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THE GENESIS OF THE CONCEPT OF ENTROPY

by

Michael J. Higltsberger
Professor of Physics
Institute of Experimental Physics
University of Vienna
Vienna, AUSTRIA

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Michael J. Higltsberger

Institute of Experimental Physics

University of Vienna

1. Historical Aspects

The laws of mechanics were formulated by HUYGENS (1629-1695) and LEIBNITZ (1646-1716). These laws were reduced to the Newtonian axioms (1687: *Philosophiae naturalis principia mathematica*) in the form that the sum of kinetic energy plus potential energy remains constant. RANKINE (1820-1872) introduced the concept of energy which was derived from the Greek words $\epsilon\rho\gamma\omega\nu$ or $\epsilon\nu\epsilon\rho\gamma\epsilon\iota\alpha$ to distinguish energy from force. H. HELMHOLTZ (1821-1894) extended the mechanical energy terms to all then known forms of energy. HELMHOLTZ did this in his famous 1847 lecture at the Physical Society in Berlin. Sadi CARNOT, however, discussed this subject already as early as 1824 in Magdeburg, when he lectured about the "motion producing force" of heat and fire and about the development of proper machineries to do this. At that time James WATT's (1736-1819) steam engine operated by thermal energy was already known. The laws of thermodynamics evolved and were governed by the principles of conservation of energy and the Carnot cycle.

Hence the terms of energy and entropy - the latter is the second law of thermodynamics - are the main issues of thermodynamics. The first law of thermodynamics simply states that mechanical energy can be fully converted into thermal energy, while, according to the formulation of Rudolf CLAUSIUS in 1850, the second law of thermodynamics determines that heat cannot by itself go from a cooler to a hotter body. The third law finally expresses the impossibility to achieve $T = 0$ by any physical process.

J. Robert MAYER published his ideas on the first law of thermodynamics in 1841 while CLAUSIUS used for the first time the word "entropy", which again originates from the old Greek words τροπή and εντροπή, for a reversible or irreversible process. He called it an equivalence value of conversion. The second law of thermodynamics was then defined in the form: The entropy of an isolated system cannot decrease but must increase in irreversible processes; in reversible processes the entropy can utmostly be kept constant.

According to Lord KELVIN (1824-1907) the Carnot principle or the second law of thermodynamics mean "degradation energy". He divided energy in high-grade energy for mechanical and electrical energy, medium-grade energy for chemical energy and low-grade energy for heat. Of course, the total quantity of energy remains constant also for Lord KELVIN. He said reversible processes are

processes of high-grade energy, while at chemical reactions chemical energy retains its grade. Heat exchange and other irreversible processes give rise to a loss of grade. The grade of energy can also be defined as corresponding to the negative entropy, called "negentropy" - S . Since the second principle states that the entropy S must always increase or remain constant, for the term negentropy the result is a decrease or a constant value at the best.

Early in our century the energy concept was further extended by A. EINSTEIN's mass energy equivalence relation $E = m \cdot c^2$.

The range of energies involved in physical processes extend over nearly sixty orders of magnitude. When the energy is measured in the SI system, where $1 \text{ Ws} = 1 \text{ Nm} = 1 \text{ J}$, a supernova explosion has an energy content of 10^{42} Ws, while a typical chemical bond is of the order of 10^{-18} Ws. The stored rotational energy of our earth is approximately 10^{30} Ws, several hundred millions more than all fossile energy fuel reserves of the world. Comparable to the rotational energy of the earth is only the fusion energy potential.

In 1872 Ludwig BOLTZMANN introduced a statistical approach to interpret the second law of thermodynamics. He gave this law a meaning of probability. The direction of entropy thus was evaluated as an order of lower or higher probability. His

well-known formula $S = k \cdot \log W$ is engraved on his tombstone at the Central Cemetery of Vienna. According to BOLTZMANN entropy is a function of state. It was his genial idea to recognize that the scheme of mechanics is not restricted to observable items like energy, momentum or others. He realized that also functional states exist which must be calculated. The Boltzmann formula $S = k \cdot \log W$ was in the explicit form not written by BOLTZMANN himself but is due to Max PLANCK (1901). It is the relative probability or more precisely the number of microstates of what BOLTZMANN called "Komplexionen", which are compatible with the macroscopic situation. It is probably not widely known that also the Boltzmann constant $k = R/N_A$ was introduced by PLANCK, who originally was sceptical to BOLTZMANN's approach, but later on became convinced and was fascinated by the new ideas.

Up to now in the majority of publications seldom the concept of entropy was unclear, however, through the many varieties in which the second and third laws of thermodynamics were interpreted, misunderstandings resulted. Despite of the fact that the terms of energy and entropy were introduced at about the same time and both terms were equally important for the formulation of the laws of thermodynamics, many people are familiar with energy but less with entropy.

In a simple analogy entropy could be attributed to the activities of a managing director of a large production firm, whose job it is to determine the kind of production and the route to produce it, while energy in such a picture would play the role of a book-keeper, who is in charge of the balance sheet in which money (energy) coming in and going out is recorded.

2. Experimental facts for the concept of entropy

If two gases, liquids or solid bodies of different temperatures are brought into contact, a temperature change is observed in both. Whenever the temperature change becomes zero, it is said, they are in thermal equilibrium. The experimental evidence is that the body with the higher temperature experiences a temperature reduction, while the body with the lower temperature experiences a temperature gain. The technical term for this behaviour is called the "transitivity of temperature". Thus temperature is a value of state. FOWLER proposed 30 years after NERNST's and PLANCK's works on the third law of thermodynamics to call the functional character of temperature the law of thermodynamics with the number zero.

The first law of thermodynamics can be written in the form

$$dU = dQ + dW. \quad (2.1)$$

Writing the law in this form dU is the change of internal energy of the system in any interval of time, dQ is the heat received by the system and dW is the mechanical energy or work received by the system from outside. All three differentials refer to the same interval of time. The equation defines dU assuming that dQ and dW can be obtained independently by suitable instrumental operations. Neither dW nor dQ taken separately are perfect differentials, except if either dQ or dW is zero. However their sum and thus dU is a perfect differential in the variables that fix the state of the system. The "concept of state" itself is assumed to be already formed and the variables which determine the state must be known by independent experiment before thermodynamics is applied. The "state of a system" is therefore the complex of all its measurable properties. A state is fixed by its appropriate variables in the sense that whenever the variables resume their former values, the body is again in this former state as determined by all its measurable properties. From the fact that dU is a perfect differential, the conservation property of energy which is change of energy, follows by conventional lines of argument.

Thermal energy dQ coming from outside implies that there is a flow of thermal energy. A similar argument holds for mechanical energy and there must exist a flow vector. The last statement is easily understood, if the energy is the energy of the electro-

magnetic field. In that case the flow vector is the Poynting vector. The first law of thermodynamics began as a formal equation to define dU . It ends as an equation in which each term has independent instrumental significance and as such it evolved from a definition to a statement of a law of nature.

The second law of thermodynamics was formulated in differential form as

$$dS = dQ/T. \quad (2.2)$$

dS stands for the change of entropy of the system in question for any reversible process during which the heat amount dQ is absorbed. Again dS is an equation of definition. But as in the case of the first law of thermodynamics the equation is supplemented by the statement that dS so defined is a perfect differential in the state variables of the system. Therefore it follows that the total increment of entropy obtained by integration passing from an initial state to a final state is independent of the details of the process and depends only on both the initial and the final states. As each state may be assigned a single entropy number, the difference of entropy between two states is found by taking the difference of corresponding entropy numbers. Considering any quasi-static process which brings the system from one state to another leads to

$$\int_1^2 dS = \int_1^2 \frac{dQ}{T} \quad \text{or} \quad S_2 - S_1 = \int_1^2 \frac{dQ}{T} \quad (2.3)$$

assuming that the process is at all times close to an equilibrium situation.

Like energy also entropy must be accorded full localization in space, because the system can be divided into subsystems in any arbitrary way and the second law applied to every arbitrary element. For this reason the existence of fluxes of entropy account for the changes of localized entropy.

Heat flow is clearly an irreversible process. If a quantity of heat Q passes by conduction from a hotter reservoir at the temperature T_2 to a colder reservoir T_1 , the loss of entropy of the hot reservoir Q/T_2 and the gain of entropy of the cold reservoir Q/T_1 results in an entropy gain

$$S = Q/T_1 - Q/T_2. \quad (2.4)$$

When the entropy of the cold reservoir increases, it is because entropy is flowing into it. The entropy of the hot reservoir decreases, because entropy is flowing out of it. Since T_2 is larger than T_1 , equation (2.4) demands that entropy is being created. This example convincingly shows that a continuous steady

irreversible process leads to a continual entropy increase. Therefore the general formulation for entropy including both reversible and irreversible processes is

$$dS \geq dQ/T. \quad (2.5)$$

The first law of thermodynamics determines the conservation of energy while the second law of thermodynamics in the above formulation shows that entropy is only conserved in reversible processes, but is not conserved in irreversible processes.

From the fact that

$$dQ = c \cdot dT, \quad (2.6)$$

where c is the specific heat, the third law of thermodynamics is derived, since for T going to zero also the specific heat c would tend to zero and would require an infinite entropy difference between the state at absolute zero and the state at higher temperatures.

In any closed system, where spontaneously irreversible processes take place and no heat is taken in from the outside, the entropy increases. HELMHOLTZ argued, if

$$dS \geq dQ/T = (dU+pdV)/T \quad (2.7)$$

than one can also write

$$dU + pdV - TdS \leq 0. \quad (2.8)$$

If the system is held at constant volume which requires $dV = 0$, equation (2.8) reduces to

$$d(U-TS) \leq 0. \quad (2.9)$$

$(U-TS)$ is called the Helmholtz free energy. On the other hand for constant pressure ($dp=0$) one gets the expression $(U+pV-TS)$ which was named Gibbs free energy.

The Helmholtz and Gibbs relations are important in determining equilibria conditions in chemical reactions and in equilibria between phases.

The equilibrium conditions of thermodynamic systems can be treated in analogy to the stability criterion in mechanics. A stable equilibrium exists in a thermodynamic system and returns to the original equilibrium conditions without being forced from the outside. A thermodynamic system is indifferent, if no changes in the equilibrium conditions take place. A thermodynamic system is unstable, if the system deviates from the original equilibrium conditions.

A convincing experiment for a typical irreversible process is the following: A stone falls in a gravitational field transforming his potential energy into kinetic energy, finally hitting the surface of the earth and changing the kinetic energy into thermal energy, deformation energy and energy of sound. It is evident that this process of events is irreversible. Also nuclear fission is a striking example where kinetic energy of the fission products is transformed into heat. But using the thermal energy produced does not lead to fuse the fission products to the uranium nucleus again.

Elaborating further the discussion, let us assume, two systems 1 and 2 are in contact and capable of exchanging work and heat. The temperatures, work outputs, heat inputs and entropy inputs are:

$$T_1, T_2; W_1, W_2; Q_1, Q_2 \text{ and } S_1, S_2 .$$

The first law requires $W_1 - Q_1 + W_2 - Q_2 = 0$

and the second law $S_1 + S_2 \geq 0 .$

A. Irreversible transformation:

$$\text{If } T_2 > T_1,$$

heat flows from the hot to the cold system with the result

$$W_1 = 0 , W_2 = 0 \quad \text{which means no work output;}$$

$$Q_2 = - Q_1 ; \quad Q_1 > 0 \quad \text{and}$$

$$S_2 + S_1 = Q_1(1/T_1 - 1/T_2) > 0 .$$

B. Friction:

When $T_1 = T_2 = T$, both systems are at the same temperature and it follows

$$W_2 > 0, \quad Q_2 = 0, \quad W_1 = 0, \quad Q_1 > 0.$$

Work is done by system 2 and heat is produced in system 1 and

$$S_2 + S_1 = Q_1/T = W_2/T > 0.$$

When heat passes into or out of a body, the quotient thermal energy divided by the absolute temperature of transition is the term entropy. The dimension of entropy is $\text{J}\cdot\text{K}^{-1}$. Heat capacity has the same dimension.

An entropy change can occur also at the same temperature, f.i. when a solid sample is melted. Computing the entropy change of 1 kg ice which is melted and converted to water at 273.16 K (0°C) leads to

$$S_2 - S_1 = Q/T = 335 \cdot 10^3 \text{ J} / 273.16 \text{ K} = 1226 \text{ J}\cdot\text{K}^{-1}.$$

Q is the heat required to transform ice at the same temperature into water and is called heat of fusion or latent heat.

3. Entropy of living systems

It is somehow surprising that the temperature window for all living systems lies within a range of approximately 100 K. At about 300 K human life exists on earth. In contrast the surface temperature of the sun is roughly 5000 K and the fusion temperature inside the sun is of the order of 2 to $3 \cdot 10^7$ K.

In our physical world natural cyclic processes obeying the first and the second law of thermodynamics use also roughly 100 K temperature difference. Incoming radiation from the sun is partly reflected and partly re-irradiated to the universe. But the rest of the energy is used for a cyclic process such as the evaporation of water from the sea into the atmosphere, then condensing in the colder atmosphere and finally coming down from the sky in form of water droplets and rain. In nature most of spontaneously started processes are irreversible.

Organic life is determined and dependent on the laws of thermodynamics. These laws have a range of experience. In quantum biology the energy of a single cell is very small and therefore the effect of the wave aspect is dominant over the corpuscular aspects. The biological energy equation is of the kind

$$\Delta E_{\text{biol.}} = \hbar \cdot 1/\tau \quad (3.1)$$

with τ being the duration of a cell working cycle or

$$\frac{\hbar}{\tau} = k \Delta T. \quad (3.2)$$

In this biothermodynamical law besides the working cycle τ and the temperature difference ΔT the Planck constant \hbar and the Boltzmann constant k are intermingled. Rewriting the equation in the form

$$\tau \Delta T = \frac{\hbar}{k} = \text{const.} \quad (3.3)$$

shows the variability of life. For living systems ΔT has a very limited range and therefore τ can only vary inversely in a similar range. Another conclusion of the equation is the requirement of energy for bringing order in human systems. If energy is not supplied anymore from the outside, the thermodynamic processes in a living system lead to an entropy increase.

In 1943 Erwin SCHRÖDINGER asked the question: What is life? He answered this question by saying: A piece of matter is alive, if it is continuously "doing something", if this matter is moving and is in a material exchange with its surroundings. Teilhard de CHARDIN said: Life is a challenge to entropy.

A state which is difficult to distinguish is low-ordered. The lowest organization of biological systems means death. In contrast an high-ordered state means life or at least supporting life.

4. Entropy in quantum physics

HEISENBERG showed in 1926 that the phase-space (position - velocity) is composed of cubicles which are quantized.

The product $\Delta x \cdot \Delta v_x = h/m$ relates the Heisenberg uncertainty relation with the Planck constant

$$h = 6.6 \cdot 10^{-34} \text{ W s}^2.$$

In how many ways can N molecules of a monoatomic gas be distributed over the cells of phase space, each with the volume $(h/m)^3$ in order that their total energy for one mole is equivalent to the macroscopic value $3RT/2$?

The answer is

$$P = \left\{ \frac{(2\pi emkT)^{3/2} V}{h^3} \right\}^N \quad (4.1)$$

with e the base of the natural logarithm and

V the volume occupied by the gas at the temperature T .

Calorimetric entropies are obtained by measuring the specific heat of a substance together with its heat of fusion and boiling. To compare these values with spectroscopic-calculated entropies, it is necessary to derive out of the probability number P the entropy value. This is best done by assuming that at low

temperatures the gas becomes solid and changes to a crystalline structure. The crystalline state is characterized by regular crystal lattices. Since N molecules occupy N fixed places, the first molecule has N places to choose from, the second $N-1$ and so on. Thus at $T = 0$

$$P_0 = N!. \quad (4.2)$$

Using the Stirling formula which is a good approximation for P_0 gives $\log P_0 = N \log N - 0.4343 N$.

Since $0.4343 = \log e$, equation (4.2) can be written in the form

$$\log P_0 = N \log N - N \log e = N \log N/e. \quad (4.3)$$

The entropy S can then be found as

$$\begin{aligned} S &= \frac{R}{0.4343} (\log P - \log P_0) = \\ &= \frac{R}{0.4343} \log \frac{(2\pi mkT)^{3/2} e^{5/2} V}{h^3 N} \end{aligned} \quad (4.4)$$

with $m = M/N$ and M being the atomic mass number of the substance and the Boltzmann constant $k = R/N$. The final result is

$$S = \frac{R}{0.4343} \log \frac{(2\pi MRT)^{3/2} e^{5/2} V}{h^3 N^4}. \quad (4.5)$$

Evaluating equation (4.5) for $T = 300$ K and the volume at one

atmosphere with $V = 0.02446 \text{ m}^3$, a comparison of spectroscopic (calculated) and calorimetric entropies according to

$$S = (108.765 + 28.715 \log M) \text{ J K}^{-1} \quad (4.6)$$

with M the mass number shows excellent agreement.

Table: Comparison of spectroscopic and calorimetric entropies of monoatomic gases at 300 K under 1 atm in J K^{-1}

Gas	M	Spectroscopic entropy	Calorimetric entropy
Helium	4	126.06	126.18
Neon	20.18	146.23	146.61
Argon	39.94	154.73	154.73
Mercury	200.6	174.87	175.49

When working on his way of formulating the second law Ludwig BOLTZMANN developed the so-called "H-function" or sometimes also named "logarithm-function"

$$H = \iiint F \log F \Delta u \Delta v \Delta w \quad (4.7)$$

with $\Delta u \Delta v \Delta w$ being the volume element of the velocity space. F is determined from the Maxwellian distribution function and $F(uvw) \Delta u \Delta v \Delta w$ turned out to be the number of molecules in the volume element of velocity space. BOLTZMANN proved that H is a pure number by integrating all velocities from $-\infty$ to $+\infty$. He also

showed that H is only dependant on the analytical form of F . For equilibrium conditions

$$\frac{dH}{dt} = 0 \quad (4.8)$$

and for all non-stationary conditions

$$\frac{dH}{dt} < 0 . \quad (4.9)$$

Equations (4.8) and (4.9) are known as BOLTZMANN's H-theorem.

5. Entropy of radiation

The entropy of black body radiation was formulated by Josef STEFAN and Ludwig BOLTZMANN using their famous radiation law in the form of

$$N = \sigma T^4 . \quad (5.1)$$

N is the radiation power density in Wm^{-2} and

$\sigma = 5.6696 \cdot 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ is called the Stefan-Boltzmann constant.

Therefore from

$$dS = \frac{dU + pdV}{T} = \frac{d(NV) + \frac{N}{3} dV}{T}$$

with $N = \sigma T^4$.

dS becomes $dS = 4 \sigma \{VT^2 dT + 1/3 T^3 dV\}$.

Integrating and using the boundary conditions that for

$$T = 0, \quad S = 0,$$

the entropy becomes the form

$$S = (4/3)\sigma T^3 V. \quad (5.2)$$

This relation can also be obtained when using Maxwell's thermodynamic relation $(\delta S/\delta V)_T = (\delta p/\delta T)_V$ and the equation for radiation pressure $p = N/3 = (\sigma T^4)/3$ as can be easily seen. The entropy density of the black-body radiation is

$$\frac{S}{V} = \frac{4}{3} \sigma T^3 \quad (5.3)$$

6. Entropy and cosmology

EINSTEIN's geometrodynamics law and the observation of HUBBLE lead to the "big bang" concept with an expanding universe, but equally to the "black hole" collapse situation. There are three levels of gravitational collapse at

1. the universe
2. the black hole and
3. the fluctuations at the Planck's scale of distances.

The collapse of the universe can be reviewed as a mirror image of the "big bang". Collapse of matters to form the "black hole" can

be reviewed as a collapse of a dense white dwarf core, when the core exceeds the critical masses $1 M$ or $2 M$ (Schwarzschild-mass) at which a neutron star is no longer a possible end-point for collapse. The other possibility is a coalescence one by one of the stars in a galactic nucleus to make a "black hole" of mass between $10^6 M$ to $10^9 M$. A black hole can be defined as a region of space time that cannot communicate with the external universe. The boundary of this region is called the surface of the black hole, or the event horizon. Finally collapse at the Planck's scale of distances is taking place everywhere and all the time in quantum fluctuations. This is called the reprocessing of the universe.

The Planck's units are

$$L' = (\hbar G/c^3)^{1/2} = 1.6 \cdot 10^{-35} \text{ m}$$

$$T' = (\hbar G/c^5)^{1/2} = 5.4 \cdot 10^{-44} \text{ s}$$

$$M' = (\hbar c/G)^{1/2} = 2.2 \cdot 10^{-8} \text{ kg.}$$

These units were chosen by PLANCK because relative to them all constants in any field of physics can be expressed as pure numbers.

The so-called "big numbers" of physics reveal interesting ratios. There are about 10^{80} particles in the universe. The ratio of the

presently known expansion of the universe to the size of the elementary particle is about $10^{26}\text{m}/10^{-14}\text{m} \approx 10^{40}$.

The ratio of electric forces divided by gravitational forces is $e^2/G \cdot mM \approx 10^{40}$.

The size of an elementary particle divided by Planck's length is

$$\frac{e^2/mc^2}{(\hbar G/c^3)^{1/2}} \approx 10^{20} .$$

Finally the ratio of photons in the universe to the number of barions in the universe is about 10^{10} .

If physical constants change by as little as one percent in one way or the other, we would have red or blue stars, in which our sun would not exist anymore and therefore no life would be possible.

In a contribution to the volume for the International Colloquium on Science, Culture and Peace in honour of V.F. WEISSKOPF, Walter THIRRING using a simple argument derived for the entropy of a black hole with the mass of the sun, the number of 10^{76} . His argument was

$$\text{energy} = c \cdot \text{momentum} = c\hbar/\text{wavelength} = c^3\hbar/kM.$$

Since entropy of radiation is

number of particles = total energy/energy per particle =

$$= Mc^2 / (c^3 \hbar / kM) = (kM/c^2)^2 / (k\hbar/c^3) = \text{surface [measured in Planck length} = (k\hbar/c^3)^{1/2}\text{]}.$$

When the surface is 10^6 m^2 and the Planck length is 10^{-35} m , one ends up with the quoted number of 10^{76} .

The application of the concept of entropy to the universe has lead to controversial views extending from "entropy death" over the opinion that other worlds exist, where the entropy constantly decreases, up to the idea of an oscillating cosmos. In this paper only a few typical applications could be dealt with, but there are numerous other applications of the entropy concept which have enriched our physical and philosophical thinking. To conclude with a representative example, the last chapter is devoted to the growing field of the theory of communication.

7. Mathematical analogy of entropy and information

It has been shown that in operations involving information a quantity appears which is the negative of the quantity usually defined as entropy in similar situations. Information can be said is the raw material and consists of a mere collection of data while knowledge requires a certain amount of thinking and an agreement to organize the data by comparison and classification. The scientific theory of information started at about the time when computers where introduced. Information is often called

negative entropy or negentropy for short. What means information? If someone informs his partner that he is going to visit him within the next week at any time at full hours from 10 a.m. to 6 p.m., the possibilities are 7 days times 9 hours = 63-fold. If the visitor announces that he will arrive on Thursday, the possibilities are reduced to 9. If finally the arrival is specified for 11 a.m., the precision of information reaches the lowest possible value of one.

The above example implicitly directs to the question of what is the unit of information. In computer science the binary numerical system has been adopted. All numbers are expressed by sums of power of two. 63 for instance is the sum of

$$63 = 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0,$$

written in the binary code as 1 1 1 1 1 1, while

$$9 = 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 2^0$$

is written in the binary code as 1 0 0 1.

The unit of information distinguishes between two possibilities (P), namely

$$I = c \cdot \log 2 = 1 = \lg_2 2.$$

This unit is called one bit. Furthermore

$$c = 1/\log 2 = 3.322 \text{ and } \log \text{ stands for the logarithm of base } 10.$$

Thus information in general is

$$I = 3.322 (\log P_{\text{before}} - \log P_{\text{after}}). \quad (7.1)$$

If the logarithm to the base 2 is used, then the above equation reads

$$I = \lg_2 P_{\text{before}} - \lg_2 P_{\text{after}}. \quad (7.2)$$

On the foregoing example

$$I_1 = 3.322(\log 63 - \log 9) = 2.807 \text{ bits} \quad \text{and}$$

$$I_2 = 3.322 \cdot \log 9 = 3.17 \text{ bits}.$$

By definition information has the value one when there are only two possibilities to choose from. $\lg_2 N = 1$ if $N = 2$.

If one asks, how many binary digits (bits) requires a pack of 32 different cards, from $32 = 2^n$ it follows: $n = 5$. Thus

$$I = \lg_2 32 = 5 \text{ bits}$$

and consequently 2 packs of cards require 10 bits, because

$$P = P_1 \cdot P_2 \quad \text{with} \quad P_1 = P_2 = 32.$$

$$I = I_1 + I_2; \quad (7.3)$$

this example shows the additivity of information.

Applying the foregoing to the Boltzmann equation leads to

$$\begin{aligned} S &= k \cdot \ln P = 2.303 \cdot k \cdot \log P = \\ &= 2.303 \cdot 1.38 \cdot 10^{-23} \cdot \log P \cdot J \cdot K^{-1} = \\ &= 3.178 \cdot 10^{-23} \log P \cdot J \cdot K^{-1} \end{aligned}$$

and $I_{\text{in bits}} = 3.322 \log P$.

Therefore

$$1 \text{ bit} = 9.567 \cdot 10^{-24} \text{ J} \cdot \text{K}^{-1} \quad \text{or}$$

$$1 \text{ J} \cdot \text{K}^{-1} = 1.045 \cdot 10^{23} \text{ bits.}$$

Entropy is found to be the logarithm of the probability number.

A further example on information can be constructed with the alphabet: The English alphabet consists of 26 letters plus one space between words to make up sentences.

Therefore $I = G \cdot \lg_2 27$ bits.

If 27 symbols would have an equal probability in a sentence of G letters, one would need

$$I = \lg_2 27 = 4.76 \text{ bits per letter.}$$

If p_j is the probability of the j^{th} letter as experimentally determined in an average text, the average information per letter can be written according to SHANNON as

$$i = -K \cdot \sum_{j=1}^{27} p_j \ln p_j \quad (7.4)$$

and

$$I = G \cdot i. \quad (7.5)$$

A telegraphic information is submitted via a combination of dots and dashes - a binary information like 0 and 1. If

$$G = N_0 + N_1 \quad \text{with} \quad p_0 = N_0/G \quad \text{and} \quad p_1 = N_1/G,$$

it follows

$$p_0 + p_1 = 1 \quad \text{and} \quad P = G! / N_0! N_1!$$

$$I = K \ln P = K [\ln G! - \ln N_0! - \ln N_1!]$$

Using the Stirling formula

$$\ln Q! \cong Q (\ln Q - 1)$$

which is a good approximation for large values of Q , we get

$$\begin{aligned} I &= K [\ln G! - \ln N_0! - \ln N_1!] \cong \\ &\cong K [G(\ln G - 1) - N_0(\ln N_0 - 1) - N_1(\ln N_1 - 1)]. \end{aligned}$$

Because of $G = N_0 + N_1$, the above formula is reduced to

$$I \cong K [G \ln G - N_0 \ln N_0 - N_1 \ln N_1];$$

again with $G = N_0 + N_1$ it follows

$$I \cong -K G \left[\frac{N_0}{G} \ln \frac{N_0}{G} + \frac{N_1}{G} \ln \frac{N_1}{G} \right]$$

$$i = I/G = -K [p_0 \ln p_0 + p_1 \ln p_1].$$

Generalization leads to $N_1, N_2, \dots, N_j, \dots, N_M$

$$G = \sum_{j=1}^{j=M} N_j ; \quad \sum_{j=1}^M p_j = 1$$

$$P = G! / \prod_{j=1}^{j=M} N_j!$$

$$I = K \ln P = K [\ln(G!) - \sum_{j=1}^M \ln(N_j!)] \cong$$

$$\cong K [G \ln G - \sum_{j=1}^M N_j \ln N_j]$$

$$\begin{aligned}
 I &\cong - K G \cdot \sum_{j=1}^M (N_j/G) \ln (N_j/G) = \\
 &= - K G \cdot \sum_{j=1}^M p_j \ln p_j .
 \end{aligned}$$

For $G = 10000$ one gets:

$$I = 10000 \lg_2 27 = 47600 \text{ bits.}$$

If a distribution of letters and spaces as experimentally determined is used, one finds because of

$$i = I/G = - K \cdot \sum_{j=1}^M p_j \ln p_j = 4.03 \text{ bits.}$$

Information is the result of a choice of possibilities. However, information is not the basis for a prediction to make another choice. Information is a function of the ratio of the number of possible answers before and after an event. The logarithmic formulation is chosen in order to permit summing up information from independent situations. Using the concept of entropy one could say, a "mirror-like entropy" measures the lack of information.

Most physical problems are incompletely defined. Normally only the values of some microscopic variables are known, but their precise position in space and the velocity of the individual particles contained in a system are not known. Since precise scientific definitions often initially formulated as postulates are needed, scientific "jargons" are employed to cope with it.

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