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THE PHYSICS OF WEAK INTERACTIONS

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The Physics of Weak Interactions

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Abstract

Early in the 20th century, studies of radioactivity led to the conclusion that energy and momentum did not appear to be conserved in Beta Decay. It was suggested that new particles were involved which had no charge and no rest mass, and interacted very weakly with other particles. If such particles existed, energy and momentum were conserved. These particles were called neutrinos.

In 1956 C. Cowan and F. Reines published results of experiments confirming existence of neutrinos.

The early theory was constructed assuming that the weak interactions conserve parity. Other research which implied that parity was not conserved, was criticized.

In 1956, T.D. Lee and C.N. Yang published a paper suggesting that parity was not conserved in weak interactions. Experiments carried out by Wu, Ambler, Hayward, Hoppes, and Hudson confirmed the Yang-Lee theory. The scientific community had been incorrect in their conclusions about parity in weak interactions for 27 years!

Parity non conservation leads to the conclusion that neutrinos and antineutrinos are different particles. Later it was discovered that there are in fact 3 different kinds of neutrinos and antineutrinos.

Introduction

Radioactivity was discovered during a study of Uranium compounds, by H. Becquerel[1] in 1896. By 1914 a considerable number of nuclei were found to be radioactive. It was shown by M. Curie[2] that if a radioactive substance is placed in a magnetic field, there are three possible effects, as shown in Figure 1. From the directions of the deflection, it can be concluded that the emitted particles may be positively charged, or negatively charged, or electrically neutral if undeflected. The positively charged particles are called α particles, and are He^4 nuclei. The negatively charged particles are electrons which are called β particles. The uncharged radiation is electromagnetic radiation which has been called γ radiation.

For a given nucleus which emits γ radiation, the radiation wavelength is confined to a fairly narrow range associated with emission of photons as the element quantum state drops from one well defined energy level to a lower well defined energy level. Similarly, in α decay, the energy range is relatively narrow because an element in a well defined energy quantum state decays to a different element also in a well defined quantum state.

By 1914, it was discovered by J. Chadwick[3] that β emission differed in a fundamental way from α and γ emission. The β electrons from a given radioactive element have a continuous distribution of energy. It was also observed that neither energy nor spin and statistics were conserved considering the decaying element, decay product and β electron. For example the reaction

$$Bi^{210} \rightarrow Po^{210} + \beta^-$$

does not appear to conserve spin angular momentum. The spin of Bi^{210} is $\hbar$, the spin of Po^{210} is 0, and the spin of β^- is $\frac{1}{2}\hbar$.

Neils Bohr[4] interpreted the Chadwick result as evidence for believing that the laws of conservation of energy and momentum may not be valid for nuclear processes in which electrons are emitted. However W. Pauli[5] proposed that energy and momentum were conserved in β decay; that another particle with zero charge, and interactions so weak that detection was difficult, was emitted. At first all such neutral particles were called neutrinos. It was considered likely that energy and momentum were conserved when contributions of neutri-

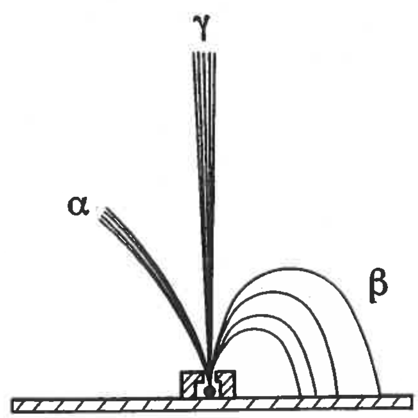

Fig. 1 Radioactivity. A magnetic field is applied perpendicular to the plane of the sheet.

nos were included. The neutrinos were assumed to have zero or extremely small rest mass, velocity equal to or nearly equal to the velocity of light and spin $\frac{1}{2}\hbar$.

Quantum Theory of Spin $\frac{1}{2}$ Particles

The Schroedinger equation for a particle of mass m, momentum $\bar{p}$, in a field with potential V is

$$i\hbar\,\frac{\partial\psi}{\partial t} = H\psi = \left(\frac{p^2}{2m} + V\right)\psi \tag{1}$$

In (1) ψ is the wavefunction, H is the Hamiltonian. The momentum is represented by the operator

$$\bar{p} = -i\hbar\,\nabla \tag{2}$$

and (1) may be written in the form

$$i\hbar\,\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \tag{3}$$

(3) gives solutions valid for particles moving with kinetic energy small compared with the rest energy mc^2, and does not satisfy the Special Theory of Relativity, for which the Hamiltonian is given by

$$H = \sqrt{p^2c^2 + m^2c^4} \tag{4}$$

for a free particle. The Schroedinger wave equation for a free particle is then, from (2) and (4),

$$i\hbar\,\frac{\partial\psi}{\partial t} = \sqrt{-\hbar^2c^2\nabla^2 + m^2c^4}\;\psi \tag{5}$$

The square root in (5) presents problems. One solution is to square both sides of (4) obtaining

$$-\hbar^2\,\frac{\partial^2\psi}{\partial t^2} = -\hbar^2c^2\nabla^2\psi + m^2c^4\psi \tag{6}$$

$\psi^*\psi$ is a scalar which cannot by itself describe spinning particles. If spinors are introduced following procedures developed for equation (3), the resulting solutions do not accurately describe atomic structure. (6) is the Klein Gordon equation[6] which describes particles having spin zero.

P.A.M. Dirac[7] decided to retain the linear time dependence of ψ and the square root, assuming for (5) the equation

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar c}{i}\left(\alpha_x \frac{\partial \psi}{\partial x} + \alpha_y \frac{\partial \psi}{\partial y} + \alpha_z \frac{\partial \psi}{\partial z}\right) + \beta mc^2 \psi \tag{7}$$

The coefficients α_x, α_y, α_z cannot be numbers because if they were, (7) would not be invariant under spatial rotations. As noted, equation (6) does not describe spinning particles. Dirac suggested that (7) be considered a matrix equation with the wavefunction ψ having N components.

$$\psi = \begin{vmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{vmatrix} \tag{8}$$

The α's and β are matrices. In matrix form

$$i\hbar \frac{\partial \psi_\epsilon}{\partial t} = \frac{\hbar c}{i} \sum_{\mu=1}^{N} H_{\epsilon\mu}\psi_\mu \tag{9}$$

For consistency with Special Relativity, each ψ_ϵ must satisfy (6)

$$-\hbar^2 \frac{\partial^2 \psi_\epsilon}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \psi_\epsilon + m^2 c^4 \psi_\epsilon \tag{10}$$

We write each side of (7) as an operator operating on ψ . Let each operator operate twice, this gives

$$-\hbar^2 \frac{\partial^2 \psi_\epsilon}{\partial t^2} = -\hbar^2 c^2 \sum_{i,j=1}^{3} \left(\frac{\alpha_i\alpha_j + \alpha_j\alpha_i}{2}\right) \frac{\partial^2 \psi_\epsilon}{\partial x^i \partial x^j} + \frac{\hbar mc^3}{i} \sum_{i=1}^{3} (\alpha_i\beta + \beta\alpha_i) \frac{\partial \psi_\epsilon}{\partial x^i} + \beta^2 m^2 c^4 \psi_\epsilon \tag{11}$$

(11) is consistent with (10), if

$$\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij} \tag{12}$$

$$\alpha_i\beta + \beta\alpha_i = 0 \tag{13}$$

$$\alpha_i^2 = \beta^2 = 1 \tag{14}$$

In (12) the $\delta_{ij} = 0$ if $i \neq j$ and $\delta_{ij} = 1$ if $i = j$. Multiply (13) by the matrix β. Using (14)

$$\alpha_i = -\beta\alpha_i\beta \tag{15}$$

Take the trace of each side of (15), obtaining

$$\sum_j \alpha_i^{jj} = -\sum_{j,k,\ell} \beta^{\ell j}\alpha_i^{jk}\beta^{k\ell} = -\sum_{j,k,\ell} \beta^{k\ell}\beta^{\ell j}\alpha_i^{jk} \tag{16}$$

Employing (14) gives

$$\sum_j \alpha_i^{jj} = -\sum_j \alpha_i^{jj} \tag{17}$$

Therefore the trace of α_i is zero. In a similar way it follows that the trace of β is zero. (14) implies that the eigenvalues of α and β are ± 1. Quantum theory requires α and β to be Hermitean. These properties require α and β to be even dimensional square anticommuting matrices. The 3 Pauli spin matrices.

$$\sigma_x = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \qquad \sigma_y = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} \qquad \sigma_z = \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} \tag{18}$$

are two dimensional matrices which meet these requirements. A fourth anticommuting matrix cannot be found. Condition (13) cannot be met. A 4×4 set of matrices, including β will meet all requirements for

$$\alpha_i = \begin{vmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{vmatrix} \qquad \beta = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \tag{19}$$

Each element in (19) is a 2×2 matrix. Later we will discuss the special case where $m = 0$ and β is not required. As already noted, a 2×2 formalism will satisfy requirements for $m = 0$.

Parity

Prior to 1956, it was believed that the laws of nature were preserved under the parity transformation

$$\bar{r}' = -\bar{r} \qquad t' = t \tag{20}$$

Either one or three reversals of coordinate directions gives the parity transformation. The one coordinate change of sign is equivalent to the statement that if a certain set of phenomena are observed in an experiment, the mirror image is also a possible experiment.

We will show that the 4 component Dirac equation does have parity conserving solutions.

Consider (7), with $\bar{r}' = -\bar{r}$, $\psi'(r', t)$

$$i\hbar \frac{\partial \psi'}{\partial t} = \frac{\hbar c}{i}\left(\alpha_x \frac{\partial \psi'}{\partial x'} + \alpha_y \frac{\partial \psi'}{\partial y'} + \alpha_z \frac{\partial \psi'}{\partial z'}\right) + \beta m c^2 \psi' \tag{21}$$

Let the parity operator be P. Let us assume

$$\psi'(\bar{r}, t) = P\psi(\bar{r}, t) = n_p U_p \psi(\bar{r}, t) \tag{22}$$

In (22), n_p is a phase factor, and U_p is a matrix. Substitute (22) into (21) factoring out the phase factor. This gives

$$i\hbar\, U_p \frac{\partial \psi}{\partial t} = -\frac{\hbar c}{i}\left(\alpha_x U_p \frac{\partial \psi}{\partial x} + \alpha_y U_p \frac{\partial \psi}{\partial y} + \alpha_z U_p \frac{\partial \psi}{\partial z}\right) + \beta m c^2 U_p \psi \tag{23}$$

Now multiply by U_p^{-1} to obtain

$$i\hbar U_p^{-1} U_p \frac{\partial \psi}{\partial t} = -\frac{\hbar c}{i}\left(U_p^{-1}\alpha_x U_p \frac{\partial \psi}{\partial x} + U_p^{-1}\alpha_y U_p \frac{\partial \psi}{\partial y} + U_p^{-1}\alpha_z U_p \frac{\partial \psi}{\partial z}\right) + U_p^{-1}\beta U_p mc^2\psi \quad (24)$$

(24) is identical with (7) if we assume

$$\begin{aligned} U_p^{-1}\beta U_p &= \beta \\ U_p^{-1}\alpha_x U_p &= -\alpha_x \\ U_p^{-1}\alpha_y U_p &= -\alpha_y \\ U_p^{-1}\alpha_z U_p &= -\alpha_z \end{aligned} \quad (25)$$

Comparing (25) with (12), (13), and (14) implies that

$$U_p = \beta \quad (26)$$

If we operate on $\psi(r,t)$ with P^2 we obtain

$$\psi(r,t) = P^2\psi(r,t) = n_p^2\beta^2\psi(r,t) = n_p^2\psi(r,t) \quad (27)$$

(27) implies that $n_p = \pm 1$

A particle with momentum $\bar{p}$ with spin in the + or $-z$ direction has 4 solutions, 2 with positive energy, and 2 with negative energy. The positive energy solutions are

$$\psi_a = \left(\frac{E+mc^2}{2mc^2}\right)^{1/2} e^{i(\bar{p}\cdot\bar{r}-Et)/\hbar} \left|\begin{matrix} 1 \\ 0 \\ \frac{p_z c}{E+mc^2} \\ \frac{(p_x+ip_y)c}{E+mc^2} \end{matrix}\right| \; ; \; \psi_b = \left(\frac{E+mc^2}{2mc^2}\right)^{1/2} e^{i(\bar{p}\cdot\bar{r}-Et)/\hbar} \left|\begin{matrix} 0 \\ 1 \\ \frac{(p_x-ip_y)c}{E+mc^2} \\ \frac{-p_z c}{E+mc^2} \end{matrix}\right| \quad (28)$$

The transformation $P\psi = \beta\psi$ simply changes P_z into $-P_z$, P_x into $-P_x$, P_y into $-P_y$ for $r' = -r$. This is equivalent to the statement that the parity operation gives a solution which is identical with the original one. However the parity operation involves the phase factor n_p, implying that the entire wavefunction may or may not have a change of sign. For an elementary particle, if the sign is not changed, the parity is said to be even, if the sign is changed, the parity is said to be odd. It is also important to note that a particle with a given spin can have momentum in any direction whatsoever.

The Hamiltonian for interaction of particles with other particles can be constructed in such a way that the parity of a given particle (+ or −) is conserved if the particle decays into other particles.

Two Component Solutions of the Dirac Equation

Suppose we have a particle of rest mass $m = 0$. Then (7) implies that there is no need for β, and as noted after (18), an $N = 2$ set of matrices is sufficient. Now $\alpha_i = \pm\, \sigma_i$ and there are two uncoupled Dirac equations

$$H\psi = -\bar{\sigma} \cdot \bar{p}\psi \tag{29}$$

$$H\phi = +\bar{\sigma} \cdot \bar{p}\phi \tag{30}$$

Suppose we have a particle with spin $+\frac{1}{2}\,\hbar$, with $\pm\, z$ direction the direction of spin, energy E and momentum $p = +p_z$

(29) gives for ψ

$$\psi = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \tag{31}$$

(30) gives for $p = p_z$

$$\phi = \begin{vmatrix} 1 \\ 0 \end{vmatrix} \tag{32}$$

Equation (29) describes a particle ($m = 0$) moving with the speed of light in the $+z$ direction, with spin vector in the $-z$ direction. Equation (30) describes a particle moving with the speed of light in the $+z$ direction, with spin vector in the $+z$ direction. Since the velocity is c, it is impossible for a moving observer to observe a change in the relative direction of momentum and spin. The parity operation clearly changes one of the 2 component equations into the other and therefore leads to different solutions, describing different particles.

We conclude, therefore, that the four component Dirac equation gives solutions which are physically the same under the parity transformation, but that the two component Dirac equation has different solutions representing different particles under the parity transformation. The connection between spin and momentum is called helicity. If the spin is parallel to the momentum, helicity is positive, if the spin is antiparallel to the momentum, the helicity is negative. +1 and −1 are eigenvalues of the helicity operator $\frac{\vec{\sigma}\cdot\vec{p}}{|p|}$.

Physics of Neutrinos and Antineutrinos

As we have noted, the four component Dirac equation gives positive energy solutions unchanged by the parity transformation. The two component theory for rest mass zero particles was proposed by H. Weyl[8] in 1929. This was criticized by W. Pauli and the scientific community on the grounds that parity conservation was believed to be valid for all physical processes. This acceptance of parity conservation continued until 1956 when T.D. Lee and C.N. Yang[9] concluded that it was not valid for weak interactions as a result of the following analysis. K mesons were observed to have two different kinds of decay. It was therefore believed that the two decays were due to two different particles which were called the θ and τ mesons.

$$\theta^{\pm} \rightarrow \pi^{\pm} + \pi^{0} \tag{33}$$

$$\tau^{\pm} \rightarrow \pi^{\pm} + \pi^{+} + \pi^{-} \tag{34}$$

In (33) and (34) the superscripts give the charges of the particles. The π meson decay products are known to have odd parity. This implies that the parity of θ is even while the parity of τ is odd. These are weak interaction processes. Continued study of the θ and τ mesons indicated that their masses and mean lives are identical within limits of experimental error. Lee and Yang proposed that θ and τ are the same particle which can decay into odd or even parity systems. This led them to conclude that parity is not conserved in weak interaction processes.

Figure 2 indicates a way of testing the parity nonconservation hypothesis. Suppose parity is conserved. Suppose the cylinder shown in Figure 2 has radioactive nuclei having magnetic moments and spin. Suppose a magnetic field is applied, in the z direction, with the system at low temperatures. A significant nuclear spin polarization occurs. Let us imagine that the spin polarization in the z direction is a consequence of nuclei rotating in the direction shown by the approximately horizontal arrows. The z direction vectors at each end are drawn with length proportional to the number of emitted β particles, with z components of momentum in directions shown. If the number of β particles with positive z momenta is equal to the number with negative z momenta, the mirror image is identical with the object and parity conservation is found valid. If the number of β particles with positive z momenta is not equal to the number with negative z momenta, as shown in Figure 3, the mirror image is different from the object and parity is not conserved.

Such an experiment was carried out by C.S. Wu, E. Ambler, R.W. Hayward, D.D. Hoppes and R.P. Hudson[10]. The radioactive element Co^{60} was introduced into a crystal $[2Ce(NO_3)_3 \cdot 3Mg(NO_3)_2 \cdot 24H_20]$ which was cooled by adiabatic demagnetization to 0.01^o Kelvin. The degree of nuclear polarization was measured by measuring the anisotropy of the γ radiation. A very large β decay asymmetry was observed consistent with Figure 3, indicating that the Yang Lee conclusion that parity is not conserved in weak interactions, was correct.

Non conservation of parity implies that neutrinos with different helicity are different particles. In ordinary β^- decay the spin and direction of motion of the emitted neutrinos are like the direction of rotation and translation of a right handed screw. These particles are called antineutrinos. (Eigenvalue of helicity operator is $+1$). The neutrinos emitted by the sun have spin and direction of motion corresponding to a left handed screw. These particles are called neutrinos. (Eigenvalue of helicity operator is -1)

In the study of μ and τ mesons it was discovered that the neutrinos associated with these mesons are different from the β decay neutrinos. It is now believed that there are three kinds of neutrinos and three kinds of antineutrinos which are called electron, muon, tau neutrinos and antineutrinos.

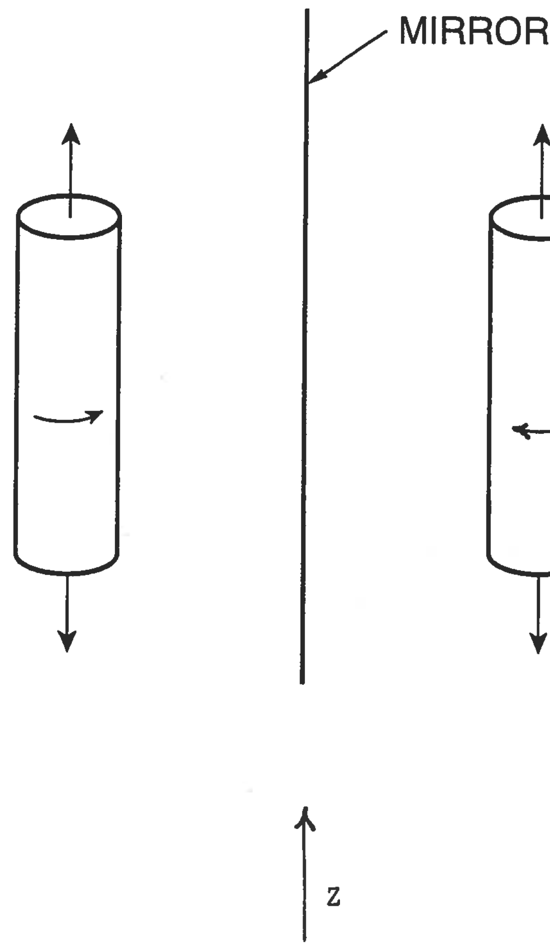

Figure 2. Parity Is Conserved

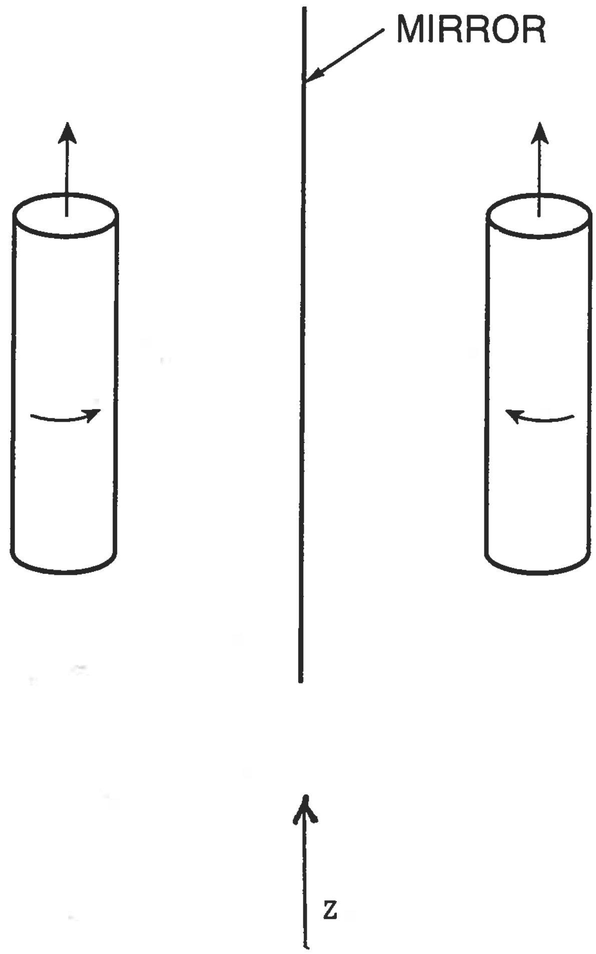

Figure 3. Parity Is Not Conserved

Conclusion

From 1929 through 1956, the scientific community believed that parity was conserved in all of physics. Theoretical research by Lee and Yang, confirmed by the experiments of Wu, Ambler, Hayward, Hoppes and Hudson, led to the discovery that parity is not conserved in the weak interactions.

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