy. The total energy of the radiation field inside the cavity decreases like $R^{-1} \sim t^{-2/3}$. The same is true for the temperature of a radiation field inside the cosmic cube.

These considerations do not only apply for a comoving cube with reflecting walls in the universe, but in the case of a thermodynamic equilibrium it applies for the universe which can be viewed as a collection of cosmic cubes. (The walls can be neglected, since in the average each cube receives photons or other particles from its neighbours, but loses an equal number of particles to its neighbours.) Since we observe that the universe is filled with photons of the 2.7 K radiation, we can say that the universe loses energy permanently as a consequence of the expansion.

The number of photons of a thermal radiation per $(\text{cm})^3$ is about $n_\gamma \approx 20 \cdot T^3$ (T measured in K). For the 2.7 K radiation one has $n_\gamma \approx 400$. The radiation energy contained in $1(\text{cm})^3$ of volume is 0.0931 eV. A cube with a volume of $1(\text{km})^3$ contains an energy of $9.3 \cdot 10^{13}$ eV = 93 TeV. Due to the expansion it loses an energy of about 100 keV per year. In 30 billion years from now, when the universe is three times as old as today and all cosmic distances have doubled, one-half of the energy contained in the 2.7 K radiation today will have disappeared.

This energy is not transformed in a different type of energy such that the total energy is conserved. There is simply no conservation law for the total energy of the universe. As long as the universe expands, and this might go on

forever, the universe loses energy. The opposite is true for a collapsing universe, i.e. a universe in which $R(t)$ decreases in time (H is negative). In this case the energy of the radiation field increases steadily. This energy does not come from another source. It is provided free of charge by the shrinking universe.

The loss of energy in the radiation field of the universe is described by the first law of thermodynamics relating the change of energy to the change of the volume: $dE + p dV = 0$ (p : pressure). This relation implies for the energy density $\epsilon = E/V$: $V d\epsilon + (\epsilon + p) dV = 0$. Thermal radiation has a pressure p which is given by one-third of the energy density ϵ . On the other hand the volume V of a cosmic cube changes during the expansion as R^3 . Thus one finds:

$$d\epsilon \cdot R^4 + \epsilon \cdot 4 R^3 dR = d(\epsilon R^4) = 0$$
, i.e. the product $\epsilon \cdot R^4$ remains constant, and the energy in the cosmic volume $\sim \epsilon \cdot R^3$ changes as R^{-1} , as discussed.

For an ordinary gas or dust, e.g. for galaxies, when treated as "dust particles" the pressure p is negligible compared to the energy density ϵ . Thus we have $dE = 0$, i.e. the energy in a cosmic cube remains unchanged. The mass density ρ decreases during the expansion like R^{-3} (such that the total mass in the cube $M = \rho \cdot R^3$ remains constant, likewise the total rest mass energy).

Today the universe loses energy as a result of the expansion, but this loss is relatively modest since the energy density is dominated by ordinary matter (e.g. galaxies), and not by the radiation field. The energy loss was much larger in the early universe, considered subsequently, where the cosmic energy density was built up by a very hot plasma of quarks, leptons, photons etc.

We have stressed above that the law of conservation of energy is not valid in cosmology. Nevertheless there have been attempts to relate the energy loss

during the expansion to an increase of energy somewhere else, in order to save the conservation law. For example, one could argue that during the expansion when all cosmic particles are receding from each other the gravitational energy of the universe increases such that the energy lost by the radiation field is exactly equal to the gain in the gravitational energy. I think such an interpretation does not make sense, since the total gravitational energy of the universe cannot be defined. Cosmology is a special application of General Relativity, in which gravity is viewed as a part of the geometry of the space-time continuum. But geometrical effects cannot be associated with dynamical quantities like energy or momentum.

The nonconservation of energy in cosmology is an important fact. If energy can disappear without leaving a notice, it may also appear spontaneously. It paves the way for the creation of all matter and energy in the cosmos out of nothing.

4. Standard Particle Physics and the Problem of Masses

The matter we observe consists of electrons, the constituents of the atomic shells, and of quarks, the constituents of the atomic nuclei. Two different quarks, the u- and d- quarks, are needed to build up the nuclear matter.

Using primarily high energy accelerators, one has found five relatives of the electron: three neutral ones, the neutrinos ν_e , ν_μ and ν_τ , and two more charged ones, the muon μ^- and the τ -lepton τ^- . Altogether there are six leptons.

Besides the "nuclear" quarks u and d four other quarks have been identified: the strange quarks s, the charmed quarks c, and the quarks t and b. (The existence of particles containing t-quarks has yet to be confirmed by experiment.)

It is useful to classify the leptons and quarks in three families:

$$\begin{array}{ccc}
 \text{I} & \text{II} & \text{III} \\
 \left(\begin{array}{c|c} \nu_e & u \\ e^- & d \end{array} \right) & \left(\begin{array}{c|c} \nu_\mu & c \\ \mu^- & s \end{array} \right) & \left(\begin{array}{c|c} \nu_\tau & t \\ \tau^- & b \end{array} \right)
 \end{array}$$

The quarks carry an attribute not carried by leptons: "color" - a nickname for the fact that each quark appears in three different editions. Due to their "color" the quarks interact with each other by the exchange of gluons. As a result quarks (and gluons) cannot be isolated in the laboratory. They are permanently bound to other quarks to form hadronic particles: protons, neutrons, π -mesons etc. . The field theory of the quarks and gluons, the theory of QCD ("quantum chromodynamics") is quite analogous to QED ("quantum electrodynamics"), the field theory of electrons and photons.

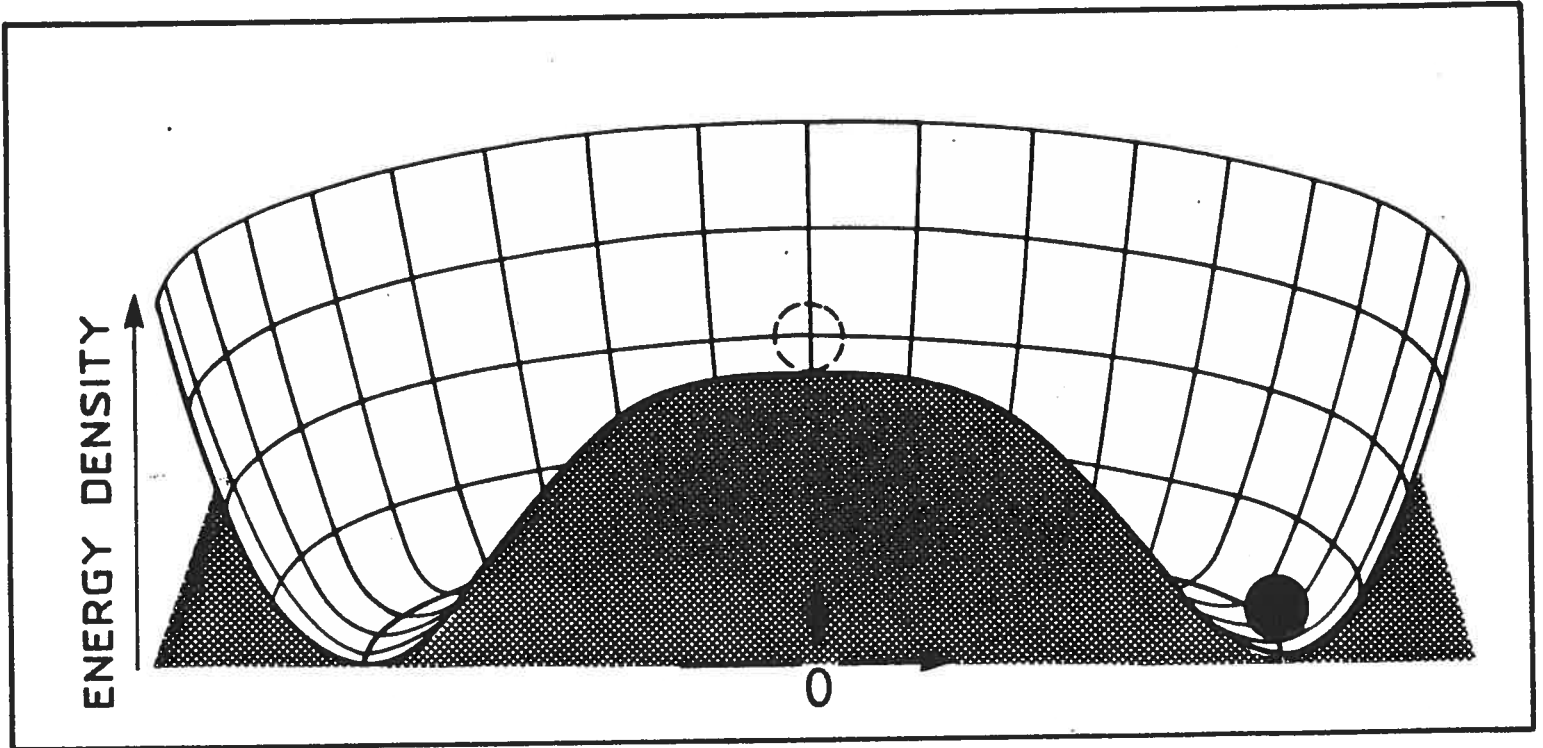
The electromagnetic and weak interactions form a bridge between the leptons and quarks in the sense that both the leptons and quarks can interact with the corresponding force-carrying particles, the photons, the charged weak bosons W^\pm and the neutral Z- Bosons, the latter being some sort of heavy partner of the massless photon. The electromagnetic and weak interactions are combined into one unified theory, the theory of the electroweak interactions.

Many features of this standard model of the elementary particles and their interactions have been successfully probed in the experiments. Only one aspect has remained essentially untested - the mechanism which generates the (rather heavy) masses of the weak bosons (of order 100 GeV), and at the same time the various masses of the leptons and quarks. The underlying idea is that the masses

of the weak bosons W^+ and Z are zero to start with (like the photon mass). In this case there would be no distinction between e.g. the electron and its neutrino. This symmetry is broken by the dynamics, in particular by the (hypothetical) interaction of the leptons and quarks with new bosons, the so-called "Higgs" bosons. These bosons are supposed to be present everywhere in space. The interaction of the bosons with the "Higgs" particles is constructed in such a way that one of the bosons, the photon, can propagate freely through space with the speed of light, while the three others, identified with the W^+ , W^- and Z bosons, are hindered in their propagation and cannot move as fast as the photon - they have acquired a mass.

The mass generation via the spontaneous symmetry breaking is an effect analogous to effects in solid state physics or other branches of physics. For example, water is homogeneous and isotropic, reflecting the fact that in the dynamical laws determining the motion of the water molecules no location or direction is preferred. However this symmetry is spontaneously broken, if the water freezes. Once ice crystals are formed, special directions or locations are singled out. This is an example of a situation, in which the underlying physical laws are perfectly symmetric, but nature decides to choose one particular solution, which breaks the symmetry. The symmetric solution is also possible, but is typically associated with a higher energy. (In our example this would mean that the water molecules do not arrange themselves in crystals, but remain a liquid - one would be dealing with a supercooled liquid, an unstable state of matter with a larger energy density.) The situation can be described by a Mexican hat type of potential (see Fig. (4)). In general one can say that the symmetric solution corresponds to an unstable equilibrium, which after a slight perturbation will turn into a stable, but unsymmetric state.

Unlike to the situation in solid state physics, where the ground state of the system is one consisting of many subsystems (atoms, molecules etc.) acting



Fig'. (4). The spontaneous symmetry breaking described by a Mexican hat type of effective potential. If the ball is placed on the top at the center, one has a symmetric situation - no direction is preferred. However this is not a stable situation. After a slight perturbation the ball will roll down the hill and come to rest somewhere on the circular bottom line. It has reached a stable situation, but the original isotropy of all directions is destroyed, since the position of the ball singles out one particular direction from the center.

coherently, in particle physics the ground state of a system is the vacuum state. In particle physics the vacuum is by no means simply the empty space-time continuum of classical physics, but due to quantum fluctuations a highly complicated system, whose properties and functions are not yet fully understood. The vacuum state is supposed to be in an unsymmetric state, which simply means that certain physical quantities, e.g. the potential of a field or the mass of a particle, are different from zero. One refers to the so-called "vacuum expectation value" of a physical quantity. In the modern theory of the electroweak interactions the mass of the W particle or of the electron are nothing but manifestations of the unsymmetry of the vacuum state.

A simple example of a physical system in which the appearance of a non-zero expectation value of a physical quantity can be seen is the Heisenberg ferromagnet - an infinitely extended system of magnetic dipoles. Magnetic dipoles have the tendency to align each other. Thus the state of lowest energy is not a state in which each dipole points in some arbitrary direction, but a state in which all dipoles point into the same direction. This direction is arbitrary, and for each direction exists a specific ground state. Thus the system has infinitely many ground states, one state for each of the infinitely many directions. We may define the physical quantity "average direction" as the average direction of the magnetic dipoles in a certain volume. If the system is totally disordered (e.g. at high temperature), this quantity is, of course, zero. However in the ground state of the Heisenberg ferromagnet the average direction of the dipoles will be a certain direction in space - it does not vanish.

The situation here is rather similar to the one in the electroweak theory, if we replace the quantity "average direction" by the quantity "mass of the weak boson".

I should like to emphasize that this mechanism of the mass generation is not without problems. The most serious one related to the so-called cosmological term in cosmology will be discussed later in connection to the "inflation" of the universe. Another problem is that this mechanism looks like a "dead end" as far as actual calculations of masses are concerned. There is, for example, no way to understand or to calculate e.g. the ratio of the muon mass and the electron mass. If this mass ratio, and many other mass ratios, just reflects the unsymmetry of the physical vacuum state, there would be no way forever to gain a more detailed understanding of the mass problem in physics. The masses of the various leptons and quarks or of the weak bosons would be "historical" quantities, i.e. numbers which were fixed in the course of the evolution of the very early universe and which depend on specific and unknowns details of that evolution.

In 1990 the first experimental results are expected from LEP, the large European electron - positron collider at CERN, near Geneva. This accelerator is able to probe the physical details how nature is able to generate the masses for the heavy weak bosons W and Z, and it remains to be seen whether the theoretical extrapolations turn out to be correct or not.

5. Elementary Particles in the Early Universe

Today the universe is relatively cold. The cosmic temperature, which is determined by the electromagnetic background radiation, is about 2.7 K. The bulk of the observed matter is given by the nuclear matter concentrated in the galaxies which is basically at rest.

At earlier times the situation was quite different. About 10^{-6} s after the Big Bang the temperature was of the order of 1 GeV ^{or $\sim 10^{13} \text{ K}$} . Nuclear matter composed of nucleons cannot exist under these circumstances. Instead it was present in form of a hot plasma of quarks, antiquarks and gluons. These particles were moving essentially with the speed of light. They contributed to the energy density in a very similar way as the photons. The electromagnetic energy density due to the photons is simply given by the Stefan-Boltzmann law:

$$\epsilon_{\gamma} = \frac{\pi^2}{15} T^4$$

(T: cosmic temperature).

The contributions of the electrons, neutrinos, quarks etc. to the energy density in the universe at high temperature (i.e. at a temperature high enough such that the particles move essentially with the speed of light) is determined analogously. Every elementary particle P which is in thermodynamic equilibrium with the other particles in the universe contributes to the cosmic energy density

the amount

$$\epsilon_p = g_p \cdot \frac{\pi^2}{30} \cdot T^4.$$

The factor g_p counts the effective number of degrees of freedom of the particle considered. Photons which can appear in two different polarizations have a g-factor of two: $g_\gamma = 2$. Fermions, i.e. particles with spin 1/2 have a g-factor 7/4. The effective g-factor of the universe, which is simply the sum of all g-factors of the individual particles, is a measure of how many particles contribute to the cosmic plasma. At $T \sim 0.3$ GeV one expects besides the photons the contributions of the electrons, positrons, neutrinos, the u and d quarks and the gluons. (The latter act in the same way as the photons.) Taking into account the color degrees of freedom of the quarks, and gluons one finds $g = 51.25$.

At earlier times when the temperature was higher the other quarks s, c, b, t contribute as well as the weak bosons. The dependence of the g-factor on the temperature T is described in Fig. (5). If the temperature of the universe is above 200 GeV, all elementary objects appearing in the standard model of particle physics (6 leptons, 6 quarks, gluons, photons, weak bosons) are present and contribute like the photons to the energy density. The effective g-factor in this case is 105.75:

$$\epsilon = 105.75 \cdot \frac{\pi^2}{30} \cdot T^4.$$

Thus at T above 200 GeV the energy density of the plasma in the universe is about fifty times larger than the energy density of a photon gas with the same temperature. At $T = 200$ GeV $\approx 2 \cdot 10^{15}$ K a cube of one (cm)³ would contain the gigantic energy of $3 \cdot 10^{60}$ eV, while today the energy content of the same

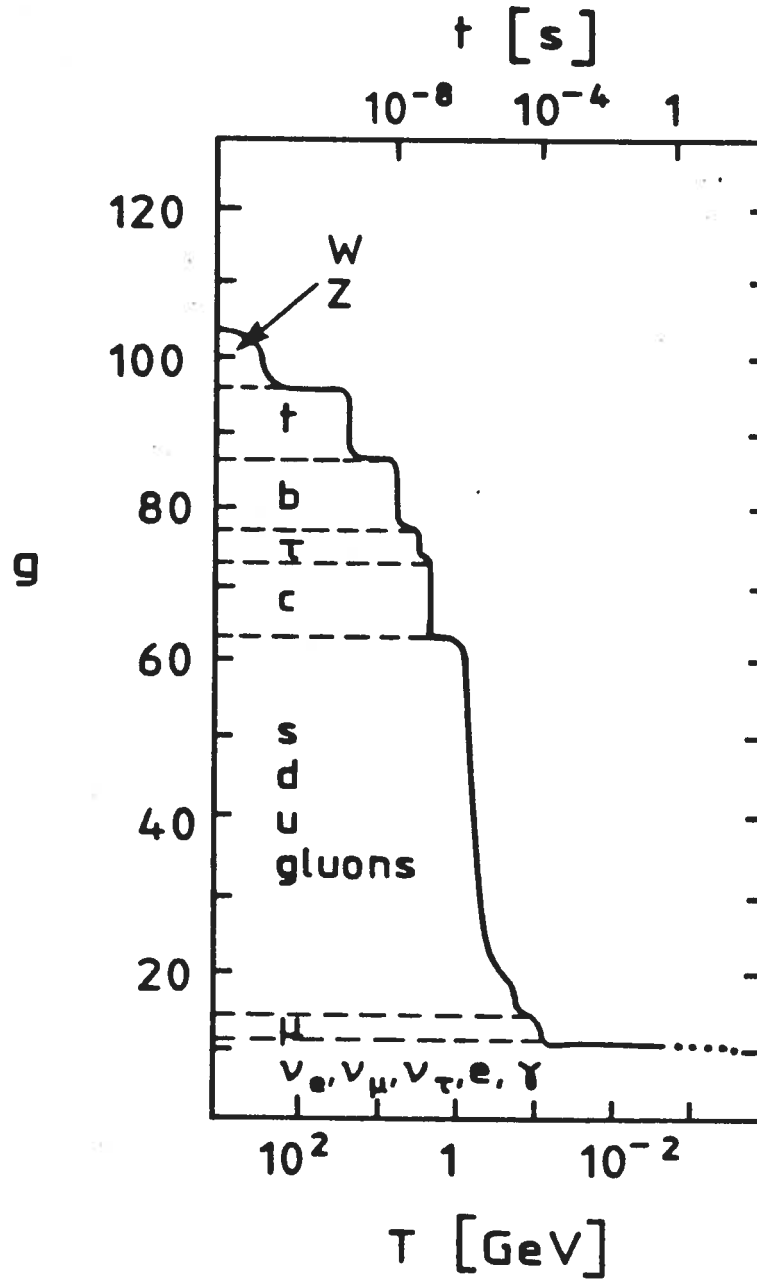


Fig. (5). The effective g -factor increases as the temperature increases. At T above 200 GeV g approaches a value of about 106.

volume due to the 2.7 K radiation is only 0.093 eV. Most of this energy disappears in the course of the expansion of the universe; only a small fraction of it can be found today in the form of normal matter or photons.

As long as the temperature is above 200 GeV, all particles in the hot cosmic plasma move essentially with the speed of light. The evolution of the universe can be described in very simple terms. For example, the expansion of the universe implies that any distance R changes inversely to the temperature:

$$\frac{R}{R_0} = \left(\frac{T}{T_0} \right)^{-1}$$

(R_0, T_0 : reference distance and temperature at a certain time; R, T : corresponding distance and temperature at another time).

The temperature at a certain time t after the Big Bang is given by:

$$t = g^{-1/2} \cdot 2.4 \cdot T^{-2} [\text{MeV}]$$

(g : g -factor; T is expressed in MeV).

In the early universe the time scale and the temperature scale are directly related to each other. Instead of the time t elapsed since the Big Bang one can just as well use the temperature which, of course, diverges as $t \rightarrow 0$.

Thus far we have assumed that we can extrapolate our knowledge about particle physics safely to energies much above 200 GeV. The question arises whether this extrapolation is correct even at energies close to the Planck energy of the order of 10^{19} GeV, i.e. the energy at which the uncertainty relations of quantum theory start to affect the structure of space and time. Once this energy is

passed, all theoretical conceptions of particle physics and of General Relativity break down, and nobody knows at present which new conceptions are needed in order to extrapolate the cosmic evolution beyond the critical temperature of 10^{19} GeV. Another question is whether something else happens before we approach the Planck energy. As discussed in the following chapter, a new development may be expected once the temperature in the universe approaches the scale of 10^{15} GeV.

6. The Grand Unification of Forces

In the standard model of particle physics there exist two different sets of matter particles: leptons and quarks. The distinction between them is provided by the chromodynamic interaction: quarks are affected by it, leptons are ignored. Perhaps the difference between these two different types of particles is not a very fundamental one, but caused by a spontaneous symmetry breaking, just like the distinction between electrons and neutrinos, which is generated as the electroweak symmetry is broken. Suppose there exists a large symmetry, which is able to unite the leptons and quarks and all the observed interactions (except gravity).

The simplest examples of such symmetries are associated with the symmetry groups $SU(5)$ and $SO(10)$. In such a "grand unified theory" the electroweak and chromodynamic interactions are related: they are different manifestations of the same underlying basic force. Furthermore the leptons and quarks are relatives: they are different manifestations of one underlying basic object. The observed differences between the electroweak and chromodynamic interactions and between the leptons and quarks are caused by a spontaneous breaking of the symmetry.

The energy scale associated with this symmetry breaking turns out to be very large (of the order of 10^{15} GeV). As a result the differences between the leptons and quarks, which are essentially nonexistent at energies above 10^{15} GeV have much space to grow and are substantial at low energies (~ 1 GeV).

A particularly intriguing feature of such a scheme of unification is the fact that the proton is not stable, but will decay, e.g. into a positron and two photons. Thus far this decay, implying, if it exists, the instability of the nuclear matter in the universe, has not been observed. The expected lifetime of the proton is model dependent and varies between 10^{28} and 10^{33} years. (The present experimental limits imply that the lifetime of the proton is larger than about 10^{30} years).

Several aspects of the ideas of a "grand unification" of all basic forces in nature are worth mentioning explicitly.

a) If protons are unstable, all nuclear matter in the universe will eventually decay, and only leptonic matter will survive. On the other hand one has found a way to understand why there is nuclear matter at all in the universe today. This matter was created spontaneously as a result of the same unsymmetry of the vacuum, which is observed today in the different properties of the leptons and quarks. If this "grand symmetry" would be unbroken, leptons and quarks would be identical objects, and there would be no need to differentiate between leptonic and nuclear matter. In fact, the leptons and quarks would be bound together by a strong force, which would be an extension of the chromodynamic interaction. Hydrogen atoms would not exist, an electron and three quarks would form a hadron - like object composed of four particles.

b) I should like to stress that in our real world both the symmetry between leptons and quarks and the breaking of the symmetry are essential. The symmetry is needed for the generation of nuclear matter, while the symmetry breaking is important in order to arrive at all the complexities which is observed today in the universe.

It is interesting to note that broken symmetries in physics are easier to recognize than unbroken symmetries. For example, the isospin symmetry of the atomic nuclei, i.e. the fact that the nuclear forces do not discriminate between the proton and the neutron, was realized by Heisenberg and Iwanenko immediately after the discovery of the neutron. However it took until 1970 to discover the unbroken color symmetry of the quarks, which is responsible for the stability of the nuclear matter and for the nuclear forces, and whose indirect effects have already been observed by Rutherford at the beginning of our century.

c) Extrapolating our present knowledge about the interactions of the leptons and quarks, one concludes that the "grand symmetry" can become relevant only at an extremely large energy (of the order of 10^{15} GeV). Thus many cosmological details of the universe, e.g. the density of nuclear matter, are tight up with the evolution of the universe immediately after the Big Bang (less than 10^{-30} s after the Big Bang), when the typical energies of the quarks and leptons were of the order of 10^{15} GeV or more.

d) The generation of nuclear matter, i.e. the generations of the quarks, present in the universe today, happens according to specific models of grand unification about 10^{-36} s after the Big Bang as the temperature drops below

10^{15} GeV ($\sim 10^{28}$ K). This process is intimately linked to the decay of nuclear matter in the distant future. Matter exists because it will decay. The same physical processes which led to the generation of matter will be responsible for its decay.

7. The Inflation of the Universe

We have emphasized in the two previous chapters that the details of the physics of elementary particles are very essential for the dynamical evolution of the universe immediately after the Big Bang. Essential features of the structure of the universe observed today depend on the aspects of particle physics, e.g. the density of nuclear matter which is concentrated today in the galaxies.

If we take the standard model of particle physics for granted and if we furthermore assume that the grand unification of forces sets in at an energy of the order of 10^{15} GeV, many aspects of the evolution of the universe are fixed. However the universe we observe today displays a number of features which cannot be explained that way. Especially one faces the following problems:

a) The Problem of Flatness

The observed expansion of the universe is such that it would come to a stop in the distant future, followed by a universal contraction if the matter density in the universe is above a certain critical density ρ_{crit} which is about 10^{-29} g cm⁻³ (see chapter 3).

The observed matter density ρ is about 10^{-30} g cm⁻³, about one tenth of the

critical density. This suggests that the universe will expand forever.

It is interesting to consider the ratio $\Omega = \rho/\rho_{\text{crit}}$ and follow its value throughout the evolution of the universe. This ratio, which today is of the order of 0.1, is an essential number. If Ω would be above one, the universe would be finite ("closed universe"). The case $\Omega < 1$ implies that the universe is infinite ("open universe").

The case $\Omega = 1$ is of special interest. It corresponds to a universe which is described by a flat ("Euclidean") three-dimensional space which expands as $t^{2/3}$.

It is rather peculiar that the value of Ω observed today is a number not far from one. In the past it must have been much closer to one. For example, when the temperature T was of the order of 1 MeV, Ω could not have been different from one by more than one part in 10^{15} . At $T \approx 10^{19}$ GeV ($\approx 10^{32}$ K) Ω could not have differed from one by more than one part in 10^{63} . Any larger deviation from one would immediately imply that the value of Ω today differs from one by many orders of magnitude. Thus in the past the ratio Ω in the universe must have been tuned precisely to specific values very close to one. Otherwise the universe would be completely different from the one we observe today. For example, if at $T \sim 1$ MeV the difference $1-\Omega$ would have been slightly larger than 10^{-15} , the present value of Ω would not be of order one, but much less - the universe would be nearly empty, and the matter density would be too small in order to form galaxies.

If on the other hand at $T \approx 1$ MeV the ratio Ω would be slightly above one (more than 10^{-15}), the energy density in the universe would be so large that the

universe would collapse before cooling down to a sufficiently low temperature such that extended structures like galaxies could form.

These considerations show that we live in a very special universe, which is old enough such that large structures could be formed during the evolution, and has enough matter and energy such that these structures could actually have been formed.

There is a special case which does not require any fine-tuning of Ω , the case $\Omega = 1$. If the cosmic energy density is at some specific time equal to the critical one, the theory of General Relativity implies that the energy density is equal to the critical one throughout the cosmic evolution. Obviously this would be a splendid explanation why the present value of Ω is of the order of one: it is precisely one. In view of the uncertainties in the determination of the cosmic matter density it is not excluded that the mass density in our universe is exactly equal to the critical one. However most astrophysicists agree that this cannot be achieved with nuclear matter. Further contributions to the mass density due to light neutral weakly interacting particles (e.g. neutrinos) are needed.

b) The Horizon Problem

The 2.7 K radiation is observed to be homogenous and isotropic. This is difficult to understand unless the radiation observed today from different regions of the sky originated from sources which were in causal contact. However in the standard theory of the evolution of the universe there is a maximal distance ("horizon distance") that a light signal could have propagated since the Big Bang. The 2.7 K radiation observed today from opposite directions in the sky originated from sources which were separated by more than 90 times the horizon distance. No communication between these regions existed, and it is difficult to understand why the physical conditions in those different regions of the universe were essentially identical, unless one supposes the uniformity of the universe as a basic principle for its evolution.

The reason for the appearance of the horizon problem lies in the fact that the size of a causally connected region of space, i.e. a region in which light signal could have been exchanged, grows with the speed of light, while the expansion of the universe is much slower. Two regions of space, which are connected by light signals just now, were never in causal contact before.

As an example we consider the region, which is causally connected with a specific point P at $t = 1$ s. This region is a sphere around P with the radius $d = 1$ light second. According to the law of the expansion of the universe at time $t = 0.01$ s this sphere had a radius $d/10$. On the other hand only a small fraction of all the points in this sphere were causally connected with P, namely all points lying inside the sphere around P with a radius of $d / 100$.

The universe which can be seen today, can be divided into 10^5 regions which were causally connected at the time of decoupling. The typical size of one of those regions corresponds to an angular width of 0.5° on the sky.

Before entering a more specific discussion, I should like to point out that both the flatness and the horizon problem would be solved by one intriguing mechanism. Suppose immediately after the Big Bang the universe went through a short period of a very rapid expansion ("inflation"), which is much faster than the expansion expected in the standard cosmological theory, where a certain distance R grows like the square root of the time t : $R \sim \sqrt{t}$. For example, such a rapid expansion would be realized if R grows for a while like an exponential of t : $R \sim e^{\text{const.} \cdot t}$. Such an exponential growth could inflate the universe by a very large factor, say 10^{50} . In this case the observed universe would evolve from a region much smaller than the corresponding region in the standard theory, whose size would have a diameter which is less than the horizon distance. The sources of the 2.7 K radiation arriving today from different directions were originally in causal contact and had time to establish a thermal equilibrium with a common temperature.

Equally elegant is the solution of the flatness problem. Suppose the universe had a very complicated structure before the time of inflation. The ratio Ω could have had any value and might even have depended on the location.

After the inflation the observed universe originating from a tiny part of the original universe would be flat to a very good approximation. This is similar to the situation on earth. Although the surface of the earth, viewed from a large distance, displays a complicated structure involving mountain chains, steep valleys etc., a small fraction of the surface, e.g. one square meter, can be regarded to a very good approximation as a flat, two-dimensional space. The inflation drives the ratio Ω to one with an extremely good precision, independent of the value of Ω before the inflation.

After this discussion the question arises whether there exists a physical mechanism to cause the inflation of the universe. Such a mechanism does exist; it is related to the breaking of the lepton - quark symmetry.

Just like the breaking of the symmetry of the electroweak interactions, which causes the masses of the weak bosons to be nonzero, this symmetry breakdown is caused by a special field, the so-called Higgs field φ . This field φ is present everywhere in space; it is an essential ingredient of what is normally called the vacuum. In the vacuum the φ -field has a certain value, which sets the scale for the breaking of the grand symmetry of leptons and quarks.

The value of the φ -field in the vacuum is associated with a certain contribution to the energy-momentum tensor, which in General Relativity determines the structure of the space-time-continuum. We know that in the observed universe this contribution must be essentially zero, since the empty space does not have any energy-momentum properties.

Nevertheless it is useful to contemplate for a moment what would happen if such a contribution of the vacuum would be present. This contribution to the energy density would have the interesting property that it is not diluted by the expansion of the universe, like the cosmic mass density. The more the universe expands, the more energy is present in a specific comoving volume. As a result the expansion of the universe accelerates exponentially. This is precisely the kind of inflation needed to explain the cosmological puzzles mentioned above.

It is rather mysterious why the energy density of the φ -field in our observed vacuum is zero. This feature of the vacuum is not yet understood, and one is required to set the energy density of the vacuum to zero by hand. Since one expects that this energy density is defined only up to a constant, this is possible. However this "tuning" of the energy density can only be made once, for a specific value of the φ -field. As soon as the φ -field takes another value, it would contribute to the energy density.

The value of the φ -field depends on the temperature. Of course, the universe today is very cold. Its temperature is essentially zero ($T \sim 2.7$ K). However immediately after the Big Bang the universe was extremely hot. If the temperature exceeds a certain critical value T_c , the value of the φ -field will disappear. This corresponds to a restoration of the quark-lepton symmetry, which was broken by the value of the φ -field in the vacuum.

This restoration of the symmetry is similar to the melting of ice. Ice is a state of matter in which the symmetry of the space with respect to rotations in all directions is broken by the ice crystals, which single out specific directions. This symmetry breaking disappears if the ice is melted; water is isotropic.

If water is cooled down, another interesting phenomenon can appear: supercooling. Water does not automatically freeze if the temperature is lowered below the freezing point. It can be supercooled to more than 20 degrees below the freezing point. Afterwards a slight breaking of the symmetry will cause a very rapid freezing of the system, and a large amount of internal energy (latent heat) is released.

In typical models of grand unified theories one finds for the critical temperature $T_c \sim 10^{14}$ GeV. Once the vacuum value of the ϕ -field disappears, the ϕ -field will contribute to the energy density. Especially this is expected to happen shortly after the Big Bang, when the universe cools below the critical temperature. There will be a supercooling effect. The value of the ϕ -field stays zero, and the lepton- quark symmetry remains unbroken. We arrive at a very strange state of matter, where essentially all the energy density is concentrated in the ϕ -field. The universe expands exponentially. This expansion cannot proceed for a long period. Typical estimates give that the inflationary era continued for about 10^{-32} s. During this time the universe expands by a factor of about 10^{50} . Afterwards the transition to the "normal" vacuum with a non-zero value of the ϕ -field (and zero contribution of the ϕ -field to the energy density) is made. The energy density stored by the ϕ -field is released, which leads to a reheating of the universe and to a tremendous production of particles. Afterwards the system cools down and expands as in the standard cosmological model.

Specific calculations show that the transition from the "supercooled" vacuum to the "normal" vacuum proceeds by the formation of bubbles of the "normal" vacuum in the "supercooled" vacuum. One is reminded of the formation of vapor bubbles once water approaches the boiling point. This bubble formation would

cause large scale inhomogeneities in the observed universe which are not observed. If one chooses a very specific form of the φ -interaction, these problems can be avoided. In this scenario ("new inflationary universe") the whole observable universe is arranged to be within one bubble. A possible evolution of the universe would be as follows. At the start the universe was extremely hot. The temperature exceeded the critical temperature T_c . Thermal fluctuations would drive the value of the φ -field to zero. The quark-lepton symmetry is unbroken, and the inflation of space sets in. As the temperature decreases, the system undergoes a phase transition. The rate of this transition is supposed to be very slow compared to the cooling rate, and one would have a substantial supercooling effect. Thus a certain part of the universe would have essentially zero temperature, with the value of the φ -field near zero. A rapid inflation results, during which a sphere with a radius of 10^{-24} cm could easily expand to a region of the size of a baseball ($r \sim 10$ cm). Afterwards the symmetry breaking sets in, and a very dense plasma of particles is produced. The "baseball" expands according to the standard model to a region as large as the observed universe.

The idea of the inflationary universe is certainly very attractive. Various cosmological puzzles, in particular the flatness and horizon problems, are solved very elegantly. However one should differentiate between the idea of an early inflation and the ideas concerning the physical mechanism causing the inflation. The latter based on the idea of the phase transition between two different vacua, must be regarded as highly speculative. Thus far it is unknown how the masses of particles are generated. The mechanism of spontaneous symmetry breaking involving an elementary field has not yet been confirmed by experiment to be the correct mechanism. A more profound understanding of the matter can only come from progress

in particle physics (especially understanding the mechanism of mass generation) and in cosmology (especially understanding the interplay between space, time and matter once the quantum effects of gravity become relevant).

8. Conclusions and Outlook

Physical cosmology, once a special field of research in gravity theory and in astrophysics, has become in the recent years a union of particle physics, nuclear physics and gravity theory. Today it is clear that important concepts in particle physics, e.g. the mechanism of mass generation and of spontaneous symmetry breaking, must have been of great importance in the early evolution of the universe. Man-made accelerators probe the structure of matter down to distances of the order of $(100 \text{ GeV})^{-1} \sim 10^{-16} \text{ cm}$. In cosmology this energy scale corresponds to a time scale of order 10^{-11} s . Thus in principle we "understand" the cosmic evolution since 10^{-11} s after the Big Bang. Further extrapolations can only be made by relying on theoretical models which allow us to go beyond the present experimental limit of $\sim 100 \text{ GeV}$ on the energy scale. As I discussed, the prospects are good that the theoretical extrapolations are not totally wrong. The consistent picture of the cosmic evolution emerging slowly may, in fact, be considered as a hint that one is on the right track.

Especially the very early phase of the cosmic evolution, the phase of a very rapid inflation, is, of course, very poorly understood. Nevertheless it offers for the first time a possibility to understand the creation of the universe. The energy and matter of the observed universe would be relics of the early inflation, and it is understood how they emerged, together with space and time, out of nothing. During the inflationary period none of the conservation laws

(energy-momentum conservation, baryon number conservation etc.) were valid. Those laws are valid approximately only in the cold universe today; they are results of the cosmic evolution, not its boundary conditions. Viewed from this point of view, a creation of the universe out of nothing, which seems to contradict common beliefs, dating back to the Greek philosophers like Parmenides, becomes a real possibility.

The inflationary scenario offers not only a possibility to understand why our universe is as large and as smooth as it appears to be, but may eventually also explain the universality of the laws of physics throughout the universe. Our standard model of cosmology does not explain, why the laws of physics, e.g. the laws of electromagnetism or the finestructure constant α , are the same on earth and at a distant quasar, separated from us greatly in space and time. Perhaps immediately after the Big Bang the universe was in a highly chaotic state. The universality of the physical laws observed today may be the result of a rapid inflation smoothing out not only the geometry of space, but also freezing out the physical laws and the constants of nature in a large volume of space. The ultimate goal of physics is not only to understand the structure of the universe in terms of a number of physical laws and a few parameters, but to understand the origin of the physical laws themselves. Whether this can be done in a consistent and scientific way, without relying heavily on pseudo-scientific principles like the anthropic principle, is totally unknown. The field of cosmology is wide open for absorbing new ideas. It provides an opportunity and stimulates to do fundamental new work; it is challenge for the young.

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