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INEFFECTIVE GENERAL EQUILIBRIA IN A TRANSITION PERIOD ECONOMY

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Ineffective General Equilibrium in a Transition Period Economy [1]

The evolution of market economy in Russia demonstrates many strange economic phenomena. Often they are supposed to lie beyond economic theory. Usually they are explained by special factors such as peculiar government policy, hostility of world market or Soviet mentality.

But we believe that traditional approaches of economic theory still can be applied to modern Russian reality. One needs only to study them in more details, to examine particular cases which are formally permitted but usually are not considered as non-essential for a "normal" market economy. These cases may well be appropriate for Russian economic situation. Here we will concentrate on five essential features of Russian economic situation in 1993:

- *absence of investments*

  During 1992-94 gross investments do not even recover amortization of production capacities.

- *high inflation*

  Since price liberalization in January 1992 up to the end of 1993 Russian economy has been characterized by high inflation of almost constant rate of about 20% per month (or 800%) per annum.

- *negative real interest rates*

  During 1992 and 1993 the interest rates of all kinds were not higher than 250% per annum that is significantly lower than the inflation rate.

- *recession*

  Since the very beginning of Russian economic reform the level of production was declining. However the decline was not selective and influenced not only inefficient industries. For example one of the main export industry i.e. the oil industry suffered severe recession as well.

- *arrears (Debtors Receivable)*

  One of the most notorious problems of Russian industry is the arrears. The arrears can occur due to objective factors such as lack of appropriate payment transfer technology or subjective factors. Subjective factors are the ones due to benefits which banks and enterprises have from the slow payment transfer or large arrears, correspondingly.

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In the present work we consider the arrears arisen because of delays in payment. These arrears are *not* controllable by enterprises. But in the final section we will show how enterprises can play around with the large arrears as mutual credit and benefit from it.

In the model below we take the first two factors mentioned as premises and obtain the rest. We emphasize that this will be done within rational market behavior framework and we will not need the assumptions on impact of government policy and foreign trade and non-economic factors.

The model uses the neoclassical general equilibrium approach. Among four major neoclassical markets Russia has got only the product market. The labor market is simply absent in its neoclassical meaning. The capital market is very unstable — there are always some rapid changes going on but one invariant is the case: the absence of production investments is persistent. Correspondingly we consider only the product market equilibrium assuming rational producer and consumer behavior. Although the model is referred to as general equilibrium model since the close financial cycle and product turnover are taken into account.

We will consider three kinds of economic agents: producers (enterprises), consumers (households) and banks interacting within a closed economy.

First consider behavior of producers. In the absence of stock an labor markets the distinction between profit and wage within enterprise net income does not influence production strategy.. This is why we assume that the enterprise is seeking for the total of net income

$$\begin{aligned} NetIncome(t) &= GrossOutcome(t) - ProductionInputCosts(t) + \\ &+ (NewLoans - DebtService)(t) \end{aligned}$$

In an economy without capital investments and consequently without long-term loans the producers still need short-term loans in order to make up for various delays in technology and financial cycles. Let us denote $\vec{p}(t)$, $\vec{x}^j(t)$, $\vec{v}^j(t)$ for prices, producer j gross outputs and producer j gross inputs, correspondingly. Vectors $\vec{x}^j, \vec{v}^j, \vec{p}$ belong to common product domain R_n^+. The production possibilities of each producer are constrained by its production possibility set

$$\left\{\vec{x}^j(t-\tau), \vec{v}^j(t)\right\} \in T^j, \tag{1}$$

where τ is duration of the production cycle.

In addition to τ we assume that producers face delays in payment θ. Due to objective factors (poor payment transfer technology) or subjective factors (unwillingness of banks to transfer money as soon as possible and absence of any means to enforce immediate transfer) in 1993 the producers received payment 1 month later than the product is delivered.

$$GrossOutcome(t) = \vec{p}(t-\theta)\vec{x}(t-\theta)$$

Although delays in payment are usually lower than the production cycle duration, they are much more important for producers under high inflation as

shown below. Besides in 1992-93 in Russia the delays of payment were quite high themselves and were the same order of magnitude as the duration of the production cycle. Due to delays of payment the producers have a special kind of assets — so called *Arrears* (Debtors Receivable) B^j.

$$\dot{B}^j = \vec{p}(t)\vec{x}^j(t) - \vec{p}(t-\theta)\vec{x}^j(t-\theta) \tag{2}$$

This asset is natural guarantee for short-term debt L^j:

$$\dot{L}^j = rL^j + K^j, \;\; L^j \leq \chi B^j, \;\; . \tag{3}$$

where $K^j = (NewLoans - DebtService)(t)$ is *Net Credit Flow*, r is the bank interest rate, $\chi \leq 1$ is creditability rate which is assigned by bank lending the loan. The equation above describes so called "credit line" and *Net Credit Flow* may be either positive or negative. If latter is the case the *Debt Service* is greater than *New Loans* received.

At every time moment the producer makes decisions upon the $\vec{x}^j$, $\vec{v}^j$ and K^j under the constrains (1)-(3). We assume that the producer maximizes Discounted Cash Flows or NPV of *Net Income*

$$\int_t^{t+T} e^{-\Delta t'} \Pi(t')\, dt' \Rightarrow max\,, \tag{4}$$

here

$$\Pi^j(t) = \vec{p}(t-\theta)\vec{x}^j(t-\theta) - \vec{p}(t)\vec{v}^j(t) + K^j(t)$$

is *Net Income*, Δ is the discount rate and T is sufficiently long time horizon. The producer is assumed to expect proportional exponential price growth $\vec{p}(t) = \vec{p}(0)e^{it}$, hence the nominal discount rate Δ consists of real discount rate δ (pure time preference) and inflation rate i

$$\Delta = i + \delta_p \tag{5}$$

As for r and χ we assume that the producer expects them to be constant and equal to their present value.

Then the optimal control problem (1)-(4) can be easily solved. If $\Delta > r$ the constraint (3) turns to an equality i.e. the producer tends to borrow as much as possible:

$$L^j = \chi B^j \tag{6}$$

$$K^j = \chi\left(\vec{p}(t)\vec{x}^j(t) - \vec{p}(t-\theta)\vec{x}^j(t-\theta)\right) - \chi r B^j \tag{7}$$

In case of high interest rate $\Delta < r$ the producer does not borrow but tends to lend money. This situation does not occur neither in current Russian economy nor in a "normal" market economy and is not considered in the present work.

As for production schedule $\vec{x}^j(t-\tau)$ and $\vec{v}^j(t)$ are determined as follows:

$$(\vec{x}^j(t-\tau), \vec{v}^j(t)) = arg \max_{(\vec{x},\vec{v})\in T^j} \kappa \vec{p}\vec{x} - \vec{p}\vec{v}\,, \tag{8}$$

where

$$\kappa = 1 - (1 - e^{-i\theta} e^{-\delta_p(\tau+\theta)}) \left(1 - \left(1 - \frac{r}{\Delta}\right)\chi\right) \tag{9}$$

The *Gross Outcome* and *Net Income* depend on κ hence both on the duration of production cycle τ and on the delays in payment θ. Attention should be paid to the fact that τ and θ enter the expression (9) in a different manner. The inflation influences production level only due to existence of delays in payment.

For the sake of clarity we assume that discount rates, production cycles, delays in payment, expectations and creditability rates are the same for all the producers.[2] It is evident that in this case the whole supply side can be described as one producer with the aggregate production possibility set

$$T = \sum_j T^j : \{\vec{x}, \vec{v}\} \in T \equiv \vec{x} = \sum_j \vec{x}^j,\ \vec{v} = \sum_j \vec{v}^j,\ \{\vec{x}^j, \vec{v}^j\} \in T^j$$

Now consider the households. We will describe their behavior in a traditional way. The households receive income Φ which consists of the producers' *Net Income* Π and the banking profit Π_B and spend it on consumption $\Psi = \vec{p}\vec{c}$ and savings S. The savings increase the households' deposits with interest rate ρ.

$$\Phi = \Pi + \Pi_B \tag{10}$$

$$\dot{D} = \rho D + \Phi - \Psi \tag{11}$$

We assume $\Psi = \vec{p}\vec{c}$, where $\vec{c}$ is current consumption vector.

In Russia consumer credit is very rare so we assume that consumers can not borrow:

$$D \geq 0 \tag{12}$$

As it is often done in general equilibrium models we assume that at every moment t the households choose the vector $\vec{c}(t)$ maximize the discounted utility of consumption:

$$\int_t^{t+T} e^{-\delta t'}\, U(\vec{c}(t'))\, dt' \Rightarrow max \tag{13}$$

where δ is pure time preference rate, $\vec{c}(t')$ — future consumption,
$U(\cdot)$ — utility function , T — pretty long time horizon.

The problem (11)-(13) can be solved easily if the utility function has constant relative risk aversion (CRRA):

$$U(\alpha\vec{c}) = \alpha^{1-\beta} U(\vec{c})\,,\ \beta > 1 \tag{14}$$

[2]If it is not the case the equilibrium may even fall inside of the aggregate production possibility set.

The condition $\beta > 1$ is necessary for utility being $-\infty$ at zero consumption and finite at $\vec{c} \to \infty$. If any component of $\vec{c}$ is negative we also define utility to be $-\infty$. This natural definition enables to leave out extra constraint $\vec{c} \geq 0$.

Note that consumption expenses $\vec{pc}$ may exceed the households' income Φ with deposits still growing $\dot{D} > 0$. This corresponds to the case when the consumer spends some part of the interest ρD.

As well as before we assume that the consumers expect the deposit interest rate ρ to be constant over time all prices and income to grow exponentially with the same rate i equal to the current inflation rate. Then solution of the optimization problem (11)-(13) determines current consumption $\vec{c}$ as function of current prices $\vec{p}$, income Φ, deposits D, interest rate ρ and inflation rate i:

$$\vec{c} = arg \max_{\vec{pc} \leq \Psi} U(\vec{c}) \,, \tag{15}$$

where consumption expenses Ψ are as follows:

- if $\rho - i - \delta > 0$

$$\Psi = \Phi(1 - \frac{1 - \delta/(\rho - i)}{\beta})) + D(\rho - i - \frac{\rho - i - \delta}{\beta}) \,, \tag{16}$$

  with savings being equal to

$$S = \Phi - \Psi = \Phi\frac{1 - \delta/(\rho - i)}{\beta}) - D(\rho - i - \frac{\rho - i - \delta}{\beta})$$

- if $\rho - i - \delta < 0$

$$\Psi = \Phi \tag{17}$$

  without savings and deposits $D = 0$

Thus the households' behavior depends essentially on the real discounted interest rate $\rho - i - \delta$. If $\rho - i - \delta > 0$ then consumers are interested in savings. If inflation is too high $\rho - i - \delta < 0$ then consumers are keen to eat up rather than to accumulate the income. This fact is valid not only for CRRA functions but also for any strictly concave utility function.

Now turn to the banking system. We do not consider financial market explicitly and take the interest rates r and ρ and creditability rate χ as given. Let us investigate the aggregate balance sheet of the banking system. The assets are the producers' debts L and the liabilities are the households' deposits D and payments being transferred that are equal to aggregate arrears B. The current banking operations are lending new loans to enterprises K, borrowing households' savings S, payment of banking profit $\Pi_B = rL - \rho D$ (we do not consider increase in the banks' equity) back to the households and transfer of payments from consumers (including intermediate consumers — enterprises) to producers. The balance of current operations is as follows:

$$S + \vec{pc} + \vec{pv} = K + \vec{p}(t - \theta)\vec{x}(t - \theta) + rL - \rho D$$

With the aid of (2),(3),(10),(11) we derive that the natural aggregate balance equation for the banking system

$$\dot{D} + \dot{B} = \dot{L}$$

is equivalent to the strong Walras law

$$\vec{p}\vec{x}(\vec{p}) = \vec{p}\vec{c}(\vec{p}) + \vec{p}\vec{v}(\vec{p}) \tag{18}$$

Using the product supply and demand functions $\vec{x}(\vec{p})$, $\vec{v}(\vec{p}) + \vec{c}(\vec{p})$ obtained from the models of producers' and consumers' behavior we are eventually able to define the problem of determination of equilibrium prices i.e. prices $\vec{p}$ which satisfy

$$\vec{x}(\vec{p}) \geq \vec{c}(\vec{p}) + \vec{v}(\vec{p})$$

However, this equilibrium is different from the neoclassic one. The matter is that the producers appreciate production inputs and outputs differently. Instead of conventional functional $\vec{p}\vec{x} - \vec{p}\vec{v}$ producers maximize functional $\vec{p}A\vec{x} - \vec{p}\vec{v}$, where $A = \kappa$ (9). As the excessive demand function satisfies the Walras law (18) the existence of equilibrium can be proven with the aid of Gale's lemma if the aggregate production possibility set $\mathcal{T}$ is productive and matrix A is not very different from the unit one E. But this equilibrium turns out to be inefficient — it is not the maximizer of social utility function $\max_{\{\vec{x},\vec{v}\}\in\mathcal{T}} U(\vec{x} - \vec{v})$.

In the dynamic model considered the equilibrium prices obtained describe a self-consistent steady inflation state of economy. In this state all real variables are constant and all nominal variables grow exponentially with the same rate i. As all the forecasts are perfectly correct this path is rational expectations equilibrium (REE).

Depending on relationships between parameters i, r, ρ, χ these equilibria are divided into two classes. The first one corresponds to $\rho > i + \delta$ and $D > 0$. In these equilibria must be $\chi > 1$ — in the absence of capital investments the savings can not be absorbed by the borrowers and the basic principles of commercial credit $\chi \leq 1$ are violated. So if by some reasons there are no long-term investments then consistent equilibrium with positive real interest rate does not exist.

The second case is $\rho < i + \delta$ and $D = 0$. These equilibria are very much similar to the situation in Russia in 1993: no savings, negative interest rate, underloaded production capacities. In this case the bank's balance equation is

$$\dot{B} = \dot{L}\,,$$

Using (6) we derive

$$(\chi - 1)\dot{B} = 0 \tag{19}$$

If $\chi < 1$ only zero-inflation equilibrium $\dot{B} = \dot{L} = 0$ is possible. More appropriate positive-inflation equilibria are the case if $\chi = 1$. These equilibria correspond to

the situation in a transition period economy. Note that due to (9) the equilibrium inefficiency $|\kappa-1|$ is decreasing with χ for all $\chi \leq 1$. This means that the positive inflation equilibria are more efficient than non-inflation ones. Thus though lower interest rates promote inflation they also stimulate production partly making up for the losses incurred by delays in payment.

Substituting $\chi = 1$ in (9), we obtain

$$\kappa = 1 - (1 - e^{-i\theta} e^{-(\theta+\tau)\delta_p}) \frac{r}{i + \delta_p} \tag{20}$$

So given constant $i, \theta, r < \Delta, \chi = 1, \rho < i + \delta$, we have got REE — real variables such as $\vec{v}, \vec{x}, \vec{c}$ and price proportions $\vec{p}/|\vec{p}|$, are constant over time and the nominal variable such as B, L, Φ, K, grow with the steady rate i. This equilibrium exists at any interest rate and inflation rate that meet $0 < r < i + \delta_p$ incl. negative real interest rate $0 < r < i$ [3].

This situation occurs chiefly because the main source of banks' credit resources is not the households' deposits but the enterprises' own arrears which are ***not*** interest-bearing. In such economy the interest rate r arises as result of political negotiations between the banking system and industry rather than that of economic mechanisms. $0 < r < \Delta$ is to mutual benefit with banking profit proportional to r and enterprises' profit proportional to $\Delta - r$. However the relation between the interest and inflation rates influences the equilibrium efficiency. The point is that in the presence of delays in payment all real variables do depend on the inflation rate i (nonneutrality of money is the case). Compare the inefficiency of equilibrium at different i:

$$|\kappa - 1| = (1 - e^{-i\theta} e^{-\delta_p(\tau+\theta)}) \frac{r(i)}{i + \delta_p}$$

Figure

The more the delays θ and interest rate r the more is the equilibrium inefficiency. This gives rise to decrease in aggregate supply and decrease in producers' $NetIncome$. If the inefficiency is small then the banking profit Π_B is increasing with r and θ. If the inefficiency is significant then the recession results in decrease of banking profits

Certainly the interest rate r increases with the inflation rate i but in this model the function $r(i)$ is exogenous. If banks and enterprises share the inefficiency premium in constant shares: $r(\Delta)/\Delta = const$, then inefficiency increases with i but it is still always less than in the absence of the credit line. If the interest rate is fixed $r(i) = const$ (all inflation bonus is given ot the producers) then the more inflation is the closer the equilibrium and the social optimum are — function

$$|\kappa - 1| = \frac{1 - e^{-i\theta} e^{-(\theta+\tau)\delta_p}}{i + \delta_p} r$$

[3] Condition $r < i$ is consistent with $\rho < i + \delta$: as $\rho < r$, holds $\rho < i$ hence $\rho < i + \delta$

decreases with i. Thus low interest rate not only pulls the equilibrium back to the social optimum but also enables normal operation under high inflation.

We considered a dynamic model of a closed market economy with delays in payment. We showed that the delays in payment shift the market equilibrium point away from the social optimum with production level being lower than in the welfare point. However the economy studied is quite sustainable. Even under high inflation despite negative real interest rate and the absence of deposits banks lend money and earn profits due to the delays in payment. As for producers they also do not suffer too badly since they can borrow money at a low rate for the payments delayed. We proved that these are negative real interest rates that lower the equilibrium inefficiency.

These inefficient equilibria occur only if there are no long-term investments.

We do not consider any other possibilities for accumulating financial capital except the arrears. That is why this simplified solution is obtained.

The model above is an essentia of a more sophisticated model that was developed in Computing Center of Russian Academy of Sciences under Prof. A.A.Petrov, Corr. Member of RAS. That model is a system model of Russian economy. It considers government and Central Bank intervention, foreign trade and export of capital and hierarchy of markets. In that model all the features described above are present but in addition the following positive feedback is taken into account: the recession stimulates inflation via credit emission. This may lead to the collapse. This factor is not considered in the simple model above.

In that model there exist REEs with $\chi \neq 1$. The arrears can be increased by enterprises in order to achieve their own goals. In the model the whole industry is considered as a consortium whose activity results in discrimination of three price systems:

1. the market prices (or consumer prices) such as considered above
2. nominal prices that are declared and used to determine nominal contract sum and hence creditability
3. internal prices that are actually used when production inputs are purchased

The difference between the nominal and internal prices are arrears. The producers receive extra loans at a negative interest rate using the arrears. It is proven that there exists equilibrium that maximizes aggregate income of industry if external credit level given.