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THE USE AND ABUSE OF MATHEMATICS: A KANTIAN CRITIQUE

by

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> I maintain however, that in every special doctrine of nature only so much science proper can be found as there is mathematics in it.[1]
>
> And since in every doctrine of nature only so much science proper is to be found as there is a priori cognition in it, a doctrine of nature will contain only so much science proper as there is applied mathematics in it.[2]

These quotations, and others of like sort to be found elsewhere, certainly establish Kant to be a radical exponent of the use of mathematics. Such pronouncements are enough to cause necessarily the heart of every Pythagorean to make a quantum leap of real proportion. As we shall see, there are here elements which will in turn cause necessarily the heart of every Pythagorean--and many another as well--to make a negative quantum leap. Mathematics is indeed a priori, purely rational, entirely certain and integral to all natural science. It is, however, restricted to natural science and it is this restriction which denies mathematice any metaphysical aspirations and which will certainly cause the Pythagoreans to form a negative unity in opposition to Kant.

In the Critique of Pure Reason Kant says that the mathematician, the student of nature and the logician, no matter how advanced in rational knowledge, are still merely artists. He continues, "There is besides, an ideal teacher, who controls them all and uses them as instruments for the advancement of the central aims of human reason. Him alone we ought to call the philosopher."[3] The highest, final aim is "the whole destination of man and the philosophy which

relates to it is called moral philosophy."[4]

So in very brief outline we have the use and abuse of mathematics. Its use is in natural science and as a tool, implement, to facilitate the real end, goal, purpose of human existence which is moral existence. Abuse appears when the instrument attempts to become the teacher, to control, to determine human destiny. This abuse in turn results in moral abuse.

Let us turn now to the why and how of all of this. The place to begin our discussion is with what Kant called his Copernican Revolution in Epistemology.[5] Experience has taught me to warn readers at this point that the language here can be misleading. We are dealing with ontology/metaphysics, with the real and not only with epistemology as such. The entry point is epistemological, die Sache, the subject itself is reality and our knowledge of that reality together with how they relate and why they relate as they do. The purpose is ultimately moral. With that word of caution, let us proceed.

The Copernican revolution Kant is talking about is a reversal of the way the relation between our knowing and what is known is conceived. The usual approach assumes there is a given to be known and somehow or other we get to know it. With this way of conceiving the knowing process there is no way to account for the a priori, universal and necessary character of mathematics and mathematical science. From the rational side there is no reason to believe that our logic

necessarily applies to the world and from the empirical (a posteriori) side there is no way to account for the universal and necessary character which Kant sees as intrinsic to science. Reverse the approach, he says. Rather than our minds adjusting to the known, assume that what can be known is what conforms to the way our minds appropriate what is given to produce what we call experience. To begin with we have the raw data of sensation. These percepts, Kant says, are blind. They do not mean anything; we can make no sense of them as such. To make sense of them the first thing we do is attempt to assign them a where and a when, a space and a time. Where do these come from? Space and time themselves are not perceived. They are not sensations or combinations of sensations. What are they? In Kant's terms they are pure, a priori forms of intuition. Observe that space and time are contributed by our minds. They are the way our minds organize the raw data of sensation and produce individual experiences. They are not concepts. Concepts are thought. The forms of intuition are our mind's way of seeing, so to speak. Time is sequential. It is the form of all experience, internal and external. It is the way our minds organize experience sequentially and is always one series with no two units ever coexisting within it. Space is the form of external experience . It is the direct intuiting of place and is the way our minds organize experience so that form/shape, size and relative position can be established. Perceptions become experiences only when they have been located in the

space-time fremework.

So far we have only discrete, unconnected experiences. We still have no way of telling how much, how intent, how long, how related or whether necessary or only actual. How do we connect together these space-time experiences to produce coherent, intelligible experience? How do we come to possess a larger context of meaning in terms of which experience is coherent and intelligible, known? Here appear Kant's categories of the understanding (Verstand). We are now dealing with concepts, not intuitions. The mind's activity is thinking, not intuiting. What are thought are the possible ways in which space-time experiences can be made definite and can be related. The four types of categories are quantity, quality, relation and modality. Quantity and quality provide for definite identity and therefore for comparison; relation provides for connections among experiences; modality concerns the status of the identity or relation as possible, actual or necessary.

Do we now have science? No. Do we have knowledge? Not yet. For knowledge we need another unity, a higher synthesis than the understanding provides. We need the ideas of reason (Vernunft) which govern how the categories are used. There are only three of these ideas: self, world and God. While no object corresponding to these ideas can ever be given in experience, the ideas are necessary to the possibility of experience. Their role is to provide the structure necessary for placing the categorized space-time experiences in a complete

context. For example, the category of cause, one of the categories of relation, only places every perception in a sequence of causes and effects. To that we need to add the idea that all phenomena, all experience,is in one universal series. The idea of world is that complete, universal idea. Without this idea the possibility of a spontaneous event remains and if one such can be, then there is no certainty that anything is necessarily in causal sequence and all coherent knowledge and therefore all science is lost. The ideas of reason are, then, the large patterns of unity which we must think in order to produce coherent experience, in order to achieve knowledge. The ability to think such ideas, ideas which range beyond all possible experience, shows the power of reason to range beyond the quantifiable.

To all of this I need to add another component, something of tremendous importance to the entire philosophy of Kant, but which I am here only going to mention. There has to be a synthesizing center of all of this. Kant, in the first edition of the Critique of Pure Reason, calls it the Transcendental Unity of Apperception. It is elsewhere referred to as the pure "I think"which is intrinsic to every experience, to all knowing.

Knowledge is exclusively of phenomena. The known world of experience is the given of sensation as it appears to our minds. Percepts without concepts are blind; concepts without percepts are empty.[6] Together they produce experience, phenomena. Knowledge is the synthesis of all these components, a synthesis

produced by our minds. The noumenal, reality as it is in itself, independent of our knowing processes, is entirely and necessarily unknown to us.

We are now in position to elucidate why mathematics is intrinsic to natural science. Natural science is the science of the externally experiencable. This means it is a doctrine of body, Kant says. A doctrine of body can, however, become science only by means of application of mathematics.[7] Why? Because in order to make our external experience coherent we have to bring it within the synthetic unity produced by understanding and reason and mathematical construction is a necessary intermediate step. This is so because a doctrine of body, of external experience, is intrinsically a doctrine of space and time and mathematics is"the construction of concepts by means of the presentation of the object in a priori intuition."[8] Mathematics is the logic of a priori intuition. The specifically relevant a priori intuition is space, the form of all external experience although time is involved at every stage because arithmetic, algebra, geometry--and calculus we should add--involve sequential synthetic connections of units and hence time. Mathematics is the instrument we use to construct possible objects of experience and possible relations among such objects and in terms of such objects to construct, by the same instrument, our questions and organize our experiments to compel nature to answer our questions. The answers appear in mathematical form when given rational expression. By mathematical

procedure we find out what body could be and how body can relate to body. By it we build precise quantity and relation of quantity to quantity. This is what it means to present an object in a priori intuition. By experience we find out which body exists and how it relates to other body.

Our mathematical procedure is one of construction of concepts as distinct from thinking conceptually and judging through ideas. Kant does not mean by this that mathematicians do not think or judge but rather that the foundation of their thinking and judging is pure intution. "The sum of the matter is this: the business of the senses is to intuit, that of the understanding is to think."[9] But understanding thinks, in natural science, in terms of what mathematics constructs. Without this mathematical construction the understanding is unable to connect with the data of sensation. How mathematical construction provides the necessary mediation is our next concern.

In all knowledge of the world of experience we begin with the raw data of sensation or at least so it would seem to naive, uncritical belief. This is not exactly true, Kant says, though sensations are necessary to the beginnings of knowing. "Without sensibility objects would not be given to us, without understanding they would not be thought by us. Thoughts without contents are empty, intuitions without concepts are blind. Therefore it is equally necessary to make our concepts sensuous, i.e., to add to them their object in intuition , as to make our

intuitions intelligible, i.e., to bring them under concepts."[10] Mathematics is the way intuitions are given to concepts and concepts are applied to intuitions. In order to receive sensations, empirical intuitions, contents, there has to be the capability to receive, to appropriate what is encountered. The forms of intuition, space and time, constitute that capability. That the very reception of sensuous experience presupposes these contributions of mind is why it is not quite accurate to say we begin with sense data. We begin with sense data as received. This receiving becomes specific, determinable, through mathematics. To make sense data into experience we space-time what is given. We construct the object in intuition is another way to say it. The construction is performed mathematically. To make sensations into objects of experience we specify when and for how long. We temporalize them, place them in temporal sequence. We measure and to measure specifically we employ discrete units. We mathematicize. We also specify how wide, how high and how long. We spatialize them, place them in spatial location and relation. To do so we meansure and to measure specifically we employ discrete unit. We mathematicize.

Discrete unit. What constitutes a unit? How do we establish what a unit is? Kant discusses this point at some length in his Critique of Judgment in connection with elucidation of judgments of the sublime. The primary unit by which we measure, and therefore the basic mathematical unit, is itself found to be an intuition which appears to be arbitrary. In a sense it

is so. It may come as a surprise that the logical structure which gives science its tremendous precision should have intrinsic to itself an arbitrariness, that the way theoretical constructs and concepts and their relationships and empirical constructs and concepts and their relationships are made clear, definite and exact should contain arbitrary intuition on our part but so it does, Kant argues. It is an arbitrariness which, however, in no way threatens the logic of quantity itself. "The estimation of magnitude by means of concepts of number (or their signs in algebra) is mathematical, but that performed by mere /bloss_7 intuition (by the measurement of the eye) is aesthetical."[11] Kant's question is, which sort of measuring is involved in the measuring which produces the fundamental unit of measure applied in our mathematical reasoning. How great something is is a function of how many units it encompasses. "But since the magnitude of the measure must then be assumed known and this again is only to be estimated mathematically by means of numbers...we can never have a first or fundamental measure and can therefore never have a definite concept of a given magnitude."[12] Kant, a few pages later, develops the theme in the opposite direction. We can take a man as measuring unit for a tree, the tree for a mountain, the mountain for the earth's diameter and that for the planetary system and so on indefinitely.[13] There is, neither in pure not in empirical intuition, neither in our logic of measure itself nor in any empirically given reality, either a smallest or a largest unit. That there is neither empirically

should cause no surprise. Kant has worked out the reasons for this state of affairs in the antinomies of pure reason. That there is none in the pure logic of measure is his point here. We appropriate measure originally by a measurement which is "in the end aesthetical."[14] We adopt, by the eye, so to speak, a unit and from there we set out to measure the universe. What we take as a unit of measure seems arbitrary. Scientific thinking eventuates in a rigorously precisely measured universe, this is true, but that rigorous measuring itself is, in part, aesthetic. Our definite and precisely measured universe rests upon no "definite concept of a given magnitude."[15] That this is so in no way undermines the exactness, precision, logical unity or validity of mathematics or science. What it does is make clear that the measuring of measure is an act of judgment. This is, in part, why Kant calls it aesthetic: no precept or rule can be given. We adopt as unit of measure what suits the measuring we are setting out to accomplish rather like the way the artist adopts a "logic" to present his or her idea in empirical form. Once adopted, a rigorous relationship is entailed.

That the measuring of measure is a direct intuition shows the power of reason to measure. Again, we see that reason is clearly more than can be encompassed by quantity and its logic. Once the original unit is adopted, a rigorous logic is entailed. "But as soon as anything is taken as a <u>quantum discretum</u> , the number of units in it is determined and therefore at all times equal to a certain number."[16] So it is, but that which constitutes

a unit is a direct intuition, a direct measuring of measure, determination of the fundamental unit by what I would call an immediate judgment of intuition. From here on we produce a synthesis in intuition which is mathematical construction. A definite temporal location is coordinated with a definite place and relation of place to place in measured space. This is still at the level of intuiting, seeing. The object is constructed as a specific space-time entity in relation to all space-time. Such a construction is a judgment of intuition. It is intuited quantity that we are working with. We are not here dealing with concepts. Kant's own example serves well to clarify the point.[17] That three and only three dimensions belong to space is in no way derivable from the concept of space. That there are but three lines which can intersect at right angles at one point becomes evident only when we construct a space. Furthermore, even though geometrical figures can be absolutely alike in concept and hence should be capable of being interchanged at will, there are instances in which, once given mathematical construction, we see at once that they cannot be interchanged. An example would be two triangles equal in angles and length of sides which share a common base on the equator of a sphere, one in one hemisphere, the other in the other. Or, in more everyday experience, the right and left hand will provide an easy example. Construction in intuition, in space and time, then, is clearly not <u>thinking</u>, not conceptualization. By means of intuitive construction we produce objects out of the raw material of sensation.

Mathematics is intrinsic to such construction for as soon as we specify space and time, we quantify it and mathematics is the logic of quantity. Observe that the first of Kant's categories of the understanding has already invaded our discussion. The rest follow closely for when we try to specify, to be determinate in our construction, we establish a definite, precise discrete quantity. We also employ what Kant calls quality when we attempt to be specific concerning the intensity of our perceptions, how full, real, what degree of content we are experiencing. Intensive quantity, Kant calls it. We want to know how this constructed perception relates to others and whether the relation is merely possible, actually existent or necessary. To be specific, to make determinate, we measure. We use mathematics once more.

Here is where use tempts toward abuse. While we again proceed mathematically, the procedure, however, must be determined by the understanding which provides the rules for what has been given space-time mathematical construction. Because mathematics has been so necessary and has performed so well and is again necessary, one is tempted to assume that it is knowledge itself rather than an implement employed by reason to produce a synthesis of reason in judgment. At this point the understanding, by its system of categories, provides the fabric of possible determinate relations among determinate space-time entities. Here we <u>think</u> rather than <u>intuit</u>. But what we think in terms of is what intuition in mathematical construction can possibly provide, for concepts in themselves are empty.

In imagination we synthesize the manifold of intuition, we construct mathematically, the objects of experience. But the construction is not yet complete. The concepts which unify these constructions to produce knowledge are provided by the understanding.[18]

For the understanding to perform its guiding, synthesis producing,function there has to be some way in which these pure, empty concepts can be joined to specific intuitions. These rather different aspects of knowledge must be brought together somehow. That somehow is produced by the imagination. Kant calls these imaginatively produced presentations of the concepts transcendental determinations of time.[19]These are the imagination's drawings, so to speak, of space-time pictures of the categories. These pictures are made specific through further use of mathematics. Kant calls this stage of our knowledge of nature the schematism of the pure concepts of the understanding.[20] Why Kant speaks specifically of a determination of time can be seen from the schema of the first of the categories, the concept of quantity. Its schema, its representative picture, is number, "a representation which comprehends the successive addition of one to one (homogeneous). Number therefore is nothing but the unity of the synthesis of the manifold (repetition) of a homogeneous intuition in general, I myself producing the time in the apprehension of the intuition."[21]Specific quantity, and so specific quantity of any sort, space included, involves successive addition of one to one. Successive means temporal.

The one is whatever we take as our unit by that measurement of the eye we spoke of earlier. What is universal and necessary is that all measuring, whatever unit we adopt, is successive addition of unit to unit and hence is temporal. The adopted unit together with the rule of successive addition of the honogeneous produces the specific time or the number. Now we can establish definite quantity. Kant follows a similar procedure to make clear how and why the schemata of quality, relation and modality are, like that of quantity, "nothing but determinations of time a priori according to rules, and these, as applied to all possible objects refer...to the series of time/quantity/, the contents of time /quality/, the order of time /relation/, and lastly, the comprehension of time /modality/."[22]

Each of these schemata presents a temporal and where appropriate spatial picture of its concept and thereby provides the needed mediation between thought concept and intuited sensation. It is, however, important to keep in mind that the concept is the rule. The synthesis in intuition is taken up into the higher synthesis of the understanding through the schemata. There is more to come. First there are the principles of the pure understanding. Each principle gives a rule as to how to proceed with further construction, with further identification and specification of experience. I will here cite only the first by way of illustration. "All phenomena are, with reference to their intuition, extensive quantities."[23] They are so because to think even the most simple

of such, a line, I must draw it in thought, that is, I must think all its parts in sequence. The same holds for any chosen time. These can be known, then, only as successive synthesis.[24] On this characteristic of all phenomena rests the entire mathematics of extension.[25] These are not mathematical principles. Rather they are the principles which make mathematics and its application to all experience possible.

A second more to come, as mentioned earlier, is the synthesis produced by reason (Vernunft) in its role as law or rule giver to the understanding. This aspect, important as it is in Kant's approach, I will here only note. Reason, through its ideas of self, world and God, guides understanding as understanding in turn guides intuition. For natural science and mathematics the idea of world is the particularly relevant one. Why so should be obvious. The world is not given in experience nor even capable of construction in intuition. It is thought. How it is thought is through the categories, as governed by the principles of pure understanding. That thinking is quantitative in nature and therefore intrinsically mathematical. The principles and schemata produce a mathematical/quantitative picture of the categories as possible determinations of time--and therefore of space as well--and the synthesis of these elements, a synthesis produced by reason's power of constructive imagination, is knowledge of the experiencable world. Percepts and concepts are united to produce phenomena in coherent, intelligible order, the phenomenal world which is studied by

natural science whose proper tool is, manifestly, mathematics.

The clarity, certainty, precision, power of explanation and prediction, the impressive success of mathematical science has an enchanting effect and tempts one to assume that mathematics itself reveals the true character of all being, all reality. Spectacular use invites equally spectacular abuse. To avoid such abuse it is necessary to show "that the art of measuring and philosophy are two totally different things, though they are mutually useful to each other in natural science, and that the method of the one can never be imitated by the other."[26] Kant continues a few lines one, "I will show at the same time that the art of measuring, or geometry, will by its method produce nothing in philosophy but card houses; while the philosopher with his method produces in mathematics nothing but vain babble."[27]

Mathematics and the portion of philosophy which is reason in its theoretical function do not differ in their object. Both in their pure rational aspects concert objects of possible experience and actually experienced objects in their applied aspects. Both, therefore, concern quantity. The differences are in form, range of application and what we may term rank. "It is owing to the form of mathematical knowledge that it can refer to quanta only, because it is only the concept of quantities that admits of construction, that is, of a priori representation in intuition."[28] Philosophy is thinking and proceeds by judging uniting representations in consciousness.[29] These judgments take

many forms and levels and therefore the range of philosophy is considerably wider than that of mathematics. One form and series of levels concerns quantity, phenomena, objects of experience, but not as constructed, which is the business of mathematics, but as thought, judged. This is theoretical reason and gives us the sequence : understanding, principles and,in the ideas of reason,rules governing the understanding. All objects of understanding can, as we have seen, be given in experience. The ideas of pure reason are of another source and type. They are pure rational ideas spontaneously produced by reason and subject only to rational elucidation, never of proof or demonstration in the mathematical connotation of the terms. This is not because they are somehow less than mathematics. They are, rather, more than mathematics because they are part of a more comprehensive rational synthesis.[30] In theoretical reason the ideas have their application solely in terms of possible experience, to the thinking of objects of experience, it is true, but their function is as law-giver, ruler, whereby sensation is brought to intelligible synthesis in knowledge. This mathematics cannot do. It constructs objects of experience in intuition; reason judges experience in thought.[31] Mathematics, as the logic of quantity, is an indispensible tool in the process. But tool it must remain.

"Pure rational cognition from mere concepts is called pure philosophy, or metaphysics; on the other hand, that pure rational cognition which is based only upon the construction

of concepts by means of the presentation of the object in a priori intuition is called mathematics.[32] "Natural science properly so called presupposes metaphysics of nature; for laws, i.e. principles of the necessity of what belongs to the existence of a thing, are occupied with a concept which does not admit of construction because existence cannot be presented in any a priori intuition. Therefore natural science presupposes metaphysics of nature."[33]

Unless this ranking, this order of priority, is kept clearly in mind in practice we get those card houses and vain babble spoken of earlier, much to the detriment of philosophy, mathematics and natural science alike. In natural science illicit, wrong-headed metaphysics creeps in unbeknownst to the mathematician who believes he has evaded all metaphysics, and even if his mathematics survives unharmed both philosophy and natural science become corrupted if not destroyed. An example cited by Kant concerns how the diffusion of light as it travels from its source is to be conceived.[34] Lines radiating from a point is frequently taken as the way to depict this phenomenon. In spite of the apparent mathematical sense and plausibility of such an approach, it is entirely inappropriate, not because it is bad mathematics, but because the mathematics implies bad metaphysics. The phenomenon is experientally wrongly understood because wrongly spatialized, but wrongly spatialized because it has been wrongly conceived metaphysically at a more primary level. Since no inquiry has been made into the origin

and nature of their own approach, since no attempt is made to discover where the concepts of space and time which they employ here come from, current rules borrowed from ordinary experience are accepted as axioms.[35] This is exactly what has happened here. Lines radiating from a point can never produce an adequate intuition of the diffusion of light because whereas the light is evenly distributed over the entire sphere which describes its outer limit, the image of radiating rays necessarily suggests rays of light which would appear as bright spots on the surface and dark areas and the dark areas would become greater and greater the further the light travels no matter how many rays one assumes at the initial source point. From a geometrical viewpoint the image is entirely plausible and apt, but it is still bad metaphysics and erroneous natural science. Where is the fundamental problem? On the surface it would appear that mathematics gave the initial orientation but that really is not so. The fundamental problem is in how objects are understood to fill space and this is purely a metaphysical question.

Almost all natural philosophers, Kant says, explain the differing gravitational and interactional behavior of bodies of equal volume by the assumption that all bodies contain, in differing proportion, empty space. "Who could have thought," he asks, "that those mathematical and mechanical philosophers should have based such a conclusion on a purely metaphysical hypothesis, which they always profess to avoid, by assuming that the real in space...must always be the same and can differ

only in its extensive quantity, that is, by the number of parts."[36] Kant here in the Critique of Pure Reason only develops his case far enough to show that a dynamic explanation in terms of quality, intensive quantity, is possible. This is sufficient for his purpose in this context because it establishes that the metaphysical notion of the real in space is an arbitrary assumption. Its simplicity and the logic of numerable discrete quantity, both of which make it easily assimilable to mathematical expression, tempt mathematicians, natural scientists and philosophers of nature, all of whom necessarily employ mathematics in their work, to accept this assumption without question. It is the philosopher's business to correct this abuse by working out the correct metaphysicsl principles upon which natural science rests.

In his Metaphysical Foundations of Natural Science Kant does just that. Whereas earlier he had only shown that an alternative to the atoms in the void metaphysic was possible, here he develops a dynamic concept of body, of matter as the real in space--and eventually space and time. Rather than discrete bits of stuff rattling around in empty space, he sees the real in space to be a dynamic relation of forces. He argues that the notion of impenetrability, solidity, of resistance to intrusion, is taken by mathematical natural philosophers as given. Whatever fills a space resists whatever would invade that space. But why so is unexplained and inexplicable and worse yet incapable of any mathematical

construction.[37] This is a particularly serious defect in a philosophy of nature which presumes to be without metaphysical assumption and to be amenable to mathematical analysis and expression throughout. This does not mean it is false. What it means is that it requires justification and cannot be used as a justification for anything. Kant argues that the notion of forces of attraction and repulsion provide an adequate concept of the real in space upon which to base natural science. In this way Kant can account for occupying space without filling it, for gravity and for attraction generally.[38]

Details we do not need to pursue here. Kant sums up this discussion well at the beginning of his "General Observation on Dynamics."

> The universal principle of the dynamics of material nature is this: all that is real in the objects of our external senses and is not merely a determination of space (place, extension and figure) must be regarded as moving force. By this principle, therefore, the so-called solid, or absolute impenetrability, is banished from natural science as an empty concept and in its stead repulsive force is posited. On the other hand, the true and immediate attraction is defended against all the sophistries of a metaphysics that misunderstands itself and this attraction is explained as a fundamental force necessary even to the possibility of the concept of matter.[39]

Kant admits that the mathematico-mechanical mode of explanation with its impenetrable bits of matter can produce an impressive picture because its initial metaphysical assumption, discrete particles, is capable of mathematical construction and its shapes, volumes and densities are readily

suited to mathematical evidence and proof. However, it pays for this advantage with a double disadvantage "in that it first of all must lay at its foundation an empty concept (that of absolute impenetrability), and secondly must give up all proper forces of matter."[40] The dynamic relation of forces has the disadvantage that its fundamental concepts, those of forces, are not suited to intuitive or mathematical construction.[41] It has the advantage, however, of providing a coherent explanation of matter and therefore provides a basis upon which to explain its behavior. In the next major section, "Metaphysical Foundations of Mechanics," Kant does explore how the dynamic metaphysical principles he has evolved are operative in a mathematically expressed natural science. It is essentially through measurement of motion.

The point of all of this is that it is an abuse of mathematics to use it as metaphysics, an abuse which corrupts metaphysics and natural science and brings the noble art of mathematice itself into disrepute. Construction of concepts in intuition is one thing, thinking is another and they must be kept each in its own proper area and function.

I return to a passage quoted in part earlier.

> The mathematician, the student of nature and the logician, however far the two former may have advanced in rational, and the last, particularly, in philosophical knowledge, are merely artists of reason. There is besides, an ideal teacher, who controls them all and uses them as instruments for the advancement of the essential aims of human reason. Him alone we ought to call the philosopher: but as he exists nowhere

> while the idea of his legislation exists everywhere in the reason of every human being, we shall keep entirely to that idea....[42]

Philosophy has the honor of being the science which relates all knowledge to the essential aim of human reason. In theoretical reason the essential aim of human reason is the complete a priori unity of all knowledge of nature. Mathematics is, as we have repeatedly seem, integral to the fulfilling go this aim. Philosophy has, in this regard, the task of seeing to it that the synthetic unity produced is indeed in conformity with the essential aim of reason, is indeed knowledge. To this end the students of nature and philosophers alike must recognize"that reason has insight into that only, which she herself produces on her own plan, and that she must move forward with the principles of her judgment, according to fixed law and compel nature to answer her questions...in order to be taught by it but not in the character of a pupil who agrees with everything the master likes, but as an appointed judge, who compels the witness to answer the questions which he himself proposes."[43]

As glorious as this all may be, we have not yet reached the nature of the true philosopher. Essential aims are not yet the highest end, the ultimate end of human reason. That end is "nothing but the whole destination of man and the philosophy which relates to it is called moral philosophy."[44] While the philosophy of nature concerns what is, the philosophy of morals concerns what ought to be. Freedom is its object.[45]

As Kant says in many ways and many places, his critical approach does seem at first wholly negative in its results, but actually is positive in its contribution because it exposes the imperialist claims of theoretical reason and opens the way to a true appreciation of nature and more importantly to a true appreciation of morality and this means, in turn, a true appreciation of the person, the existing human individual.

All thinking is judging. In reason's theoretical application that judging is tied to possible experience and is therefore narrowly restricted in its judgments. In aesthetics we encounter ideas too rich in content to be reduced to concept, but as the beautiful they are produced in intuition. [46]There are also ideas which are too rich in content to be given in intuition.[47]Freedom is an idea too rich to be captured in either fashion. The thinking which determines for itself what to be, what ought to be and therefore what to do is the highest, the ultimate end or aim of human reason. It is reason in its purely rational function, synthesizing being.

The radical exponent of mathematics turns out to be a radical exponent of the intrinsic worth of the person. By elucidating the difference between construction of concepts and thinking, Kant has elucidated the range of the power of human reason. By revealing the intrinsic limits of mathematics and mathematical philosophy he has revealed the limitless worth of the person. That worth centers in the

in the power to create order in being and to begin with to create order in one's self so as to be a center of order, meaning, beauty and goodness. By measuring measure he reveals the measurelessness of the one who measures.

Notes

1. Immanuel Kant, Metaphysical Foundations of Natural Science in Philosophy of Material Nature, trans. James Ellington (Indianapolis: Hackett Publishing So., 1985), p. 6. Hereafter cited as Kant, MFNS.

2. Ibid., p. 7.

3. Immanuel Kant, Critique of Pure Reason, trans. F. Max Müller (2nd. ed. rev.; Garden City: Doubleday and Co., 1961), p. 490. Hereafter cited as Kant, CPR.

4. Ibid.

5. Ibid., p. 505.

6. Ibid., p. 55.

7. Kant, MFNS, p. 9.

8. Ibid., p. 5 and Kant, CPR, p. 421.

9. Immanuel Kant, Prolegomena to any Future Metaphysics, trans. Lewis White Beck ("Library of Liberal Arts;" Indianapolis: Bobbs Merrill Co., 1959), p. 52.

10. Kant, CPR, p. 55 .

11. Immanuel Kant, Critique of Judgment, trans. J.H. Bernard (New York: Haftner Publishing Co. , 1951), p. 89. Hereafter cited as Kant, CJ.

12. Ibid.

13. Ibid., p. 95.

14. Ibid., p. 89.

15. Ibid., p. 89.

16. Kant, CPR, p. 320.

17. Kant, Prolegomena, pp. 31-34.

18. Kant, CPR, p. 71.

19. Ibid., p. 107.

20. Ibid., pp. 104-05.

21. Ibid., p. 107.

22. Ibid., p. 108.

23. Ibid., p. 118.

24. Ibid.

25. Ibid., p. 119.

26. Ibid., pp. 428-29.

27. Ibid., p. 429.

28. Ibid., p. 422.

29. Kant, Prolegomena, p. 52.

30. Ibid., pp. 76-77. See also Kant, CPR, pp. 423-25.

31. Kant, CPR, p. 195 and p. 208.

32. Kant, MFNS, p. 5.

33. Ibid., pp. 5-6.

34. Ibid., p. 71.

35. Kant, CPR, p. 428.

36. Ibid., p. 124.

37. Kant, MFNS, p. 42.

38. Ibid., p. 67.

39. Ibid., p. 77. I quote here the original as found in Metaphysiche Anfangsgründe der Naturwissenschaft ("Kant's Gesamelte Schriften;" Berlin: Prussian Academy of Sciences, 1905-1956), IV, 523. Das allgemeine Princip der Dynamik der

materiellen Natur ist: dass alles Reale der Gegenstände äusserer Sinne, was nicht bloss Bestimmung des Raums (Ort, Ausdehnung und Figur) ist, als bewegende Kraft angesehen werden müsse; wodurch also das so genannte Solide oder die absolute Undurchdringlichkeit, als ein leerer Begriff, aus der Naturwissenschaft verwiesen und an ihrer Statt zurücktreibende Kraft gesesst, dagegen aber die wahre und unmittelbare Anziehung gegen alle Vernünfteleien einer sich selbst missverstehenden Metaphysik vertheidigt und, als Grundkraft, selbst zur Möglichkeit des Begriffs von Materie für nothwendig erklärt wird.

40. Kant, MFNS, p. 80.
41. Ibid., p. 79.
42. Kant, CPR, p. 490.
43. Ibid., pp. 503-04.
44. Ibid., p. 490.
45. Ibid., p. 491.
46. Kant, CJ, p. 157.
47. Ibid., p. 157 and pp. 188-89.